teacher's resource book and guide

part IV

physics

SECOND EDITION

PHYSICAL SCIENCE
STUDY COMMITTEE

Physical
Science
Study
Committee

physics

SECOND EDITION

teacher's resource book and guide

part IV

D. C. HEATH AND COMPANY

Published by D. C. HEATH AND COMPANY

under arrangements with

EDUCATIONAL SERVICES INCORPORATED

Copyright © 1966 by Educational Services Incorporated. No part of the material covered by this copyright may be reproduced in any form without written permission of the publisher. Printed in the United States of America.

Printed in May 1966

INTRODUCTION TO PART IV

Part IV returns to a consideration of the atomic nature of matter, a topic which was introduced in the latter chapters of Part I, and carried further in Part III in the kinetic theory of gases. Now, however, with the material that has gone before, and the development of electrical forces in this part, we have the tools to look quantitatively at atomic structure. The wave kinematics from Part II and the particle mechanics from Part III are used to arrive at the modern wave-mechanical description of nature which is required for a detailed understanding of the world of the atom, a world in which "particles" (like electrons) often behave like waves, and waves (like light) frequently act like particles. The great concepts of mechanics, such as the laws of conservation of momentum and energy, still hold; but to explore the domain of the very small, we must enlarge our view. At the end of this volume we are in the realm of "modern" physics. We have arrived at the base on which current research in nuclear and solid-state physics is built.

The course ends with a detailed description of the hydrogen atom. Some may wonder why we do not exploit our new knowledge and explore more complex atoms and molecules, why we do not return to a description of crystal structure, investigate atomic nuclei, mesons, strange particles . . . the list could become quite long. It may be well to restate one of the intents of the course. Rather than trying to give the student a general acquaintance with the myriad facets of physics, the course was designed to explore deeply a few of the most fundamental ideas of physics. Students will not learn "all about atoms" in Part IV. They should, however, have a good understanding of the fundamental reasons for believing that atoms exist, and an understanding of why the mere existence of atoms demands that the laws of physics be recast to include wave properties of particles and particle properties of waves. If they really understand this much, they will have gone far enough. Thorough exploration of more recent developments in physics requires considerable mathematical sophistication. On many points physicists are not really sure — not sure enough to be able to express their ideas simply.

Before beginning the study of atoms, we must first explore quantitatively the subject of electricity, since electric forces are the glue which binds together the components of atoms and brings about the coalescence of large groups of individual atoms in matter as we commonly see it. Accordingly, the first chapters of Part IV deal with electricity and magnetism, but of a very special kind. We are concerned with the atomistic nature of electrical forces from the outset. The early chapters of Part IV have a dual role: exploring the electrical constituents of matter (hence developing ideas of the structure of atoms) and presenting a quantitative statement of the laws of electrical forces.

The approach to electricity is fundamental. While this approach provides a powerful base for the analysis of electrical devices, electrical technology is not developed as a topic. In directing our attention to the properties of atoms, a study of complicated electrical circuits like those involved in radio and television is a digression. Even such an old standby as Ohm's law is considered in perspective, simply as a useful relation which holds under special circumstances for some special types of circuits, certainly not as *the law* of electricity.

SUMMARY OF PART IV

The volume divides naturally into two halves. The first half, Chapters 26–31, develops static and magnetic electrical forces. Chapters 26–29 deal with the electrical interactions between elementary charged particles, ending with the concept of an electric circuit. Chapters 30 and 31 are concerned with the magnetic fields of currents and with electromagnetic induction, culminating in a description of electromagnetic waves. Chapters 32-34 explore the world of the atom, probing into the structure of atoms and the question of the quantum nature of light. The content of the individual chapters is summarized below.

Chapter 26, Some Qualitative Facts About Electricity, lays a foundation for a particle view of electricity that is basic to an understanding of the atom. The chapter begins with electrostatics but quickly goes beyond electrostatics by getting into the movement of charges during charging by contact and by induction, and in weak currents. The electroscope and electrometer are introduced as measuring instruments. The picture of electricity as subatomic particles is filled out through an investigation of the transport of charge in different media: gases, solutions, and metals.

In Chapter 27, Coulomb's Law and the Elementary Electric Charge, electrostatic forces are given quantitative meaning through the experiments which lead to Coulomb's law. $F = kQq/r^2$. These relationships make it possible to give a general description of the pattern of the electrostatic field surrounding simple arrangements of charges. The atomistic nature of electrical charge is disclosed through a quantitative discussion of a variant of Millikan's famous oil drop experiment. The force constant in Coulomb's law is established through scaling up the Millikan-type experiment. Finally, we note that charge is conserved in the universe, and that all "elementary" particles known have a charge $+1$, -1, or 0 electrical charges.

Chapter 28, Energy and Motion of Charges in Electric Fields, begins by extending the results and apparatus of the Millikan experiment of the previous chapter to a determination of the energy of an electric particle accelerated between charged parallel plates. A

574110

time-of-flight experiment is described for measuring the masses of the proton and electron. Currents and ionic charges are discussed in preparation for the introduction of batteries as sources of energy. The electromotive force, or EMF, of a battery is defined. The general idea of a potential difference and the potential in the field of a point charge is developed. Finally volts and amperes are introduced as units.

Chapter 29, Electric Circuits, is primarily concerned with the analysis of simple circuits. In this analysis the ideas of electrical energy are used extensively. Ohm's law is introduced as a special, though important, case describing the relation between voltage across a circuit element and the current in that element.

In Chapter 30, The Magnetic Field, magnetism is explored quantitatively, starting with the familiar compass needle. Magnetic fields associated with magnets and currents, and the vector nature of a field are examined in text and laboratory. The force on currents in magnetic fields is investigated, applying the results to a consideration of meters and motors and to the motion of a charged body in a magnetic field. This leads to a description of a mass spectrographic method for determining the masses of atomic particles. We then investigate the magnetic field near a long, straight wire, define the "circulation" of B around the wire, and apply this concept to the field inside a long solenoid. An experiment in the laboratory then allows us to redefine the units for magnetic field in terms of fundamental quantities.

Chapter 31, Electromagnetic Induction and Electromagnetic Waves, completes the treatment of classical electricity and magnetism. Here we note that a potential difference will be generated across the ends of a wire which is moved through a magnetic field, derive the quantitative relation between the parameters involved, and note the conservation of energy in the induction process. A general statement of the induction law is derived in terms of the rate of change of flux. We note that the direction of induced fields is always such that energy is conserved, as stated by Lenz's law. Finally, noting the symmetry between changes in electric fields and changes in magnetic fields, we show that energy can be propagated by electromagnetic waves, and discuss the general characteristics of the electromagnetic spectrum. At this point, the course has begun to bind together optics, dynamics, electricity, and waves in an integrated picture.

Chapter 32, Exploring the Atom, begins a study of the structure of the atom. Using high-energy alpha-particles in scattering experiments, we probe the structure of gold atoms, arriving at the Rutherford model of the atom. The trajectories of charged particles scattered by a Coulomb field are calculated and compared with the observed angular distributions of the alpha particles. We find that Coulomb's law holds to distances as small as 10^{-14} meter. Scattering experiments with other elements disclose the number of elementary charges contained in various nuclei and the approximate size of the nucleus. Finally, we note some fundamental difficulties with the Rutherford model of the atom, since observations of atomic spectra and atomic stability do not fit the model according to the classical electromagnetic theory.

Chapter 33, Photons and Matter Waves, introduces evidence that light and matter show both wave and particle properties. Evidence for the "grainy" nature of light comes first from Millikan's experiment; then Taylor's experiment suggests that interference patterns may result from the cumulative effect of many tiny impulses of light. The photoelectric effect is investigated in detail and Einstin's analysis of this phenomenon is discussed. We conclude that a photon of frequency ν has an energy $h\nu$ and a momentum $h\nu/c$. The momentum relation is verified by study of the Compton effect. We next note the "wavy" character of moving matter, as evidenced by electron diffraction experiments, and conclude that a particle with a momentum p has a wavelength $= h/p$, according to the de Broglie relation. We then look for the situations in which the wave aspects will be important, finding for example that wave behavior should be important for electrons inside of atoms.

Chapter 34, Quantum Systems and the Structure of Atoms, applies the discussion of matter waves specifically to the structure of the atom. Analysis of the Franck-Hertz experiment shows the existence of discrete energy levels in atoms. The existence of internal energy states is further confirmed by analyzing emission and absorption spectra. The hydrogen spectrum is examined in some detail. Finding that energy levels can be pictured in terms of standing wave patterns brings us to a point where both wave and particle characteristics are essential to a description of the structure of a simple atom.

RELATED MATERIAL FOR PART IV

LABORATORY. For the first four chapters of Part IV, laboratory experiments play the same fundamental role as in the development of the preceding three parts of the course. For the last four chapters, the laboratory plays a less prominent part because significant laboratory experiments are quite complex. The Lab Notes contain suggestions on handling the laboratory experiments.

DEMONSTRATIONS. Demonstrations should play a greater part in the classroom work of Part IV than in any part of the previous work. Suggestions are included in the discussions of the individual sections throughout this Guide.

FILMS. The PSSC films intended for use with Part IV are listed below with the chapters to which they relate. They show a number of essential experiments most of which are too difficult to perform in school. Individual

films are discussed at more length at appropriate points in the Guide.

Chapter 27, Coulomb's Law and the Elementary Electric Charge.
Coulomb's Law; 30 min.
Millikan Experiment; 30 min.
Coulomb's Force Constant; 35 min.
Electric Fields; 25 min.
Electric Lines of Force (lab film); 7 min.

Chapter 28, Energy and Motion of Charges in Electric Fields
Counting Electrical Charges in Motion; 23 min.
Elementary Charges and Transfer of Kinetic Energy; 33 min.
EMF; 20 min.
Electrical Potential Energy and Potential Difference; Parts I and II; 54 min.

Chapter 30, The Magnetic Field.
A Magnet Laboratory; 22 min.
Electrons in a Uniform Magnetic Field; 11 min.
Mass of the Electron; 18 min.

Chapter 31, Electromagnetic Induction and Electromagnetic Waves
Electromagnetic Waves; 33 min.

Chapter 32, Exploring the Atom.
Rutherford Atom; 40 min.

Chapter 33, Photons and Matter Waves
Photons; 18 min.
Interference of Photons; 13 min.
Photo-electric Effect; 28 min.
Matter Waves; 28 min.

Chapter 34, Quantum Systems and the Structure of Atoms.
Franck-Hertz Experiment; 30 min.

Science Study Series. Currently available titles which are related to Part IV are included in the end-of-chapter "Further Readings."

SCHEDULING PART IV

Many teachers are likely to have more difficulty scheduling Part IV than other parts of the course. This will usually be due to the fact that this is the last part of the course, and the semester will be running out. All the delays have accumulated, and now many will be faced with a fascinating bit of subject matter and a limited amount of time. The following table suggests possible 10- and 15-week schedules. Suggestions for possible variations of these schedules follow the table.

CHAPTERS	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
	CLASS PERIODS	LAB PERIODS	EXPERIMENTS	CLASS PERIODS	LAB PERIODS	EXPERIMENTS
26	2	1	IV-2	5	1	IV-1, IV-2
27	4	3	IV-3, IV-5 IV-6	6	4	IV-3, IV-4 IV-5, IV-6
28	4	0	—	5	1	IV-7
29	3	1	IV-9	5	2	IV-8, IV-9
30	4	3	IV-10, IV-11 IV-12	8	4	IV-10, IV-11 IV-12, IV-13
31	5	0	—	8	0	—
32	5	0	—	8	0	—
33	5	1	IV-14	8	1	IV-14
34	6	1	IV-15	8	1	IV-15

Part IV has been written so as to permit a certain amount of flexibility in scheduling. Historically, and practically, the mass measurements on atomic particles have been made with a combination of electric and magnetic fields (Sections 30–7). However, the text introduces time-of-flight measurements of mass (Section 28–1) in order to investigate these fundamental atomic quantities without needing to study the magnetic field. This, then, makes it possible to delete all of Chapters 30 and 31, although in this case Section 31–9 should be assigned as reading.

Another part of the course which can be skipped if necessary is Chapter 29. This deals with circuits, and its deletion will not seriously interfere with the understanding of atoms. If you choose to delete *Chapter 29* and *study Chapters 30 and 31* you must pick problems in these latter chapters with care, since many of them involve circuit concepts.

There are certain sections scattered through Part IV which can be skipped or given minimum treatment to save time without serious loss of continuity. These are indicated by bracketing in the individual chapter schedules.

In summary, the major time saving may be accomplished by skipping immediately from the end of Chapter 28 to Section 31–9. This, plus skipping a few other bracketed sections can cut the time required for Part IV to a minimum of about 6 weeks.

If schedule disaster has overtaken you, it might be possible to get by with a good class by: a) Reading Chapter 26; b) Studying 27–1, 2; c) Showing the Millikan Oil Drop film; d) Studying 27–7, 28–1, 28–6, 31–9; e) Going on to Chapters 32, 33, and 34. We hope you don't need this drastic advice!

chapter
26

Some Qualitative Facts About Electricity

This chapter divides naturally into two halves: 1) an introduction to electrical phenomena which shows that there are two kinds of charge, and how these charges can be produced and detected; and 2) a qualitative investigation of the charge carriers in gases, solutions, and metals, leading to the conclusion that charge is carried in nature by subatomic electrical particles.

Chapter 26 is an *introductory* chapter. Most of the topics in the chapter could be expanded considerably. Indeed, in following chapters, many of them are. Thus, while it is important to ensure that students understand the chapter, it will be well to stay at the level presented and avoid extensions in depth and quantification. If you have time for extensions, it can be used more profitably on the work that follows.

The chapter lays a foundation for a particle view of electricity that is basic to an understanding of the atom. While the chapter begins with electrostatics, it quickly goes beyond classical electrostatics by getting into the movement of charges. Thus, electric currents are introduced early. Current is developed as the motion of electrical particles rather than the "fluid" of Benjamin Franklin's day or even the later ideas of Faraday and Maxwell.

The student should begin to see electricity as a subatomic phenomenon — the ions as a kind of atomic fragment, and the electron as one of the building blocks of matter. He should see that electrons are common to all kinds of substances, and that electrons are all alike in that the chemical composition of a substance does not change when electrons are transferred from one substance to another (as in thermionic emission).

CHAPTER SUMMARY

SECTIONS 1 THROUGH 7. These sections deal with charge separation, the forces between charges, and the detection of charges.

Objects may be electrically charged by rubbing them together. There is a force of repulsion between like charges, and attraction between unlike charges. A model is proposed which pictures electrically neutral matter as containing equal numbers of positive and negative charges, and charged bodies as containing an excess of one sign of charge.

Charges move freely in some bodies (conductors) but not in others (insulators). A simple electroscope is introduced as a charge detector. Charges can be separated through electrostatic induction. Through induction, a force of attraction develops between a charged body and a neutral body, whether it be a conductor or an insulator. Electroscopes can be shielded, made more sensitive, and calibrated. The battery is introduced as a chemical mechanism for separating charges.

SECTIONS 8 THROUGH 13. The remainder of the chapter fills out the picture of electricity as subatomic particles through a description of the transport of charge in different media: gases, solutions and metals.

Gases, which are ordinarily good insulators, may be made conducting by exposure to ionizing radiation. This leads to the conclusion that while a molecule is neutral, its parts are charged (or can be). A cloud chamber is described briefly as a device that permits us to "see" the paths of charged particles. Some solutions do not need an outside ionizing agent to become conductors — ions seem to be formed by a spontaneous breakup of the molecules of the dissolved substance.

Conduction in metals is another case of charge transport. In conducting metals, the charge carriers must be free to move inside the metal, but since they don't ordinarily fly away from the conducting material, we get a picture of the charge carriers in metals as a sort of contained electron gas. Through thermionic emission we see which particles are in motion and can be boiled out.

Knowledge of thermionic emission and the forces between like and unlike charges enables us to introduce the essentials of the vacuum tube and cathode-ray oscilloscope. The principles of these devices will be used in later chapters.

COMMENT

One of the reasons that the movement of both positive and negative charge carriers is introduced in this chapter is to facilitate seeing electric current as not limited to the motion of electrons. This general approach to electric current — the transport of charge in matter, not just in metals — contributes to visualizing and understanding definitions that come later [Chapter 28, Section 6, where electric field is defined as force (and electric potential difference is defined as work done) per unit *positive* charge]. In the world of gases, liquids and semi-conductors, as well as conducting metals, this is the way things are. Both positive and negative charges move in gases and solutions, the negative charges move in conducting metals, and the positive holes move in some semi-conductors.

SCHEDULING CHAPTER 26

SECTIONS 1 THROUGH 6 make a reasonable single reading assignment. Such an assignment might well be preceded by performing Experiments IV–1 and IV–2. Both experiments can be handled in a single laboratory period. Class discussion can be developed around these experiments supplemented by a few demonstrations suggested below. Some HDL problems should be assigned in the course of work on these sections. Class discussion of a few problems such as 12, 15, or 22 should be helpful. Depending on how much lab work is done by students and how many supplementary demonstrations are performed, from 2–3 periods will be needed to cover these sections.

SECTIONS 7 THROUGH 10, together with a few more problems on the earlier work, might constitute a second assignment for this chapter. Class discussion of this assignment might include demonstrations such as the discharge of an electroscope through a high resistance, the transfer of charge by an ionized gas, and tracks in a cloud chamber.

SECTIONS 11 THROUGH 13 comprise another possible reading assignment. A demonstration of the conductivity of liquids might be worked into a discussion of these sections.

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the volume summary section. Sections which can be deemphasized or abbreviated without loss of continuity are enclosed with brackets [ ].

RELATED MATERIALS FOR CHAPTER 26

LABORATORY. Experiment IV–1, *Electrified Objects*, uses plastic strips rubbed with wool and cotton to investigate qualitatively the characteristics of static charges. It may be done at home, or if your students have had other courses which familiarized them with the behavior of charged objects, the experiment may be done as a demonstration. Assigning this experiment to be done at home is probably the simplest way to get it done at the optimum time — before getting into Chapter 26.

Experiment IV–2, *Electrostatic Induction*, familiarizes students with charge separation and conduction resulting from induction. Desirably, this experiment should be scheduled with Section 5.

See the HDL Notes for suggestions on these experiments. The discussion of the experiments includes suggestions on handling static electricity experiments under conditions of high humidity.

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

DEMONSTRATIONS. If your schedule allows time, good demonstrations of electrostatic phenomena are desirable for this chapter. The presentation and discussion of demonstrations on the work of this chapter should focus on the qualitative nature of the effects observed — the existence and movement of charges and the forces between charges. Quantitative work will come later.

Suggestions for demonstrations are given in the following Section discussions. The demonstrations with the electroscope and the cloud chamber are particularly recommended. Construction notes are included where appropriate.

Chapter 26	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTIONS	CLASS PERIODS	LAB PERIODS	EXPERIMENT	CLASS PERIODS	LAB PERIODS	EXPERIMENTS
1, 2, 3, 4, 5, 6,	1	1	IV–2	3	1	IV–1 IV–2
7, 8, 9, [10]	½	—	—	1	—	—
11, 12, 13	½	—	—	1	—	—

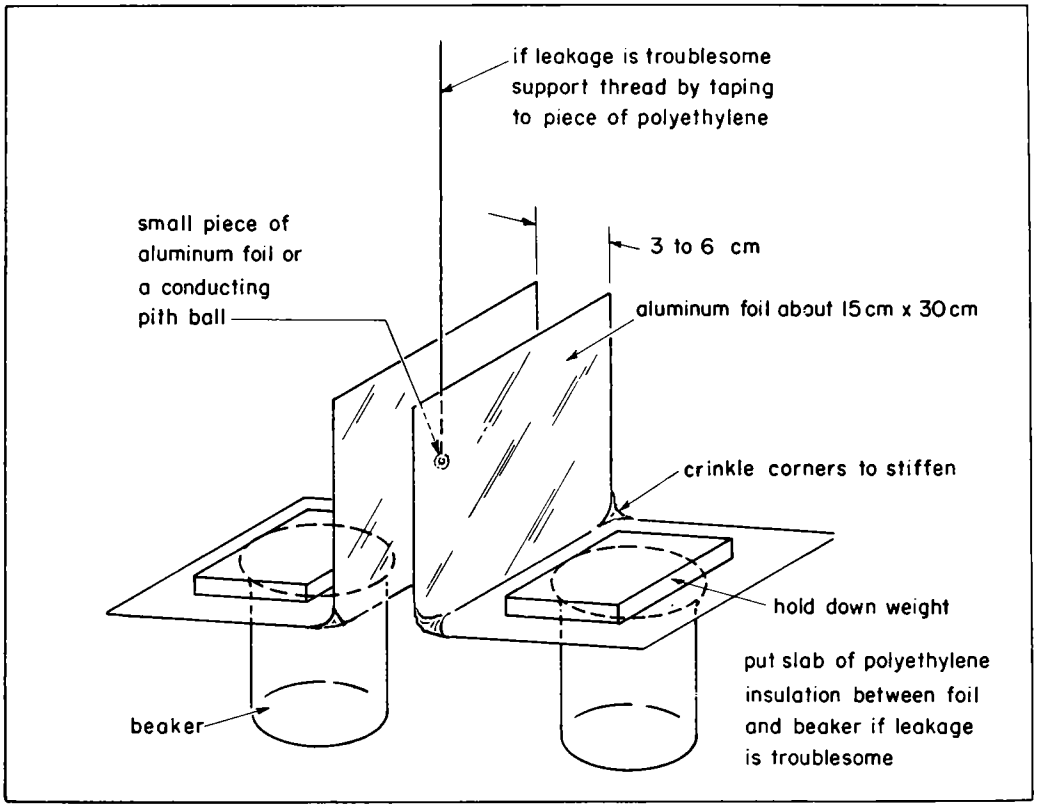


Figure (a)

26-1 Attraction and Repulsion Between Electrified Objects

26-2 Electric Forces Between the Building Blocks of Matter

PURPOSE. 1. To show that there are two kinds of electric charge.

2. To study qualitatively the forces that act between charged bodies.

3. To show that the forces between charged bodies can be understood in terms of charged particles in atoms.

EMPHASIS. See suggestions on scheduling. Sections 1–6 make a reasonable single reading assignment which can be developed in 2–3 days of laboratory and demonstration and discussion, together with some problem solving.

DEVELOPMENT. The ideas of these first two sections are entirely qualitative. They rest upon the development of a simple model from straightforward observations. Many teachers have found that a session which is a blend of laboratory work and discussion is an effective way to handle this kind of material. If you have time for Experiment IV–1, do it before starting this section.

DEMONSTRATION: *Charge Transfer by an Oscillating Pith Ball.* This demonstration can be done after Experiment IV–1 to illustrate the transfer of charge by a conductor. Suspend a conducting pith ball or a small square of aluminum foil midway between two aluminum foil plates as shown in Fig. (a). Give the plates opposite charges by using charged plastic strips. Allow the pith ball to touch one of the plates. It will become charged by conduction, be repelled, and fly across to touch the opposite plate. There it will lose its charge, become oppositely charged, and again be repelled. It will oscillate back and forth until the plates are nearly neutral.

If the humidity is high, the plates can be charged from a 500 micro-microfarad, 20,000 or 30,000-volt capacitor that has been charged from a Ford ignition coil as described in Experiment IV–2. Hold the charged capacitor by its insulating case, briefly touching its terminals simultaneously to the plates.

COMMENT. The introductory paragraphs before Section 1 introduce the idea that electric forces are of great magnitude. This will be important later to an understanding of atomic structure. You may not want to emphasize the relatively huge magnitude of the electrical forces at this point, but you should do it sometime during the first two chapters. The first paragraph states that "Sometimes other forces appear which are enormously greater" (than gravitational forces). In our everyday (macroscopic) experience, we only occasionally notice examples where electrical forces are greater than gravitational ones. The reason

for this is simply that gravitational mass is the "charge" associated with gravitational force and, here on the earth, we are very near a huge "gravitational charge," the earth itself. Thus the forces between ourselves and the earth are large. However, the gravitational attraction between ordinary-sized objects is trivial compared to the electric forces that can be developed between them. Even further, on a microscopic scale, consider the forces between subatomic particles. If all the positive and negative charges in a pinhead could be shoved to opposite sides of the pinhead, the force of attraction between them would be about 10^{17} tons — truly enormous. The nearly exact equality of positive and negative charges in most matter makes these potentially tremendous forces seem quite small. Quantitative statements must be left for later, but the electrical forces must not be treated as if they were weak and relatively unimportant.

* * *

The text uses the traditional glass rod rubbed with silk to get a positive charge. Plastic rods are now easier to obtain than the traditional hard rubber; so the text uses them, rubbed with a piece of fur, as a source of negative charge.

* * *

Some student might ask why like charges don't attract and unlike ones repel. The only answer is that they don't. With gravitational forces, however, all masses attract. While no one has ever seen a type of mass that repels, we sometimes speculate about the possibility of its existence. Presumably such "unlike" masses would be repelled by mass as we know it. Another possibility might be a kind of inconsistency, for example, *A* repels *B*; *A* repels *C*; but *B* attracts *C*. It happens that nature is kind in this regard. All this is just to emphasize that it is an *experimental fact* that all charges can be separated into two classes called positive and negative. There is no logic that says things *have* to be that way; they just are.

26-3 Insulators and Conductors

PURPOSE. To introduce the idea that electric charge flows readily in some materials, not so readily in others.

CONTENT. a. A metal and a glass rod differ in the degree to which electric charge can flow through them.

b. The difference can be ascribed to the fact that in metal, charges are free to move, and that in glass they are not.

c. Metal and similar materials are called conductors; glass and materials like it are called insulators.

DEVELOPMENT. In the course of laboratory work, or demonstration, the experiment shown in Fig. 26-2 of the text can be performed and analyzed. Other materials such as a paper tube, or a wood dowel can be tried.

COMMENT. The students have all they need from this section when they realize that charged particles are free to move in some materials, while in other materials they are not free to move.

* * *

(You will not want to go into the following in detail, but it may be worth mentioning in passing.) Like most categorical definitions, the classification of all materials into either conductors or nonconductors is an oversimplification. Metals are better conductors than anything else by many orders of magnitude. Some materials like polystyrene are such good insulators that they will hold a charge placed on them for months.

Some materials, such as germanium and silicon (from which transistors are made), and copper oxide are much better conductors than plastics but much poorer than metals. They are often called semiconductors. Salt solutions are poor conductors, but many orders of magnitude better than plastics. Gases are sometimes nearly perfect insulators, and sometimes nearly as good conductors as metals, depending on the degree of ionization.

Conductivities in amperes per volt-meter:

Silver	61.4×10^6	metals
Steel	5×10^6	metals
Carbon	2.8×10^4	a case by itself
CuO	10^{-3}	semiconductor
Bakelite	10^{-10}	plastics.
Glass	10^{-12}	plastics.
Rubber	10^{-16}	plastics.
Polystyrene	10^{-18}	plastics.

* * *

The mechanisms for conduction are quite complicated. We believe that electrons move about in metals, but a quantitative treatment of just how easily they move is difficult. Semiconductors are even more complicated. Ionized solutions and gases are treated later. In general, class discussion of these mechanisms should not be undertaken at this time. The principal mechanisms are treated briefly in the last sections of this chapter (Sections 9, 11, 12).

26-4 Some Experiments With an Electroscope

26-5 Electrostatic Induction

26-6 Improved Electroscopes and Electrometers

PURPOSE. 1. To introduce the electroscope as an instrument which can be used to detect the presence and sign of an electric charge.

2. To introduce the idea that electric charge can be moved by induction as well as by direct contact.

3. To show that induction phenomena can be understood in terms of the simple model of charged particles within atoms.

4. To show how the electroscope can be refined to make it usable for accurate quantitative measurements.

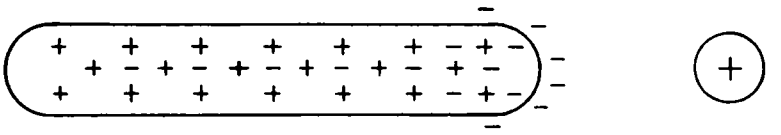
EMPHASIS. The important new phenomenon to be understood in these sections is that of electrostatic induction. The techniques of electroscopes are quite simple and involve principles which have already been presented.

DEVELOPMENT. In addition to performing Laboratory Experiment IV–2, one of the best ways to handle this material is to demonstrate various instances of charging by conduction and induction, and ask the students for a step-by-step analysis of the effects in terms of changes in the distribution of charges and the consequent forces. Another alternative is class discussion of such problems as numbers 12, 13, 14, 15, and 16.

COMMENT. The text points out that the attractive force between a charged body and a neutral conductor is somewhat larger than the force between a charged body and a neutral insulator. This is not important enough to warrant full class discussion, but in case it stimulates questions, here is the picture.

The difference is not large, and for certain insulators may be almost undetectable. How this can be when the charges are perfectly free to move in a conductor and completely unable to move any large distance in an insulator may seem paradoxical.

Without excessive mathematical detail, only a qualitative answer to this paradox can be given. Consider a long conductor near a positively charged body as at (a). The positive charge attracts negative charge toward the near end as shown. Even though the charges are perfectly free to move on the conductor, the negative charge that can build up on the end of the conductor nearest the positive charge is limited because the negative charges repel one another and cannot completely bunch up at one end. In fact, the negative charge on the end of the rod can never build up to a magnitude greater than the attracting positive charge. The charge induced on a conductor, then, is limited by other factors than how easily it can move in the conductor.



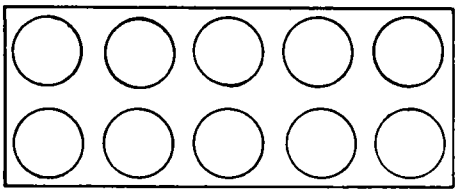
(a)

Consider the insulator at (b), made up of neutral atoms (rather large!). When a positive charge is placed near it as at (c), the negative charges in all the atoms are displaced ever so slightly toward the positive charge.

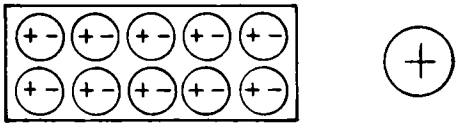
The net result is a neutral region in the center and a net negative charge at the end closest to the external positive charge. There is a net positive charge at the opposite end. An easy way to see the effective charging of the two ends of the bar is to regroup the adjacent + and − charge pairs, omitting those closest to the ends, as in (d). Now we see that one of the end surfaces is predominantly positive, the other negative. In between, all charges are neutrally paired. Thus the "dielectric polarization" of the entire rod is equivalent to a migration of charges from one end to the other, as occurs in the case of a conductor.

* * *

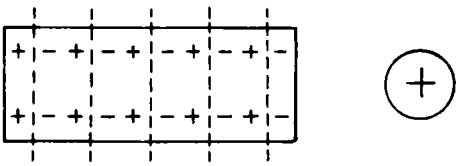
The metal case of an electroscope performs two principal functions which are somewhat related. The first is that of shielding. A change completely surrounded by a conducting case can have no forces exerted on it by charges outside the case. This shielding is perfect if the case completely surrounds the



(b)



(c)



(d)

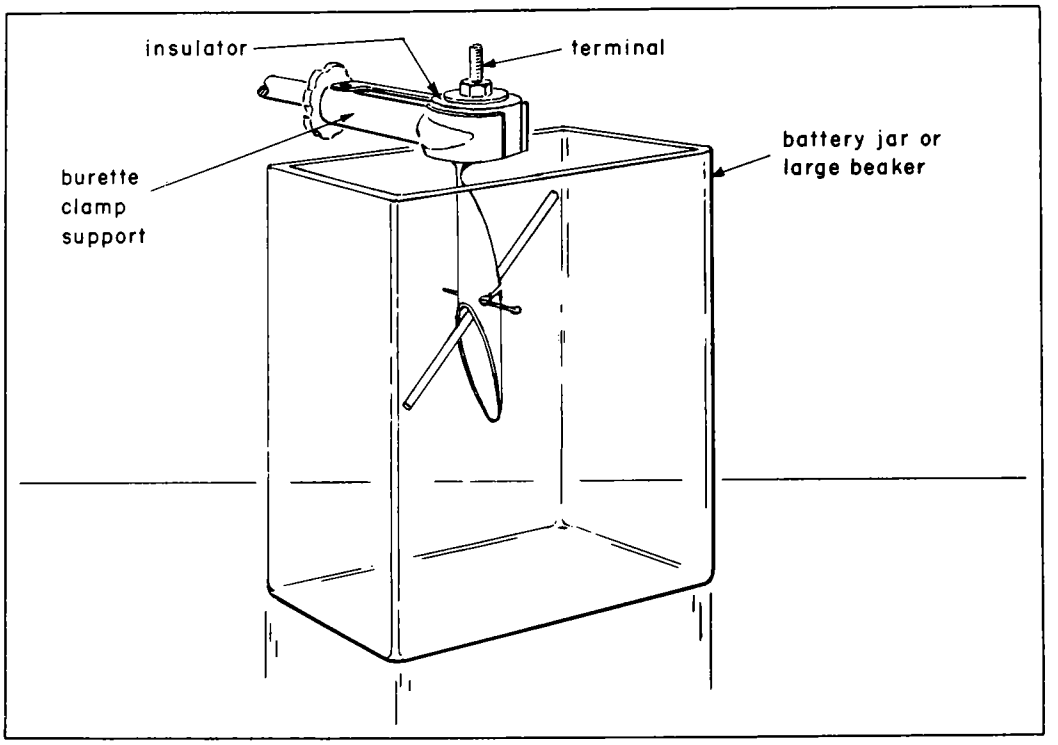


Figure (a)

charge, but it is very good even if rather large openings are left in the case. Even if charges are placed on the case the force on a charge inside remains zero.

The text points out that perfect shielding depends on the inverse r^2 law presented in the next chapter. A proof of this is not difficult, but if it is developed, might be more appropriate in connection with Chapter 27 than at this point. A proof is given in Appendix 1 of this volume of the Guide. A proof is also given in the PSSC film, "Coulomb's Law."

The second function of the metal case is to increase the reproducibility of measurements made with the electroscope. As long as charges from rubbed rods and combs are beind detected, no great gain is made with a metal case. But if charges are to be placed on the electroscope with a battery, then there must be something to which each end of the battery can be connected. In principle, one battery lead could be connected to the electroscope and the other to the earth, but then the reading would vary with the distance of the leaves from the earth. The metal case provides a constant reference point to which the other end of the battery can be connected, thus making the readings reproducible.

DEMONSTRATIONS WITH AN ELECTROSCOPE

A Shielded Electroscope

The effect of shielding may be shown by using two coffee-can electroscopes. Leave one of them intact and remove the soda straw and its mount from the other, supporting it with a ringstand and burette clamp in a battery jar as shown in Fig. (a).

Pass a charged plastic strip around the battery-jar electroscope, letting the class see that the soda straw moves whenever the strip is brought near any part of the glass case. Now pass the strip around the coffee-can electroscope to show that the soda straw moves only when the strip is near the terminal or directly in front of the electroscope.

Separation of Charge Using Tape

Two electroscopes, each with a horizontal metal plate attached to the insulated terminal, are needed for this demonstration.

Fasten a 1 to 2-inch piece of vinyl electrical tape to the plate of one electroscope. Hold the electroscope with one hand and peel the tape off without touching the plate. The electroscope will become charged. Without touching the bottom of the tape or the plate of the second electroscope, stick the tape to the plate of the second electroscope to show that the tape is also charged. If the two plates, with the tape still stuck to the second, are then touched together, both electroscopes will discharge.

This demonstration shows that charge separation depends on close contact rather than on rubbing against a force of friction. It also shows that whenever a given amount of charge of one sign is separated

from a body, an equal amount of charge of the opposite sign is left on the body.

Discharge of an Electroscope through a High Resistance

Students too often think of electrostatic phenomena as having little relation to the phenomena of current electricity. This demonstration will show that the charge from rubbed plastic can flow in a current similar to the current produced by a battery.

First, show that the rate of discharge from a charged electroscope is negligible. Then, with pinch clothespins, connect a 3 to 10-inch piece of magnetic recording tape between the insulated terminal of the electroscope and the case. When the electroscope is charged, a current will begin to flow through the tape, discharging the electroscope in 5 to 100 seconds, depending upon the initial charge, length of tape, and humidity conditions.

The Transfer of Charge by an Ionized Gas

Hold a burning candle or match near the terminal of a charged electroscope. The positive and negative ions produced by the chemical reactions taking place at high temperature in the flame will discharge the electroscope rapidly.

An intense radioactive source will also ionize the gas and discharge the electroscope. However, sources strong enough to discharge an electroscope rapidly are dangerous to handle. The luminous face of a clock, placed close to the open chamber of an electroscope, will discharge it in about 10 minutes. The glass face should be removed, since it stops alpha particles which produce a large fraction of the total ionization.

Deflecting a Stream of Water

A simple demonstration showing the effect of induced charges can be done by bringing a charged rod near a fine stream of water. An eyedropper tube, mounted in a one-hole rubber stopper and held in a burette clamp supported by a ringstand, can be used as a nozzle to give a fine stream from the faucet. When a charged rod is brought near the stream, the stream will be deflected and drops may tear away from it and circle the rod.

Other, more complex experiments involving the effect of charged objects on streams of water will be found in the book "Soap Bubbles" by C. V. Boys, in the Science Study Series.

26-7 Batteries

26-8 Electric Currents

PURPOSE. To introduce the battery as a device which will separate charges by chemical means. To point out that when the terminals of a battery are connected through an external circuit, the charge separation function of the battery results in an electric current, i.e., the flow of charge around the circuit.

EMPHASIS. See suggestions on scheduling. Very little classroom discussion is required, and little should be allowed. Extended discussion is almost sure to be a digression.

COMMENT. Since a battery is a very complicated device, it is probably not wise to attempt a detailed explanation of the mechanism by which it works. It should simply be explained as a device which transfers negative charge from its positive to its negative terminal. (Actually positive charges may also be moving within the battery.) The battery continues this transfer of charge until the repulsion between the accumulated charge on the terminals and any additional charge of the same sign, counteracts the chemical forces. Then the process stops. If some process, external to the battery, removes negative charge from one terminal and places it back on the other terminal, the battery will keep on transferring charge until it runs down.

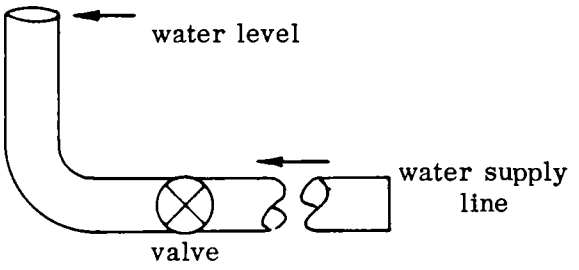
The students soould be brought to understand that the battery does not produce charge. It simply moves it. At no time does the battery accumulate a net charge. With batteries, the words "charge" and "discharge" are unfortunate since they seem to imply the storing of charge within a battery. What is stored is chemical energy which can be used to do work, i.e., move charge even against the electric forces tending to recombine positive and negative charges. *After the students have studied the next chapter* it might be well to emphasize this point with the following: A typical 100 ampere-hour automobile battery will transfer $100 \times 60 \times 60 = 360{,}000$ coulombs during one discharge. The force between two 360,000-coulomb charges placed a foot apart would be roughly 10^{18} tons. Surely that net "charge" was never in the battery. You wouldn't dare comb your hair near it, or put two "charged" batteries anywhere near each other!

* * *

Often students wish to know how fast charges flow. Each case is a complicated problem in itself, but a few order of magnitude statements can be made. The individual charges usually move very slowly; drifting along in a typical electrical circuit at a few centimeters per second or less (see Section 29-4C). On the other hand, patterns of charge rearrange themselves on conductors very rapidly, a good order of magnitude estimate being with the speed of light. If a charge were put on one end of a metal rod from, say, a piece of fur, that amount of charge would be distributed more or less evenly along the rod in the time it takes light to travel the length of the rod, sometimes somewhat longer (perhaps 100 times as long), never less. Now the individual charges from the fur do not travel along the metal with this speed. One of the original charges pushes a like charge in the metal which in turn

pushes its neighbor so that the *electrical effect* travels with a speed which approximates that of light.

One way of clarifying this point is to present the example of an open standpipe full of water, with a closed valve at its bottom end, as shown.



When the valve is opened, water begins to overflow the top of the pipe. From the rate of overflow and size of pipe, we can calculate the speed of flow in the pipe:

$$\frac{\text{volume}}{\text{second}} = (\text{area of pipe}) \times (\text{speed of flow}).$$

We may now ask the question, "How long a time lapse is there between the instant that the faucet is opened and the instant when water overflows the pipe?" If the standpipe was initially empty, the answer is simple:

$$\text{time lapse} = \frac{\text{length of pipe}}{\text{speed of flow}}.$$

A different answer is obtained if the pipe was initially full. There is a time lag between opening the valve and the beginning of the overflow. The lapsed time is the time required for the pressure wave to travel from the faucet to the open end. This time is determined essentially by the length of the pipe and the speed of sound through water.

In the case of the flow of electrical charges through conductors, the conductors are already "full of charges" (free electrons). Thus when new charges are fed in one end, other charges flow out the other end almost immediately. The lag is determined by the speed of propagation of an electrical impulse through the metal. This is of the order of the speed of light. No single charge moves nearly that fast, a typical "drift speed" being about 0.02 cm sec.

26-9 Conductivity of Gases; Ionization

26-10 The Cloud Chamber

26-11 Conductivity of Solutions

26-12 Electrons in Metals

26-13 Diodes, Electron Guns, Cathode-Ray Oscilloscopes

PURPOSE. To discuss briefly the mechanisms of charge transport (electric current) in gases, liquids, and solids and to present briefly the evidence for these mechanisms. The aim in presenting these sections is not to provide an encyclopedic coverage of electrical conduction but to make it clear that there are many ways in which electric charge can move in addition to conduction in metals, that there are various charge carriers and conducting mechanisms, and to further familiarize students with the atomistic picture of electricity.

EMPHASIS. See suggestions on scheduling. These sections will stimulate as much discussion as you wish to devote to them, but keep an eye on your time schedule. Limiting your development to the material presented in the text will provide enough depth to prepare students for the material which is to come. Probably the most important material for future reference is the evidence presented in Section 12 for charge transport in metals by relatively free electrons.

CONTENT AND COMMENT. The principal function of Sections 9, 11, and 12 is to back the earlier assertion that charge can be pictured in terms of subatomic particles. Sections 10 and 13 introduce some of the devices that will be employed later.

In Section 9 the transport of charge in gases is discussed. This transport occurs only when the gas is *ionized* and this means that there are atoms or parts of molecules some of which are positive and some negative (these are the ions). Atomic structure will, of course, be much more thoroughly developed in later chapters.

Section 10 is, in a sense, a footnote which indicates how the paths of charged particles in gases can be "seen". The mechanism of a cloud chamber is not particularly important. The section is probably best handled as a kind of commentary accompanying a demonstration. Your laboratory equipment should include a simple cloud chamber. Use it for demonstration in connection with this section.

Section 11 develops the idea that the carriers of current in liquids are also ions. It is pointed out that metals may be electroplated out of solution, verifying this idea. A background in chemistry would be helpful to the teacher in carrying on a class discussion about this material. We can hear the question: Why doesn't sodium plate out of sodium chloride solution the way that copper plates out of copper sulphate solution? (A first suitable answer: Sodium metal reacts with the water as fast as it is formed to make hydrogen gas and sodium hydroxide, but copper metal doesn't react with the water.)

Section 12 develops a picture of conduction in metals as due to charged particles which are free to move internally. Thermionic emission shows that it

is the negative particles that are free to move. The section introduces the idea that the negative particles, the electrons, are the same in all materials since the transfer of electrons from one metal to another does not alter the chemical composition of either metal.

The devices introduced in Section 13 do not require extensive development, but should be understood at the level presented since they provide some of the basic mechanisms used in many of the experiments which are analyzed in later chapters.

DEMONSTRATION FOR SECTION 11: *The Conductivity of Liquids.* This series of demonstrations, designed to accompany Section 26–11, shows that some liquids contain ions and are conductors while others are insulators.

Most of these demonstrations utilize the apparatus shown in Fig. (a) connected to a 110-volt A.C. line. To reduce the hazard of electric shock, the carbon rods can be fitted into a pair of holes drilled in a block of wood resting across the top of the container. The rods will stay in place if tape is wrapped around them close to their tops. If small arc-light carbons are used, connections to them can be made with battery clips. The carbon electrodes from No. 6 dry cells are more convenient, since they have screw terminals on their tops. However, dry-cell carbons should be thoroughly cleaned before use.

A clean container, pure water, and clean electrodes must be used for each demonstration. If your tap water contains too high a concentration of dissolved salts, use distilled water.

Fill the cell with water. The lamp will not light. Show that the circuit is in operating order by shorting the cell with an *insulated* screw driver. (It should be pointed out to the students that most water contains some dissolved salts and therefore cannot be trusted as an insulator.) Add some sugar to the water and stir with a glass rod. The lamp will not light, since sugar does not form ions when dissolved in water.

With the electrodes rinsed and in pure water, add drops of concentrated sulfuric acid from a pipette or piece of glass tubing until the lamp lights.

Table salt added to pure water will show that it also forms ions, making a conducting solution. Acetic acid can be used and will show only slight conductivity, barely lighting the bulb because of the small amount of ionization.

If the carbon electrodes are placed in a saturated solution of copper sulfate and supplied with low-voltage D.C., copper will deposit on the cathode. Reversing the current will remove the copper from one electrode and deposit it on the other. This suggests that copper ions are positively charged.

Some students will probably want to electroplate various objects. Results, in general, will be disappointing unless special precautions are taken. Handbooks and other references give details on various plating solutions.

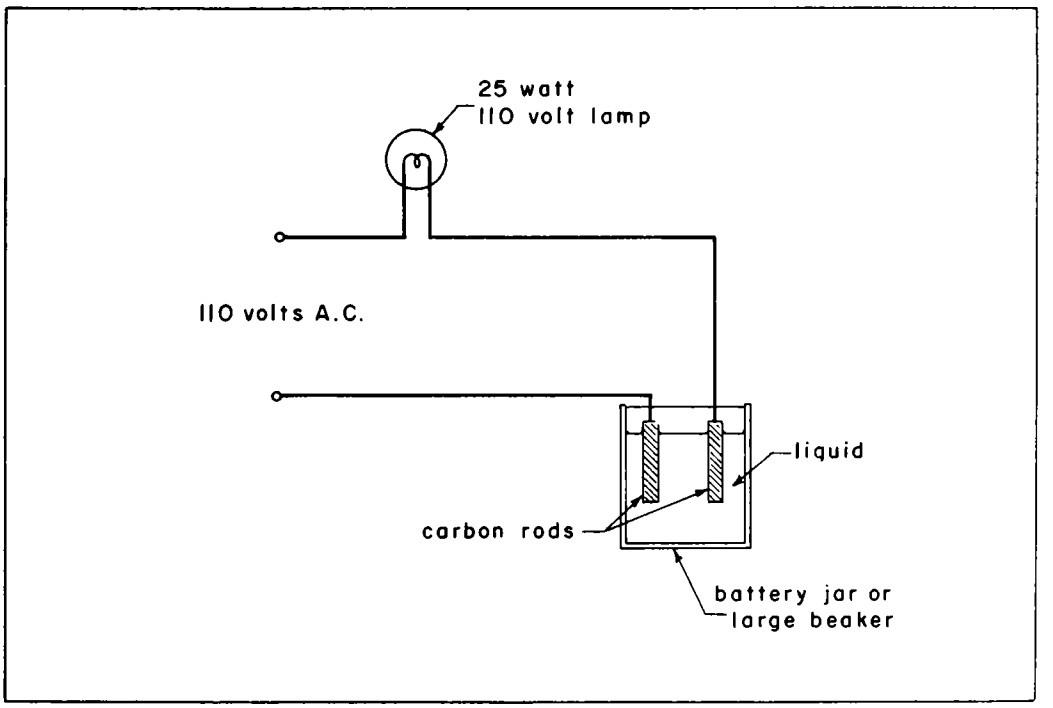


Figure (a)

A More Complex Demonstration for Classes That Have Studied Chemistry. Prepare, before class, solutions of barium hydroxide and sulfuric acid as follows: Dissolve 6.4 gm of $Ba(OH)_2 \cdot 8H_2O$ in 200 cm^3 of water to make 0.2N solution. Mix 5.6 cm^3 of concentrated sulfuric acid with 1,000 cm^3 of water to make about 0.2N solution.

Using the same apparatus as before, half-fill the cell with water and, with 110 volts across the apparatus, add 10 cm^3 of the barium hydroxide solution and stir. The lamp will now light.

Next, add (while stirring) about 10 cm^3 of the 0.2N sulfuric acid. Barium sulfate will precipitate, fewer and fewer ions will remain available for conduction, and eventually the lamp will become extinguished.

If you now add more sulfuric acid, the lamp will light up again; this time you can extinguish it by carefully adding barium hydroxide solution. The process can be played back and forth as long as your patience, your supply of acid and base, and your time permit.

Don't attempt to do all the above demonstrations unless you have plenty of time and, in any case, don't try to explain all the details of what happens when salts, bases, and acids ionize and conduct current. At this stage in the course, the only object of these demonstrations is to show that some solutions contain mobile charged particles that make them conductors.

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The following table classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1.* The sign is the same, but one cannot tell whether it is positive or negative.
2.* Negative.
3. (a) Wool is charged; (b) Test with electroscope.
4.* No. The removal of any number of negative charges from a neutral object gives it a net positive charge.
5.* (a) Charge flows through the metal rod and the body.
(b) Insulate rod from body.
6. Prevents spark when tanks are open by providing a conducting path for charges that may exist.
7. See discussion on page 26–12.
8.* The sign of the charge is the same on the leaves as on the electroscope knob.
9.* (a) Nothing;
(b) Nothing;
(c) Both electroscopes will discharge completely.
10.* Positive.
11.* Negative.
12. (a) Center block is positive. End blocks are negative.
(b) Negative charges attracted to both ends.
13. See discussion on page 26–13.
14. See discussion on page 26–13.
15. See discussion on page 26–14.
16. Use an electroscope which carries a charge known to be positive.
17.* They will fall.
18.* In a good vacuum there are hardly any charged ions which can remove charge from the leaves.
19.* The negative terminal.
20. The current is proportional to the *rate* at which copper is deposited.
21.* The electroscope leaves would fall.
22. (a) $F = 3n \times 10^{-10}$ newton.
(b) $n = 2 \times 10^{4}$ particles.
23. (a) $F = (3P - 3M) \times 10^{-10}$ newton.
(b) $(P - M) = 2 \times 10^{4}$.
24.* Positive.

COMMENTS AND SOLUTIONS

PROBLEM 1* (See Short Answers.)

PROBLEM 2* (See Short Answers.)

PROBLEM 3

Suppose that you have electrified a plastic rod by rubbing it with wool.
(a) Do you expect the wool to become charged?
(b) How could you find out if it does?

If this problem is assigned and discussed before the electroscope (Section 4) is studied, solution (1) is appropriate. If the problem is handled after the electroscope is understood, solution (2) is also appropriate.

(1) Put the plastic rod in an insulated sling hung on a light silk or plastic thread. The rod will be attracted to the wool, indicating the presence of a charge.

(2) (a) When an object is charged by friction, electric charges of one sign are transferred from one to the other of the rubbed objects. Thus one body is left with an excess, and the other object with a deficiency, of one kind of charge. If the plastic rod has become electrified by rubbing it with wool, then the wool has become electrified by rubbing it with the plastic rod. The charge on the wool is opposite to that on the rod.

(b) If an electroscope is charged with the rod, then the leaves will tend to collapse as the wool is brought near the knob, because the wool contains the opposite charge.

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1–3	1, 2, 4	(3)			
4	5, 8, 9	3	6	7	3, 7
5–7	10, 11	13, 16	12#, 14, 15		12#
8–11	17, 18, 19		20		20
12–13	21, 24		22#, 23		

PROBLEM 4* (See Short Answers.)
PROBLEM 5* (See Short Answers.)
PROBLEM 6

When the gasoline tank of an airplane is filled, the metal nozzle of the hose is always carefully connected to the metal of the airplane by a wire before the nozzle is inserted in the tank. Be prepared to explain why this procedure is followed, and to describe how it accomplishes its purpose.

Since the plane and the gasoline truck usually have rubber-tired wheels, they are insulated from the ground. Moreover the service ramps are usually concrete and or asphalt, and are not necessarily good conductors to the earth. Therefore, in spite of grounding chains, the plane and truck could remain charged. A spark could jump when the metal nozzle was brought near the plane. The connection with the wire insures that any charge unbalance will be removed before the tanks are opened. A spark when the tanks are open might ignite the explosive mixture of gasoline and air.

When the gasoline flows, it is possible for the stream of liquid to become charged by friction with the nozzle. The charged gasoline would go into the plane and charge it oppositely to the nozzle. Unless a good conducting path is provided so the charges on the plane and nozzle could recombine, a spark could result — with disastrous consequences.

PROBLEM 7

(a) When you touch a metal object like a door handle on a dry winter day, you are likely to see a spark and feel a definite shock. We usually explain this by saying that we have built up a static charge. How could you tell the sign of the charge you are carrying?

(b) If you accumulate a static charge and then touch the wooden frame of a door, you often find no spark or shock — although there would be if you touched the metal handle. Why?

(c) Sometimes, if you touch the wooden frame and then touch the metal handle, there is no spark or shock in either case — although there would have been one if you had touched the handle first. Suggest an explanation.

(a) If you are already charged, you could charge an uncharged electroscope by touching it. The electroscope would now carry the same kind of charge that you carried. You then discharge yourself by touching ground or a grounded object. The electroscope is being used as a storage device for the charge you carried. The nature of the charge on the electroscope can be found in the following way.

Charge a plastic rod with wool. By the convention described in Section 1, this charge is negative. If, when the plastic rod is brought near the knob of the electroscope, the leaves diverge even more, the charge on the electroscope is the same as on the plastic. This is negative, so *you* were charged negatively. If the leaves collapse, the electroscope charge is the opposite of the charge on the plastic. In this case you were charged positively.

If, instead of plastic and wool, glass and silk were used, the glass rod becomes positive, and the analysis above is reversed.

The build-up of charge can occur when a substance is well insulated from its surroundings. In humid weather an insulator may become a conductor through its surface dirt being hygroscopic and picking up moisture from the air. Thus, it becomes a quasi-conductor allowing the charge to leak away as fast as it builds up. Some automobile ignition failures in damp weather are traceable to this cause. The insulating porcelain of the spark plugs accumulates a film of dirt which is a better conductor when damp than when dry. This allows an appreciable proportion of the spark to "leak" across the surface of the plug instead of discharging across the gap.

(b) When you are the bearer of a sizeable charge and touch a metal object, the transfer of charge is the spark. Because metal is a conductor, the charge can distribute itself quickly throughout the conductor.

When an insulator is touched, the charge is unable to flow through or over it. Hence, little charge can be transferred and there is no spark.

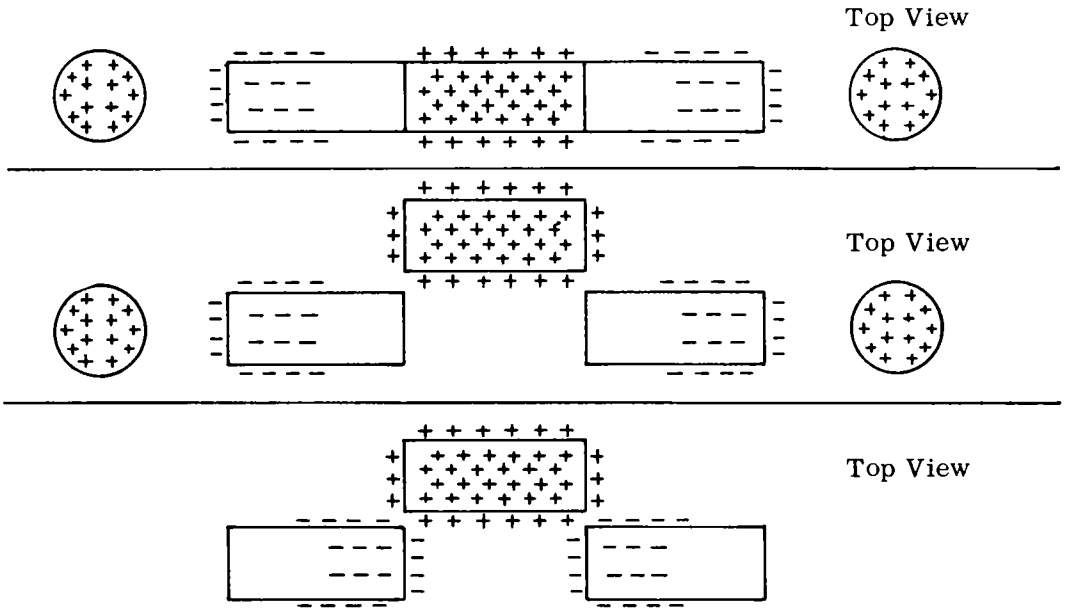
(c) Sometimes the insulator is not perfect and is really simply a very poor conductor. At the instant the wood is touched, the insulator does not accept charge fast enough to cause a spark; yet the contact may last long enough to allow the charge to flow slowly away from you. When you subsequently touch the door handle, there is no spark because there is no charge left on you to be transferred.

PROBLEM 8* (See Short Answers.)
PROBLEM 9* (See Short Answers.)
PROBLEM 10* (See Short Answers.)
PROBLEM 11* (See Short Answers.)
PROBLEM 12

Three metal blocks in contact are resting on a plastic tabletop. You place two objects with strong positive charges, one at each end of the line of blocks, close to but not touching the blocks. You then poke the blocks apart with an (uncharged) insulating rod, while the objects with strong positive charges are nearby. Finally you remove the two positively charged objects.

(a) What charge is now on each block?

(b) Explain how the blocks acquired these charges by describing the motion of negative particles.



(a) As shown in the above diagrams, when the three blocks are separated, the two outer blocks will have negative charges and the middle block will have a positive charge.

(b) The combination of three blocks in contact is presumed to be electrically neutral at the outset. The two positively charged bodies cause a redistribution of charge. In metals, it is the negative charge that is free to wander, so the middle block loses negative charge to the two outer blocks. Thus, when the blocks are separated, the middle one is left with a net positive charge. A return to the neutral distribution is prevented by the insulating properties of air and the table top. If the air is ionized or if the table top is slightly conducting, there would be a gradual drift back to neutrality.

(Initially the forces due to the two positive charges do not cancel each other at all points, because the force due to each decreases with increasing distance. The exact relation, Coulomb's law, is discussed in the next chapter.)

PROBLEM 13

You have two metal spheres of the same size mounted on insulating stands, a strip of plastic, and a piece of cotton cloth. Describe what you would do to give the two spheres equal electrical charges (a) of the same sign, (b) of opposite signs.

(a) Charge the plastic strip by rubbing it with the cotton cloth. Then charge one of the two spheres, either by touching the charged plastic strip to it or, preferably, by induction. Remove the charged plastic strip and touch the two spheres together. Since they are of the same size, the charge will be divided evenly between them.

(b) This must be done by induction. Bring the two metal spheres into contact. Charge the plastic strip and hold it near, but not touching, one of the spheres. Now separate the spheres and then remove the plastic strip. The sphere farther from the plastic strip will carry a charge of the same sign as the strip; the nearer sphere a charge of the opposite sign. Since the two spheres were originally neutral, the separated charges are equal in magnitude.

PROBLEM 14

(a) Describe the steps you would take to charge an electroscope positively by induction.

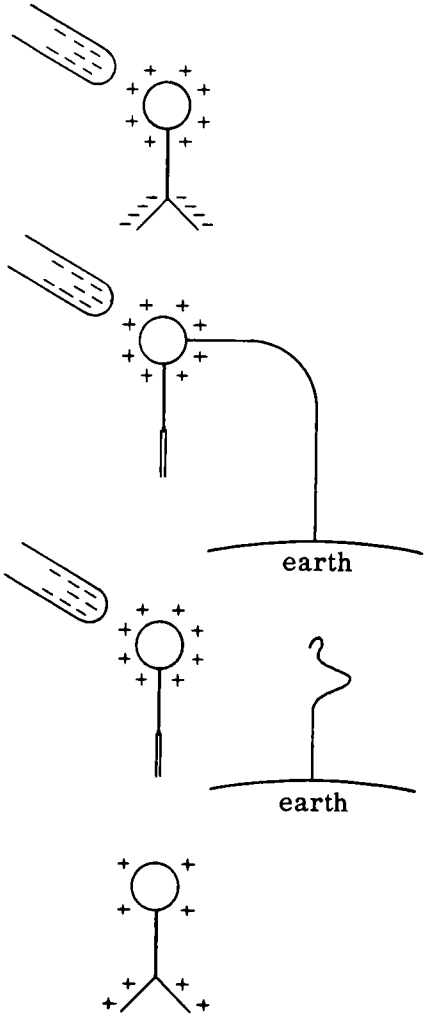
(b) Using labelled sketches, describe the movement of negative electric particles during the charging process.

The first step is to obtain a negatively charged object. If you rub a plastic rod with fur, the rod will become negative.

When a negative object is brought near an electroscope, negative charge is repelled from the knob, leaving the knob positive. This negative charge is pushed toward the leaves of the electroscope causing them to diverge. If the negative object is now removed, the charges on the electroscope redistribute evenly, and the leaves collapse.

To prevent this recombination of the charges, we make the electroscope of a much larger conductor (usually a wire to the earth, or even "you" to the earth) by touching the knob. The negative charges get as far away from the negatively charged object as

possible, consistent with other forces. That is, they flow down the wire to the earth. Students may wonder why the negative charges don't go to the leaves as before. There is some tendency for them to go to the leaves, but many more of them will go to the much larger earth.



The wire is removed — or you take your finger off the knob — while the negative object is in place. The positive charges are trapped and the negative object can be removed, leaving the electroscope positively charged.

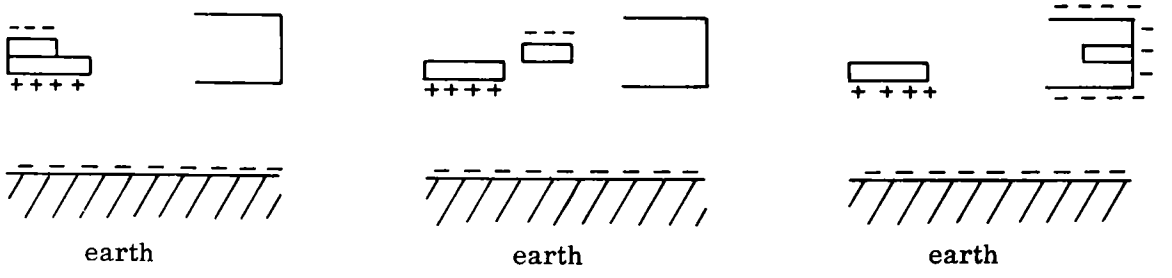
PROBLEM 15

We think the earth has a slight electric charge. How would you go about testing for it?

The difficulty of this problem stems from the fact that the charge is slight. If one scaled up the static electricity experiments in which charged conductors bristle with dust and lint particles, or charged fur shows each hair standing up erect and distinct, then, given sufficient charge on the earth, one would expect to notice this same effect. Dry hay and straw would tend to stand instead of lie in the fields, dropped hairpins and needles or lengths of wire would tend to stand up instead of lying flat on the ground.

Since the charge is "slight," then the effect is "slight." Actually, none of the above effects is observed. Can we say that since no effect is observed, there is no charge on the earth? No. The only conclusion we can reach is that this class of observations is too insensitive to detect whatever effect there might be. We can build more sensitive detectors. One possibility would be a needle mounted on a fine bearing so that it is free to rotate in a vertical plane. If the needle has been carefully balanced while inside a metallic box, no torque will act on it when it is taken out of the box unless the earth has a charge. If the earth does have a charge, the needle will tend to line up vertically. One of the difficulties involved in the measurement of a small charge is that unless one uses extreme care it is easy to mask out the effect, or create a false effect larger than the true one.

Here is another alternative. For this, we would like to have a piece of equipment which enables us to amplify whatever effect there may be. One method of doing this is to put two conductors in contact, one above the other, so that if the earth does have a charge, they will be charged by induction. Next, separate them and transport one of them to the interior of an open can. When a charged object is placed inside a closed conductor and contact is made, instead of charge sharing, all the charge originally on the inner conductor will flow to the outer one. Through repeating the experiment, the hollow conductor is caused to serve as a charge accumulator or amplifier. The figure below may help clarify this method of detection. Our open can is not a perfect closed shell, but it will serve the purpose, and you can think of simple, convincing demonstrations that you can use in class.



PROBLEM 16

How can you find out with an electroscope which terminal of a battery is positive?

Charge the electroscope with a known positively charged object. Bring the charged electroscope near one of the battery terminals. If the leaves start to diverge farther, the terminal is positively charged (see Figure 26–8). If the leaves start to collapse, the terminal is negatively charged and the other terminal must be the positive one.

PROBLEM 17* (See Short Answers.)
PROBLEM 18* (See Short Answers.)
PROBLEM 19* (See Short Answers.)
PROBLEM 20

We have seen that electric charge will flow through certain solutions, such as copper sulfate in water. In this case there is a deposit of copper built up on the negative electrode. Recall the explanation given for this in Section 26–11, and think how this process might be used as a current-measuring device. Be prepared to discuss your ideas in class.

The solution to this problem involves the assumption that an ion of copper, in solution, always has the same charge. However, even without detailed knowledge about ion formation, it can be argued that, on the average, a certain mass of copper is associated with a corresponding ionic charge. Since the number of individual ions involved is very large, deviations from the average value will be negligible.

This relationship could be written:

charge carried = constant × mass deposited.

Then it follows that:

charge carried per second = constant × mass deposited per second.

Thus the electric current flowing during the process of electrolysis is proportional to the rate of deposition of copper on the negative electrode. We could therefore use an electrolytic cell as a current measuring device by putting it in an electric circuit, then measuring the mass of copper, M, deposited during a time interval, T. The electric current that was flowing is proportional to M/T.

PROBLEM 21* (See Short Answers.)
PROBLEM 22

Suppose we have a large number of identical particles. Any two of them at 10 cm separation repel with a force of 3×10^{-10} newton.

(a) If one of them is at 10 cm from a group of n others, how strongly do you expect it to be repelled?

(b) Suppose you measure the repulsion and find it is 6×10^{-6} newton; how many particles were there in the group?

We assume that the dimensions of the particles are negligible in comparison with 10 cm. Thus the n particles can be considered to be collected at a single point 10 cm from the one particle in question.

(a) Therefore the force is n times the single force, or

$$F = n \times 3 \times 10^{-10} \text{ newton.}$$

(b) If $F = 6 \times 10^{-6}$ newton, then

$$n = \frac{F}{3 \times 10^{-10}} = \frac{6 \times 10^{-6}}{3 \times 10^{-10}} = 2 \times 10^{4} \text{ particles.}$$

Some student might point out that it is possible to consider a group of n particles distributed around the circumference of a circle of 10-cm radius and these particles are then all 10 cm from a particle at the center. The stipulations of the problem are still satisfied but now the answer to part (a) can have any value from zero (the particles *evenly* distributed about the circumference) to $3n \times 10^{-10}$ newton (the particles clumped together on one side of the circle).

PROBLEM 23

Suppose we have a large number of positive and negative electric particles, all exactly the same (except for sign). This means that any pair of them either attract or repel depending on their signs, but the magnitude of the force between them is the same as long as the separation distance is the same. For example, the force between any two of them at 10 cm is 3×10^{-10} newton.

A single positive particle is 10 cm from a group containing P positive particles and N negative particles.

(a) How strongly will it be repelled?

(b) You are unable to count the particles, but you measure the force and find it is 6×10^{-6} newton. What does this tell you about P and N?

(a) We again assume that the dimensions of the particles are small with respect to 10 cm. The group of particles then can be considered to be at a single point 10 cm from the given positive particle. In this case the forces all lie along the line joining the group with the given particle. The vector addition of the forces becomes merely algebraic addition:

The repulsive force is $3P \times 10^{-10}$ newton;
The attractive force is $3M \times 10^{-10}$ newton;
The net repulsive force is

$$(3P - 3M) \times 10^{-10} \text{ newton.}$$

(b) If P is less than M, the force is attractive. If the net force is measured as 6×10^{-6} newton, then we have

$$3(P - M) \times 10^{-10} = 6 \times 10^{-6}, \text{ or}$$
$$P - M = 2 \times 10^{4}.$$

This single measurement permits only a determination of the *difference* between P and

M. We are unable to determine anything about the value of P or M separately.

Note on Problems 22 and 23:

Problems 22 and 23 require, for their solutions, the *assumption* that charge is a linearly additive quantity. This assumption is implicitly mentioned in the second paragraph of Section 26–2. The question of its validity will not be thoroughly discussed until the next chapter. However, if a student questions this point, his attitude of cautious doubt should be encouraged rather than discouraged.

When the point is raised in Chapter 27, it might be interesting to return to these problems and to point out that an assumption had been made without experimental proof. If students then feel that no justification is required, you might point out that in the case of the force between bar magnets, the net force depends not only on the number of magnets but also on their relative orientations.

PROBLEM 24* (See Short Answers.)

chapter
27

Coulomb's Law and the Elementary Electric Charge

Chapter 27 puts electrostatics on a quantitative footing. Through a discussion of the Coulomb experiment leading to Coulomb's law and a discussion of charge sharing and the forces between charges, the idea of charge is made quantitative. The existence of a natural unit of electric charge is established through analysis of a Millikan "oil drop" experiment. A large-scale version of this experiment then enables us to determine the electric force between two elementary charges. This is equivalent to determining k in Coulomb's law. This makes it possible to measure ordinary sized charges in terms of the number of elementary electric charges. Finally, the chapter discusses experiments leading to the determination of the number of elementary electric charges on particles such as the electron, alpha particle, and proton. Fields are introduced at a *qualitative*, descriptive level. The sections of this chapter may be grouped in four units.

CHAPTER SUMMARY

SECTIONS 1 THROUGH 3 describe Coulomb's experiment showing that the force between two point charges varies directly as the product of the charges, and inversely as the square of the distance separating them, $F = kQq/r^2$. Section 3 introduces the vector addition of forces from several charges and includes an introductory concept of the electric field around a group of charges as the pattern of forces on a movable charge at various points in space.

SECTIONS 4 AND 5 describe the Millikan "oil drop" experiment in detail, showing the effects of the electrical force, the gravitational force and air friction on the motion of small charged particles. We find that small charged bodies in the electric microbalance move only at certain definite speeds which bear whole-number ratios to one another, never at intermediate speeds. Thus they are subject to electrical forces which are simple multiples of a smallest unit force. It follows that electric charge exists only in multiples of a naturally-occurring smallest unit of charge. This natural unit is called the elementary charge.

SECTIONS 6 AND 7 show how the force between two elementary charges can be determined through a combination of a Coulomb experiment and a large-scale Millikan experiment. This provides the constant k in Coulomb's law. This constant is the force between two elementary charges at a separation of one meter. To place the electrical force in the natural scale of things, it is pointed out that the electrical force between two elementary charges is 10^{36} times greater than the gravitational force between two hydrogen atoms. Section 6 also points out that, when a battery is connected between two conducting plates, the strength of the electric force on a charge between the plates varies inversely with the separation of the plates and directly with the number of batteries connected in series.

SECTIONS 8 AND 9 note that charge is conserved in nature, since any charge lost by one body is gained by another. Experiments are described which demonstrate that the electron carries a single elementary charge and the alpha particle two elementary charges.

SCHEDULING CHAPTER 27

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the volume summary section. Sections which may

Chapter 27	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTION	CLASS PERIODS	LAB PERIODS	EXPERIMENTS	CLASS PERIODS	LAB PERIODS	EXPERIMENTS
1, 2, 3	1.5	1	IV-3[P]	2.5	2	IV-3[P] IV-4
4, 5	1.5	2	IV-5 IV-6	1.5	2	IV-5 IV-6
6, 7	1	0	—	2	0	—
[8], [9]	0	0	—	0	0	—

be abbreviated or omitted without loss in continuity are enclosed with brackets [ ]. Experiments which are specifically intended to precede class discussion of related material are indicated by the superscript *P*.

RELATED MATERIALS FOR CHAPTER 27

LABORATORY. Experiment IV–3, *The Force Between Two Charged Spheres*, is a high-priority experiment and should be done, if possible, as an introduction to Chapter 27. The experiment uses charged pith balls to investigate the relation between electric force and separation, leading to the conclusion that there is an inverse-square relation. Incidentally, the experiment employs an elegant measurement technique — measuring the displacements of shadows — for a situation in which the charge would be lost if a measuring instrument were put directly on the objects.

Experiment IV–4, *The Addition of Electric Forces*, shows the vector nature of electric forces. If done, it should follow Experiment IV–3. See the Lab Notes for suggestions.

Experiment IV–5, *Driving Force and Terminal Velocity*, should be done in connection with Section 27–4 and is a prerequisite to the major experiment, IV–6. Here the students establish the linearity between terminal velocity and driving force which forms the basis for the Millikan Experiment measurements.

Experiment IV–6, *The Millikan Experiment*, affords the students an opportunity to discover for themselves the existence of the elementary charge. This experiment must be preceded by Experiment IV–5 and goes with Section 27–5 of the text. The discovery of the existence of a natural unit of charge was one of the most fundamental and important discoveries in physics in the 20th century, and this experiment allows all students to rediscover this important fact for themselves.

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILMS. "Coulomb's Law," by Eric M. Rogers of Princeton University. This film shows the inverse-square dependence of electrical forces, and demonstrates the consequences of this law. It can be used with Sections 1–3. Running time: 28 minutes.

"Electric Fields," by Francis Bitter of Massachusetts Institute of Technology. This film shows how the presence of electric fields in space may be detected by various means, including lighting a lamp. This film is descriptive, and may be used before starting class discussion of Section 1 or Section 3. Running time: 24 minutes.

"The Millikan Experiment," by Alfred Redfield of IBM and Francis Friedman of Massachusetts Institute of Technology. This film presents a direct photographic record, with commentary, of the experiment described in Sections 4 and 5 of the text. All pertinent quantities are measured. This film can be used to great effect as a summary after the two related student experiments, IV–5 and IV–6. Running time: 30 minutes.

"Coulomb's Force Constant," by Eric M. Rogers of Princeton University. This film shows how macroscopic experiments are interpretable in terms of Millikan's experiment, and derives the value for the constant k in Coulomb's law. This film parallels the development of Sections 6 and 7. Running time: 34 minutes.

"Electric Lines of Force," by Alexander Joseph of Bronx Community College. This film is a short laboratory film showing how the grass seed patterns of Fig. 28–10 were made. Running time: 7 minutes.

27–1 Force vs. Distance

PURPOSE. To show that the electrical forces of repulsion or attraction decrease as the distance between charges is increased. The force is proportional to the inverse square of the distance of separation.

EMPHASIS. One to two class periods should suffice for this and the following two sections. The material is important for the understanding of macroscopic as well as atomic electric forces.

DEVELOPMENT. Students who have played with a charged rod and a suspended pith ball should have a good qualitative awareness that the electric force is proportional to the inverse square of the separation. They will have seen how rapidly the force goes up as the distance decreases.

The main ideas of this section have been mastered when students can readily answer questions such as the following:

A piece of fur and a piece of plastic attract each other with a force of 10^{-2} newton when they are 10 cm apart. With what force do they attract each other when they are 200 cm apart? 50 cm apart? 1 cm apart? 0.1 cm apart?

This last question might serve two distinct purposes. On the one hand the answer of 100 newtons derived from the strict inverse $(\text{separation})^2$ law serves to point out the qualitative fact that these forces become enormous at small distances and that the inverse square is a very rapid function. On the other hand, most students will be quick to point out that you "couldn't get all of the fur and plastic that close." This might provide an introduction to the idea of "point" charges as described in the footnote on page 463 of the text and as elaborated in the sample problem in Appendix 2, Supplement to Section 3, at the back of this volume of the Guide.

* * *

Be sure to point out that this is not the first inverse-square law of force that we have met. In this respect gravitational and electrical forces are alike. You cannot elaborate on this similarity, but it is none the less worthy of note. Students have also seen an inverse-square relationship in the discussion of light intensity in Part II of the course.

* * *

Although methods of measurement are of secondary interest at this stage, the text does note that the law of electric force is known to $r^{-2.000000000}$ — which is very accurate indeed — and students who are on their toes will see immediately that no such accuracy could come from a direct measurement of forces. A discussion of the method that is used to establish the value of the exponent is presented in Appendix 1, at the back of this volume of the Guide, and is also demonstrated and explained in the film on Coulomb's Law.

* * *

The next to last paragraph of the section probably will not require specific comment. It is included for logical reasons. In the Coulomb experiment we are not examining directly the force between *individual* electric particles but rather the force between *bodies* made up of particles. We infer that what we observe is attributable to the forces between the particles. We assume, as it were, that the whole is no greater (or less) than the sum of its parts. This is the kind of paragraph that some students will say "Obvious" to, often without understanding its logical significance. At least a few will appreciate this kind of detail.

QUIZ PROBLEMS. See Guide for Section 3.

27-2 The Electric Charge

PURPOSE. To establish quantitatively the relation between force and charge. The charge dependence and the inverse-square distance relationship give the Coulomb force, $F = \frac{kQq}{r^2}$.

EMPHASIS. One to two class periods should be enough for the first three sections of this chapter.

COMMENT. To say that a particular "charge" can be cut in half by charge sharing, presupposes the idea of conservation of charge which is mentioned explicitly in Section 8. If we touch an uncharged metal sphere to an identical charged one, symmetry tells us only that the final charges on the two spheres are equal but not that their sum is equal to the original charge. The text has already developed the idea of electrical *particles* quite thoroughly and from this point of view conservation of charge reduces almost to conservation of electrical particles which should be accepted quite readily. Do not worry your class with this unless they raise questions.

DEVELOPMENT. Understanding charge sharing is important in order to follow the development of Coulomb's law in the text. Students' understanding of charge sharing can be checked by asking how to divide a charge into thirds. An arrangement like is satisfactory. Such an arrangement as is not. Recognizing the symmetry in a situation is important in physics and particularly in electricity. In the second situation above, not all three balls are equivalent. In fact, the students should know that the two outer balls will receive the greatest charge. Dividing a charge into fourths can be accomplished by halving, then halving the "halves." An arrangement like , and a tetrahedron will also work; but not .

Classwork on Coulomb's law is probably best handled by working through such problems as:

Two charged metal balls A and B repel each other with a force of F newtons.

(a) Suppose we cut the charge on *one* ball in half. Will it make any difference which ball we choose to divide the charge? (No.) What will be the force between the balls if the charge on one ball is cut in half? ($\frac{1}{2}F$)

(b) What will be the force between the balls if the charge on *both* balls is cut to half its original value? ($\frac{1}{4}F$)

(c) With the charge on each ball reduced to half the original value, how much closer will the balls have to be moved to restore the original force? ($\frac{1}{2}$ the distance)

Problem 5 is a good discussion problem at this point. Problems 2, 5, and 6 can be assigned.

27-3 Electric Force Fields

PURPOSE. This section introduces the word *field* to describe the pattern of electric forces around various charge distributions. Electric fields will be studied quantitatively in Chapter 28. This section is intended simply to provide a feeling for the distribution through space of the force patterns around charged objects.

CONTENT. (a) The word *field* is used to describe the pattern of electric forces on a movable charge in a given region of space.

(b) Electric forces add vectorially.

(c) The electric field over a charged plane is uniform in magnitude and direction.

(d) Electric field patterns can be demonstrated graphically using small visible particles such as grass seeds.

(e) Electric field lines are always at right angles to an electric conductor.

(f) The electric field between charged parallel metal plates is uniform.

EMPHASIS. This section is not the place to go into a definition of field as force per unit charge. This will

come in Chapter 28. Here we want to get a feeling for the field idea — the three-dimensional space pattern of the relative magnitude and direction of electrical forces around different arrangements of charge.

If you are rushed for time, this section, along with the previous two can be adequately covered in two to three periods. In that case this section can be treated in a descriptive way. Your aim would be (simply) to show

(1) the general nature and patterns of fields, and

(2) the plausibility of the idea that the electric force on a charged particle placed between parallel charged metal plates is everywhere the same.

On the other hand, if you have time for a more detailed treatment of electricity you can treat the vectorial addition of electric forces — a thing which may not come easily to your students — and some of the characteristics of field line patterns, their perpendicularity to conducting surfaces, etc.

DEMONSTRATION: ELECTROSTATIC FIELDS. This demonstration is described briefly in Section 27–3. A 4–15 kilovolt neon sign transformer, suitably protected, is the source of the electric fields. The output voltage can be varied by means of a Variac autotransformer connected as shown in Fig. (a); for most patterns an output of 4 kilovolts is quite adequate.

The 20-megohm, 1-watt resistor in each high-voltage lead is connected directly to the high-voltage terminals of the transformer and is wrapped with several layers of vinyl electrical tape so the transformer terminals cannot be touched. Do not use resistors smaller than 1 watt; they may be so short that current will flash over and short-circuit them. It may be advisable to enclose the transformer and protective resistors in a box, bringing the insulated high-voltage leads out through holes in the box.

The grass seeds must be small enough to be easily aligned by the electric field (maximum length about $\frac{1}{8}$ in.). A type called "Colonial Bent" works well.

Pour carbon tetrachloride to a depth of about 5 to 15 mm in a Pyrex glass pie plate or similar insulating container. Sprinkle about $\frac{3}{4}$ of a teaspoon of grass seed over the liquid surface (about 1 level teaspoon for each 500 cm^2 of surface). The seed will gather in clumps, but this is normal.

Since carbon tetrachloride evaporates rapidly and its vapor is toxic, it is necessary to cover the carbon tetrachloride with a layer of another liquid. Mineral oil, which is less dense than grass seed and carbon tetrachloride, is used for this purpose. Pour mineral oil slowly down a stirring rod held in the liquid so the oil flows evenly over the carbon tetrachloride, forming a layer 5 to 15 mm deep. The grass seeds will be pushed out to the edges of the container. Gentle

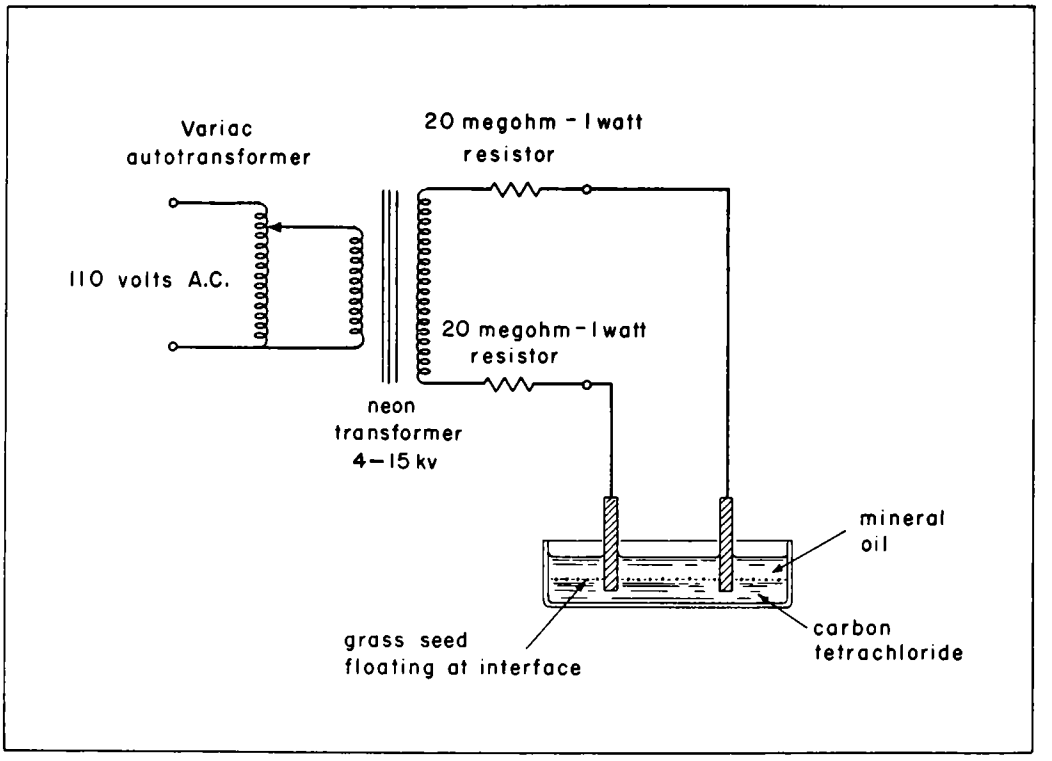


Figure (a)

stirring will distribute the seeds evenly over the entire area.

Place the electrodes in the container so that they extend below the interface of the two liquids. The electrodes can support themselves in the container; can be supported by a wooden frame from above; or can be held in place by stiff wires which also connect them to the transformer.

When using two parallel plate electrodes or concentric cylindrical electrodes, you should place identically shaped pieces directly beneath the container and connected to the corresponding voltages. This prevents the fringing field from the bottom of the electrodes in the dish from distorting the predicted field pattern. One of the electrodes must be grounded (the outer one when concentric cylindrical electrodes are used).

High temperatures from high-power illumination may cause boiling; it is better to use fluorescent lamps for direct observation with a black background and side lighting. The patterns can be projected as shadow patterns on the ceiling, but the electrodes may not show up clearly, especially if auxiliary electrodes beneath the container are used.

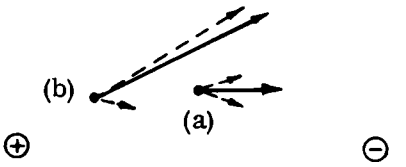
COMMENT. In this section the words *electric field* or *electric force field* are used to describe the pattern of forces on a movable charge in the region around some other charges which are thought of as fixed. The *field* is considered to exist in the space around fixed charges even in the absence of the movable or "test" charge. Some students have objected to the use of the word *field* in this connection. They ask why electric force would not be enough. The reason for using the term *field* at this point is largely one of vocabulary. In advanced work it is easier to describe the fields than the forces, and under certain circumstances (radio and light waves, described in Chapter 31) the fields get "shaken off" the charges, and seem to have an independent existence of their own. These concepts are so important in modern physics that it is regarded as important to introduce this vocabulary early, even though the course could be taught without the use of the term.

* * *

This section is too early a point to strive for an exact definition of field. In a sense, the definition is getting built in as we go along. It was already started in the work on Coulomb's law (purposely without saying so). It will be carried further in the Millikan experiment, and so on. In Chapter 28 we formally consider fields. Here we seek simply a qualitative feeling for the general character of a field — a picture of the anisotropy in space of the electric forces, a pattern which shows the ways charged objects will be pushed in the space around other charged bodies.

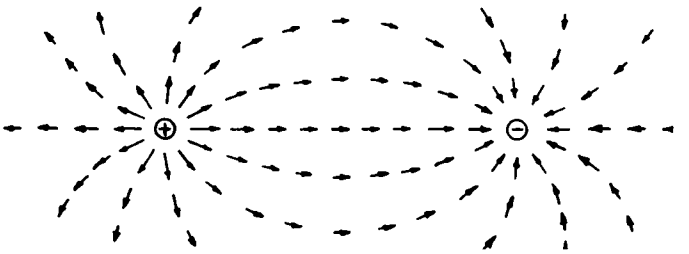
DEVELOPMENT. The word "field" is rather hard to introduce. At this point it is used to describe the "electrical condition" of some region in space. Perhaps it is best introduced with a specific example:

Suppose we have equal positive and negative charges as shown below, and we ask what the



force would be on a small positive charge placed at (a). Even without performing a detailed vector addition one can see that the positive charge pushes up and to the right, and the negative one pulls down and right. The ups and downs cancel, leaving a force to the right as shown. At (b) the force up and right due to the positive charge is so much stronger that it produces the net force shown. Continue drawing "vectors" for a few more points until you can draw the force pattern on the small positive charge *anywhere* as shown below.

This *pattern* of forces is called the electric force field or even just the electric field in the region of the positive and negative charge.



Only two electric field configurations will be used extensively during the remainder of the course: (1) the field due to a point charge (this should be well understood from Sections 1 and 2); and (2) the field between two parallel conducting plates which are oppositely charged. Without the tool of calculus it is difficult to do more than is done in the box on page 491 in proving the uniformity of the field over (or between) charged plates. Be sure to refer to Figure 27-9e of the text for experimental verification.

If you have done vector addition, Problems 3 and 4 can now be assigned.

SUPPLEMENTARY WORK. If time permits, the supplementary work in Appendix 2 may be helpful.

QUIZ PROBLEMS (for Sections 1, 2, and 3 of Chapter 27).

(1) Three identical metal spheres have equal charges. When they are spaced 10 cm apart along a straight line, the electric force on the left-hand one is 10^{-5} newton.

(a) In which direction? (Left.)

(b) What is the electric force on the right-hand charge? (10^{-5} newton to the right.)

(c) What is the net electric force on the center one? (0.)

Now two of the spheres are moved very close together and placed 10 cm from the third.

(d) What is the electric force on this third sphere?

Ans.: In the first configuration we know that the force on the left-hand charge is $k\frac{Q^2}{10^2}$ from the center one and $k\frac{Q^2}{20^2}$ from the right-hand one. Since

$$k\frac{Q^2}{10^2} + k\frac{Q^2}{20^2} = 10^{-5},$$

then

$$k\frac{Q^2}{10^2} = \frac{4}{5} \times 10^{-5}.$$

In the new configuration, the force is:

$$2 \times k\frac{Q^2}{10^2} = \frac{8}{5} \times 10^{-5} \text{ newton.}$$

(2) Two parallel metal plates are oppositely charged. A small point charge is placed midway between the plates and is found to experience an electric force of 2×10^{-8} newton. What is the force on this same particle after it is moved so that it is twice as close to one plate as to the other?

Ans.: 2×10^{-8} newton. The force field between the two plates is uniform, i.e., the force is the same everywhere between them — except, of course, near the edges.

(3) Two equal point charges A and B, are spaced 1 cm apart. They repel each other with a force of 10^{-6} newton. A third equal charge, C, moves along the perpendicular bisector of the line joining A and B. Find the force on the third charge, C.

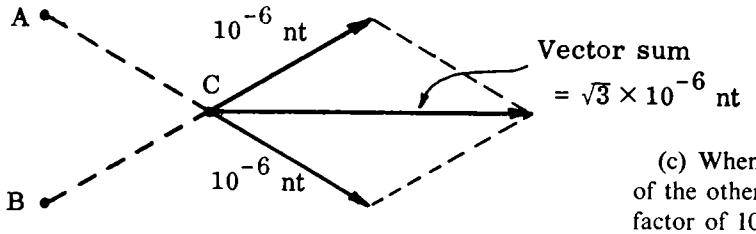
(a) When C is directly between A and B.

(b) When C is 1 cm from both A and B.

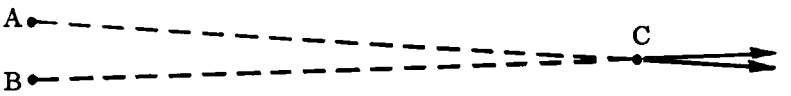
(c) When C is 10 cm from both A and B.

Ans.: (a) When C is directly between A and B, the forces cancel and the net force is zero.

(b) When C is 1 cm from A and B, the force of both A and B on C is 10^{-6} newton. However, only the component along the perpendicular bisector of the line joining A and B is effective, and the net force is $2 \times 10^{-6} \times \cos 30° = \sqrt{3} \times 10^{-6}$ newton.



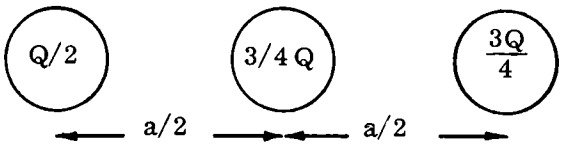
(c) When the third charge is 10 cm away from each of the other two, the forces are less than in (b) by a factor of 100 (inverse square), or 10^{-8} newton each. The angles are so small that the cosines of the angles are nearly 1 and the net force is 2×10^{-8} newton.



(If the angles are taken into account the answer is 1.998×10^{-8} newton.)

(4) (This problem is probably as hard as any which a student should be expected to solve at this level.) Two identical metal spheres carry equal positive charges so that they repel each other with a force of 10^{-5} newton when they are placed a certain distance apart. A third identical but uncharged sphere is touched first to the one on the left and then to the one on the right and then is placed midway between the first two. What is the electrical force on this third sphere?

Ans.: Suppose that the charge on each is Q and that the distance apart is a. We have then that $10^{-5} = k\frac{Q^2}{a^2}$. After touching the one on the left, the third sphere has a charge of $Q/2$ and the one on the left has $Q/2$. When the third sphere is touched to the one on the right there is a charge $Q + Q/2 = \frac{3}{2}Q$ to be shared equally — $\frac{3}{4}Q$ each. The final arrangement is then:



It is worth noting that the total charge is still $2Q$ as it must be. There is now a force to the right of

$$F_{\text{right}} = k\frac{\left(\frac{Q}{2}\right)\left(\frac{3Q}{4}\right)}{\left(\frac{a}{2}\right)^2} = 4 \times \frac{1}{2} \times \frac{3}{4} k\frac{Q^2}{a^2} = \frac{3}{2} \times 10^{-5} \text{ newton},$$

and a force to the left of

$$F_{\text{left}} = k\frac{\left(\frac{3Q}{4}\right) \times \left(\frac{3Q}{4}\right)}{\left(\frac{a}{2}\right)^2} = 4 \times \frac{3}{4} \times \frac{3}{4} k\frac{Q^2}{a^2} = \frac{9}{4} \times 10^{-5} \text{ newton}.$$

The net force is thus $\frac{9}{4} - \frac{3}{2} \times 10^{-5}$ nt or $\frac{3}{4} \times 10^{-5}$ newton to the left. This answer is not exactly right since the presence of the third charge affects the charge sharing of two of them somewhat. If the spheres are small, the effect is negligible.

A good student might short-cut the formal steps in this problem, reasoning the forces in terms of their relation to the original force between the two original charges as follows:

The force on the third charge to the right:

On the grounds of charge, is decreased $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ of original force.

On the grounds of distance, halving the distance increased the force by 4. $\frac{3}{8} \times 4 = \frac{12}{8}$ of original force to the right:

The force to the left:

On the grounds of charge, is decreased $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$ or original force.

On the grounds of distance, halving the distance increased the force by 4.

$\frac{9}{16} \times 4 = \frac{36}{16}$ of original force to the left.

$\frac{36}{16} - \frac{12}{8} = \frac{3}{4}$.

$\frac{3}{4}(10^{-5})$ to the left is the required force.

27-4 Measuring Small Electrical Forces

PURPOSE. Sections 4 and 5 describe a Millikan experiment, showing that electric charge comes in fundamental natural units. Section 4 discusses the design and calibration of the apparatus. The section is introductory in the sense that it lays the foundation for the Millikan experiment. However, the apparatus and technique are so elegant that they are of interest in their own right as an example of experimental design.

CONTENT. This section develops the idea that a small sphere driven in air by a constant force has a constant velocity which is directly proportional to the driving force. Since the electric part of the driving force is proportional to the electric charge, we can compare electric charges by observing the velocities with which a standard sphere is driven. Knowing the weight of the sphere and observing its velocity under the driving force of gravity makes possible the calibration of the apparatus in familiar force units.

The process of calibration also experimentally confirms that the electric force on a charged object between two charged plates is the same at all points, and is directly proportional to the number of batteries (later voltage) across the plates.

EMPHASIS. Since this material develops a *tool* for the Millikan experiment, it should be developed as an introduction to the next section. However the power and beauty of the elementary charge experiment of the following section will not be realized unless this simple material is thoroughly understood. The facts presented on the configuration of electric forces between parallel plates should not be emphasized as they are an aside to the central argument. They should be mentioned as they will be useful in Sections 6 and 7 and later chapters. Not more than one period should be spent on discussion and one period in the lab for this section.

DEVELOPMENT. The Laboratory Experiment IV–5, *Driving Force and Terminal Velocity*, can carry most of the teaching of this section and will provide the background necessary for the following sections. If the Millikan apparatus is not available, the film "The Millikan Experiment" provides an excellent coverage. The requirement for class discussion of Sections 4 and 5 will be sharply reduced by using the lab experiment or the film early in the study of this material.

COMMENT. Basically the electric force is not vital to the material of the first part of this section. The whole point is to prove that the velocity of a small sphere moving through air is proportional to the driving force acting on it. The physics involved is really hydrodynamics rather than electricity. The electrical force is simply used as a convenient way of doubling and tripling the force. Of course the use of the electric force prepares the way for Section 5 where it will be all important. Your class might enjoy knowing that Millikan knew that velocity was proportional to driving force before he began his experiment. The viscous drag of the air on a sphere moving through it is known to be $6\pi krv$ where r is the radius of the sphere, v is its velocity, and k is the viscosity of the air. It is this drag force which balances the gravitational and electrical forces so as to produce uniform velocity.

* * *

A word of warning with regard to the dynamics involved in the motion of the plastic spheres: Be careful in your choice of the words for describing the forces that act on a sphere; the *net* force on the sphere is zero when it is moving at constant velocity. Consider a sphere moving upward with constant speed, v_0. The gravitational force acts downward and has a magnitude F_g. The electrical force acts upward and has a magnitude F_{el}. It is then temping to say that there is a net force acting on the sphere, the net force being given by $(F_{el})_{up} - (F_g)_{down}$. *This is not true. The net force acting on any object which is moving with constant speed is zero.* To calculate the true net force, one must take into account, besides the gravitational and electrical forces, the retarding force of the air resistance: $[(F_{el})_{up} - (F_g)_{down}] - (F_{air})_{down} = 0$. It may then be useful to use the term "driving force" to indicate the resultant of gravitational and electrical forces. The net force will be zero.

* * *

The figures on page 494 can be used as a basis for questioning to determine whether students understand the logic of the section.

The fact that connecting two batteries in series to the plates produces twice the electrical force of one battery *cannot* be derived at this stage as a consequence of other knowledge. It is directly demonstrated experimentally and is easy to believe.

DEVELOPMENT. The ideas in this section are simple. They are understood when students can answer the following kinds of questions:

A steel marble will drop through syrup at a constant speed (why constant?) of 10 cm per second. How fast will a glass marble fall if it is of the same size but half the weight? Ans.: Half as fast — though bright students should probably see that the buoyant force of the syrup should be taken into account, just as the text mentions that the buoyant force of air must be considered for extreme accuracy in the plastic sphere case.

If the steel marble is pulled upward, by a string, with a force equal to its weight, how fast will it move? Ans.: Zero speed. The driving force is zero; if the ball had been moving, the viscous force would immediately stop it.

If the string pulls up with a force twice as great as the weight, how fast does the marble go? Ans.: The driving force is an upward force as large as the weight; consequently the marble moves up at 10 cm per second.

27–5 The Elementary Charge

PURPOSE. To describe an actual experiment which shows that charges occur in multiples of some natural unit (the elementary charge).

EMPHASIS. Students should leave this section with an understanding that all charges *do* occur as multiples of some elementary charge unit, but this is not all. Hopefully they should get a deep intellectual appreciation of this beautifully simple, but very significant experiment.

DEVELOPMENT. The Laboratory Experiment IV–6, The Millikan Experiment, should be used if at all possible to carry the teaching load of this section. Direct student experience with the equipment and the measurements indicating the existence of an elementary unit of charge cannot be surpassed by any other technique of teaching. The experiment, using plastic spheres, is now simple enough that good measurements can be obtained in one laboratory period if preceded by Experiment IV–5. If the equipment is not available to you, the film "The Millikan Experiment" should be used instead. If you do perform the experiment, this film makes an excellent summary and ties down any "loose ends" that may still exist.

COMMENT. Students should not be allowed to get so involved in the descriptive material of this section that they lose sight of what the experiment shows. Some of your students may have heard the Millikan oil drop experiment described as an experiment to measure the charge on the electron. That is the way it is usually described. It is *not* to be thought of in that way here. This experiment shows that in a particular electric field, a particular plastic ball shows

changes in velocity and hence, changes in the force on it. These changes in force come in small integral multiples. Since the force is proportional to the charge on the ball, the conclusion is that the charge cannot have just *any* value, but must be subject only to discrete changes which are integral multiples of a fundamental charge unit. In our development we have not set up the coulomb, so we need not measure our elementary charge in terms of any existing unit. Instead, we adopt this elementary charge as our fundamental unit charge.

Millikan, who had no plastic spheres, was faced with a much more formidable task than the authors of the text. He used small oil droplets whose size was unknown. If a drop is made of oil of density d and has a radius a, it has a mass $\frac{4}{3}\pi a^3 d$ and gravity exerts a force on it of $\frac{4}{3}\pi a^3\, dg$. If it falls with a velocity v, the retarding force on it is, from hydrodynamic considerations, $6\pi kav$, where k is the viscosity of the air. Then if the drop falls with a constant velocity v

$$\tfrac{4}{3}\pi a^3\, dg = 6\pi kav.$$

Measuring the velocity of fall allows a calculation of a, the radius of the drop. The experiment proceeds from this point on in a way very like that described in the text.

A fine first hand account of Millikan's experiments is given in his book, listed under "Further Reading" on page 505. Some of your more enterprising students might be interested.

DEVELOPMENT. It will be wise to take a little time to emphasize the generality of the conclusions drawn from the Millikan experiment (the last paragraph in this section).

* * *

Problems 13, 14, and 15 refer to different aspects of the Millikan experiment and can be assigned at this stage. Problem 18 could have been worked with the material of Section 3, but has a bearing on the Millikan experiment.

QUIZ PROBLEMS. (To follow the Millikan film.)

(1) In a certain Millikan experiment a plastic sphere is held motionless between charged plates by the electric force when 3 batteries are connected across the plates. When exposed to X rays, the sphere starts to fall and it is found that 5 batteries must now be used to counteract the force of gravity. What is the smallest number of elementary charges that could have been on the sphere to begin with? Ans.: 5.

Solution: Since the electric force is proportional to the number of batteries, and the number of charges on the sphere, with 3 batteries the electric force that is equal to the gravitational force is

$$3cq_1 = mg,$$

where c is a constant of proportionality and q is the number of charges on the sphere. With 5 batteries, the force is

$$5cq_2 = mg.$$

Therefore, assuming that there is no change in mass with change in charge,

$$3cq_1 = 5cq_2,$$

and

$$\frac{q_1}{q_2} = \frac{5}{3}.$$

Thus 5 charges is the smallest number possible in the case of no motion with 3 batteries.

(2) In a certain Millikan experiment the batteries are adjusted until a given charged sphere is motionless. The batteries are switched off, and the sphere falls 1 millimeter in 3 seconds. The sphere is raised with a strong electric field; X rays are applied; and the original field is restored. The sphere now falls 1 mm in 9 seconds. The procedure is repeated several times, yielding the following data:

Time to fall 1 mm with no field: 3.00 seconds.
With the standard field applied:
(1) does not fall or rise
(2) falls 1 mm in 9.00 seconds
(3) rises 1 mm in 3.00 seconds
(4) rises 1 mm in 9.00 seconds
(5) falls 1 mm in 2.25 seconds.

(a) What was the probable initial charge on the sphere?

(b) What was the probable last charge on the sphere?

Ans.: (a) 3. (b) -1. Of course we were not given enough data to tell whether the answer should be $+3$, -1 or -3, $+1$. We do know that the sign is different in parts (a) and (b). Of course, the answers *could* be 6 and -2, but considering the other values found, this is not likely.

27-6 A Large Electrical Balance

PURPOSE. Sections 6 and 7 show how the constant of proportionality in Coulomb's law can be measured. Section 6 describes the experimental arrangement for this measurement.

EMPHASIS. Sections 6 and 7 should be discussed together. If you are hard pressed for time, you might treat these sections as a reading assignment — making certain, however, that students learn the one or two simple facts about forces on charges between parallel plates and know where to find the value for the constant in Coulomb's law. If you use the film

"Coulomb's Force Constant," you should be able to cover these two sections in a single period.

DEVELOPMENT. For the discussion of the large scale micro-microbalance see the next section.

This section extends two ideas on electric forces between parallel plates: how they change with the area of the plates (they don't), and how they change with separation of the plates (inversely as the separation for the same battery applied). It might be well to ask students to summarize what they know about electric forces on charged objects between metal plates. They should come up with a statement something like the following:

The electric force on a charged object placed between two parallel metal plates is:

(1) independent of the position of the object between the plates, i.e., the same everywhere (as long as the plates are large, and if the image forces are negligible).
(2) independent of the area of the plates as long as the plates are large compared to their separation.
(3) inversely proportional to the separation between the plates for the same battery applied.
(4) directly proportional to the number of batteries connected in series between the plates (later this will be defined as proportional to the voltage across the plates).
(5) directly proportional to the charge on the object.

Students should not be left with the idea that any of this is proved logically. The best we can do is state the facts and verify them by observing charges between plates. You can tell your students that all these statements can be proved, though the development is too long for the present text and properly involves the use of calculus or differential equations. At this point it will be well to leave the proportionality of force between the plates and the number of batteries as simply experimentally demonstrable, and not go ahead to the addition of voltages.

COMMENT. All the statements made about forces on charges between parallel metal plates apply only to the case where the battery is kept connected. Consider the case given in the text. A battery is connected between two metal plates, resulting in a force being exerted on a test charge placed between them. The distance between the plates is doubled, the battery remaining connected. The force on the test charge is halved. The charge which is now on the *plates* has decreased to half its former value. As the plates were moved apart, charge actually flowed back through the battery.

On the other hand, if the battery is disconnected and the plates are then moved apart, the force on the test charge remains unchanged. In this case the charges on the plates had no place to go and hence remained the same.

QUIZ PROBLEM

A plastic sphere is placed between two metal plates 1 cm apart, with no battery connected. The sphere falls 1 mm in 10 seconds. A battery is connected, and the sphere is found to move up 1 mm in 20 seconds.

(a) With the battery connected, how far apart should the plates be in order to hold the charge motionless?
(b) With the plates at the new distance how many batteries must be connected to make the charge rise 1 mm in 10 seconds?
(c) If the connection of this last set of batteries is reversed, how fast will the sphere fall?

Ans.: Let the weight of the sphere be W. In part (a) the driving force is $\frac{1}{2}W$ upward, so the electrical force is $\frac{3}{2}W$ up. To balance gravity the upward force should be equal to W. Thus it must be reduced to $\frac{2}{3}$ its initial value. It follows that the plates must be $\frac{3}{2}$ as far apart, i.e., 1.5 cm.

The driving force required in part (b) is W up, or an electrical force of $2W$ up. Thus 2 batteries are needed.

For part (c), if the batteries are reversed the driving force is $3W$ down. With this force the sphere will drop 1 mm in 3.33 seconds.

27-7 The Constant in Coulomb's Law; the Force Between Elementary Charges

PURPOSE. This section describes a method for establishing the constant of proportionality in Coulomb's law.

EMPHASIS. It is highly important to understand the strategy of the experiment. The details are secondary. If you are pressed for time, you may treat this and the preceding section as a reading assignment only. In this case students should understand that the force constant in Coulomb's law *can* be measured, even though they do not follow closely the details of how it can be done. The film "Coulomb's Force Constant," using the remainder of a single period for discussion, should suffice.

DEVELOPMENT. You might introduce the discussion of the measurement of k in Coulomb's law by asking your class to calculate the force between two elementary charges placed 0.1 mm apart. When they find they lack knowledge of the constant, ask them how to measure it. The problem always comes down to how to count or measure the number of excess elementary charges on an object which is large enough to permit making a force measurement. In order to discourage a proposal of a direct measurement you may have to give the class the value of $k = 2.3 \times 10^{-28}$ newton meter^2/elem charge^2. As soon as the class realizes that the measurement is not easy, you can discuss the experiment in the text.

You will probably wish to summarize the logic of the measurement something like this:

(1) We know the force per elementary charge between two standard plates (from the Millikan experiment).

(2) We scale up the experiment, multiplying both the plate separation and the number of batteries by 100, so that the *force per elementary charge is the same* as before (and hence known).

(3) We place one of two large identically charged spheres between the charged plates and use a spring balance to measure the electrical force exerted. This force divided by the force per elementary charge gives us the number of elementary charges on the sphere.

(4) We measure the force of repulsion between the two identically charged spheres at a given distance. We can then solve for k in

$$F = k \frac{q_1 q_2}{r^2}.$$

Students should see that k, 2.3×10^{-28}, is the measure in newtons of the force between two elementary charges at a distance of one meter. It will be important to place this force in scale by showing how large it is compared to gravitational force between small particles at comparable distances.

The film "Coulomb's Force Constant" shows the type of experiment described in the text. Discussion of this experiment can be materially shortened through the use of this film.

COMMENT. Notice the difference in development used in this text and that usually encountered. Most books *define* an electrostatic unit of charge to be that charge, q, which, when placed 1 cm from an identical charge, exerts a repulsive force of 1 dyne (10^{-5} newton). That is, Coulomb's law is written $F = q_1 q_2 / r^2$. (The constant k is thus set equal to one.) Then the charge on the electron, the elementary charge, must be measured as some fraction of an electrostatic unit (4.8×10^{-10}). Other books define a unit of charge called a coulomb in terms of magnetic forces on currents. Since the unit of charge is found in some other way, the constant k must be measured. Also the charge on the electron must be measured (1.602×10^{-19} coulomb).

Our book adopts the idea that nature supplies a natural unit of charge and that we must measure the factor k thus defined by Coulomb's law. In this scheme there is no such thing as "measuring the charge on the electron." *This section describes a means of measuring k.* For the purposes of atomic physics, this scheme is superior both conceptually and practically. For the use in electrical engineering the charge unit is inconveniently small.

It is probably best to tell students that the scheme for measuring k as described in this section is not one that is actually used. As the movie shows, this experiment is possible, but much more accurate methods exist. The actual methods involve a whole series of very indirect measurements which would be impossible to describe at this level. This is in contrast to the Millikan experiment which, at least when it was first performed, was the best method for determining the charge on the electron in terms of the previously defined electrostatic units of charge.

* * *

Problem 16 shows a plausible way for measuring the force on a large charge between two metal plates. Problems 19, 21, and 24 are appropriate. Problem 21 is useful for discussing the small fraction of the molecules in macroscopic charged bodies that have changed their charge.

An interesting type of problem is to consider the force of attraction between a proton and an electron at a distance of 0.5×10^{-10} meter, which is their separation in a hydrogen atom. The answer is 10^{-7} newton. This is quite a tremendous force compared to that on the plastic sphere in the Millikan experiment (greater by a factor of about 10^7).

27-8 Conservation of Charge

PURPOSE. To state the conservation of charge.

EMPHASIS. Descriptive reading. At this point, conservation of charge should seem fairly obvious. It can be passed with little more than noting it.

COMMENT. The material of this section can easily involve you in rather deep discussion of pair creation and other related subjects. Don't get into that unless you are competent to get out!

The text states that sometimes a neutron turns into a proton and an electron (a neutrino is given off too). A rather common statement in high school texts is that a neutron *is made* of a proton and an electron. This last statement is not correct according to present day ideas.

* * *

The basic notion of a conservation law is probably very easily accepted by the majority of high school students. They have an intuitive feeling for the conservation of objects which can be exploited here. The nomenclature "plus" and "minus" makes the idea of pair creation less distasteful.

27-9 The Electric Charge of Electrons and Other Particles of Matter

PURPOSE. This section indicates how the charge carried by elementary particles can be determined.

EMPHASIS. Descriptive material. Can be covered as part of a reading assignment.

CONTENT. (a) The electron carries a negative charge of one elementary unit.

(b) The alpha particle carries a charge of $+2$ elementary units.

(c) All "elementary" particles known today have a charge of $+1$, -1, or 0 elementary charges.

COMMENT. This section confirms what most students will have already suspected, that the electron carries a single elementary charge unit. As a matter of fact all elementary particles we know today carry no charge or a single elementary charge. Physicists know of no reason why this is so. It is simply a fundamental, empirical fact. Problem 17 is useful here, both for home assignment and for class discussion.

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The following table classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

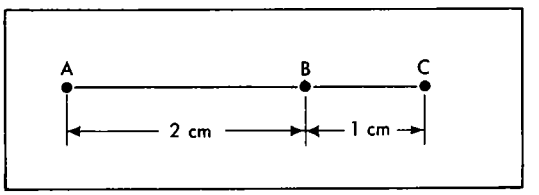
1.* 1.0×10^{-5} newton.
2. (a) 1.2×10^{-5} newton.
   (b) 9×10^{-6} newton to the left.
3. (a) 3×10^{-6} newton.
   (b) 5×10^{-6} newton, 37° toward the right from the line $\overline{CB}$.
4. (a) See diagram on page 27-14. Resultant force = $0.91\ kq^2/r^2$.
   (b) All will move toward center.
5. (a) $\vec{F}_A = 1.5 \times 10^{-5}$ newton to the left.
   (b) $\vec{F}_C = 2.0 \times 10^{-5}$ newton to the left.
6. (a) $F_e = K/X^2$.
   (b) $F_e/mg = X/l$.
   (c) Increases by factor 8.
7. (a) $F(3)/F(5) = 2.7$.
   (b) See detailed discussion on page 27-17.
8.* The net force would change in both magnitude and direction.
9.* 27-8(a) and 27-9(a)
   27-8(b) and 27-9(b)
   27-8(c) and 27-9(c)
   27-8(d) has no analog in 27-9.
10.* No. If they did, at the point they crossed the net force on a movable charge would have to be in two directions at once!
11.* Only if the line is straight and the particle is moving along the line. If the line is curved, the trajectory of the particle will, in general, be different from the direction of the force on the particle.
12. The terminal velocity of the sphere in each case is proportional to the driving force on the sphere.
13. (a) 7.0×10^{-14} newton, up.
   (b) 1.4×10^{-14} newton, down.
14. Charge on the plates is directly proportional to the number of batteries.
15. Terminal velocity increases with size.
16.* $F = 1.4 \times 10^{-15}$ newton.
17. (a) Use two batteries in series.
   (b) Three batteries.
18. (a) 5.0×10^{-5} newton; (b) 2.5×10^{-5} newton; (c) 5.0×10^{-5} newton, 2.5×10^{-5} newton.
19. (a) 2×10^{-4} newton; (b) 6×10^{-4} newton.
20.* $F = 2.3 \times 10^{-16}$ newton.
21. (a) Three positive elementary charges.
   (b) 1.02×10^{11} molecules.
   (c) 2.94×10^{-11} or 1 in 3.4×10^{10}.
22. (a) 4.4×10^{-4} newton; (b) 1.5×10^{10} elementary charges.
23. 5.6×10^{10} elementary charges.
24. (a) One negative elementary charge; (b) Electrons.

COMMENTS AND SOLUTIONS

PROBLEM 1* (See Short Answers.)

PROBLEM 2

Three equally charged objects are located as shown in Fig. 27-25. The electric force exerted by A on B is 3.0×10^{-6} newton.
(a) What electric force does C exert upon B?
(b) What is the net electric force on B?



SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1	1	2, 3			2, 3
2		4, 5		6, 7#	5, 7#
3	8, 9, 10, 11	3			
4–5		13	12, 14, 15		13, 14
6	16	17, 18	19		
7	20		21, 22	23	21, 22
9			24		

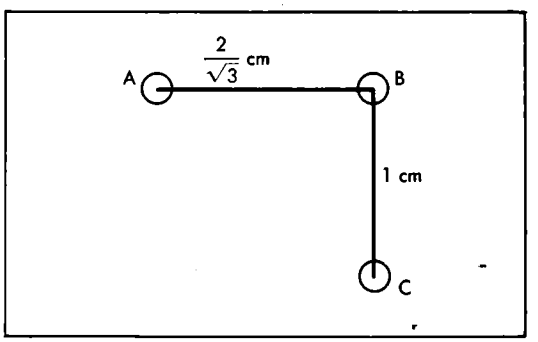
(a) Since the distance between A and B is twice that between C and B, the force of C on B must be four times that of A on B; $F_{BC} = 4 \times 3.0 \times 10^{-6}$ newton = $\underline{1.2 \times 10^{-5} \text{ newton}}$.

(b) Assuming that the three charges are of identical sign as well as magnitude, the forces are both repulsive. The force of A on B is given as 3.0×10^{-6} newton to the right. From (a), the force of C on B is 12×10^{-6} newton to the left.

$$\therefore F_{net} = 12 \times 10^{-6} - 3 \times 10^{-6}$$
$$= \underline{9 \times 10^{-6} \text{ newton to the left}}.$$

PROBLEM 3

Suppose that we place three small charged spheres, with equal charges on them, as shown in Fig. 27-26. Sphere C exerts a force of 4×10^{-6} newton on B.
(a) What force does A exert on B?
(b) What is the net force on B?



(a) Since Coulomb's law states that the force is inversely proportional to the square of the distance, then

$$\frac{F_{AB}}{F_{BC}} = \frac{(1/1 \text{ cm})^2}{\left(1/\frac{2}{\sqrt{3}} \text{ cm}\right)^2} = \frac{3}{4}.$$

But $F_{BC} = 4 \times 10^{-6}$ newton, so $F_{AB} = \frac{3}{4} \times 4 \times 10^{-6} = \underline{3 \times 10^{-6} \text{ newton}}$.

(b) Since the two forces acting on B are perpendicular, the magnitude of the net force will be:

$$F_{net} = \sqrt{(4 \times 10^{-6})^2 + (3 \times 10^{-6})^2}$$
$$= \underline{5 \times 10^{-6} \text{ newton}}.$$

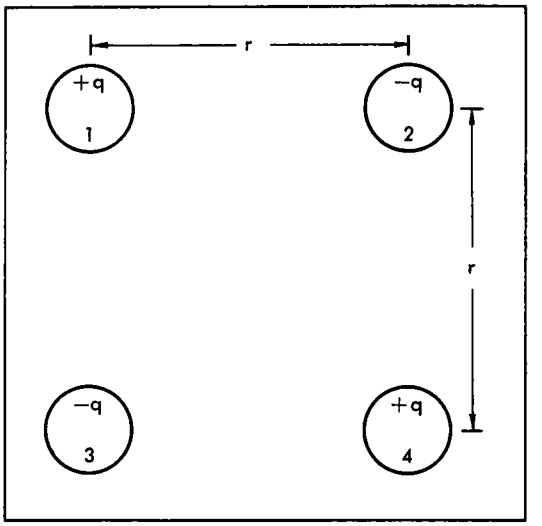
The direction of the net force is given by the vector sum, as shown at the right.

PROBLEM 4

Four identical conducting spheres mounted on Dry Ice pucks are held on a smooth level table at the four corners of a square. Two of them carry a charge of $+q$ and the other two carry a charge of $-q$ each, as shown in Fig. 27-27.

(a) Make a vector diagram, to scale, showing the electric forces on sphere 1.

(b) If all the pucks are released at the same time, In what direction will each of them move?



(a) Since spheres 2 and 3 are each a distance r from sphere 1, the forces they exert on it are equal in magnitude:

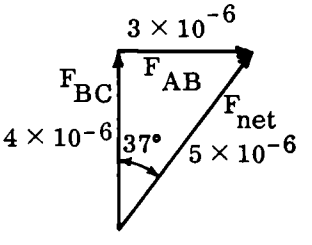
$$F_{2-1} = F_{3-1} = k\frac{q^2}{r^2}.$$

Because the diagonal of a square is $\sqrt{2}$ times as long as one side,

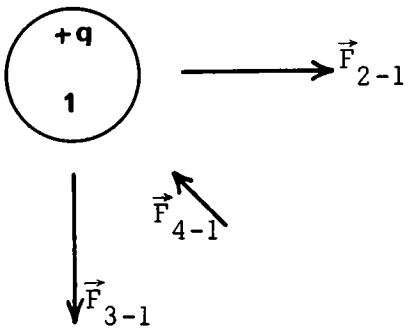
$$F_{4-1} = k\frac{q^2}{(\sqrt{2}\,r)^2} = \frac{kq^2}{2r^2}.$$

The force exerted by sphere 4 is only half as great as the force exerted by sphere 2 or 3.

The arrows on the diagram indicate the direction and relative magnitude of each force on sphere 1.



The resultant force $\vec{F}_R$ equals $\vec{F}_{2-1} + \vec{F}_{3-1} + \vec{F}_{4-1}$.



From the diagram on the right, one can see that

$$(F_R + F_{4-1})^2 = F_{2-1}{}^2 + F_{3-1}{}^2 = 2F_{2-1}{}^2.$$

Therefore,

$$F_R = \sqrt{2}\,F_{2-1} - F_{4-1}$$
$$= (\sqrt{2} - \tfrac{1}{2})F_{2-1} = \underline{0.91\,kq^2/r^2}.$$

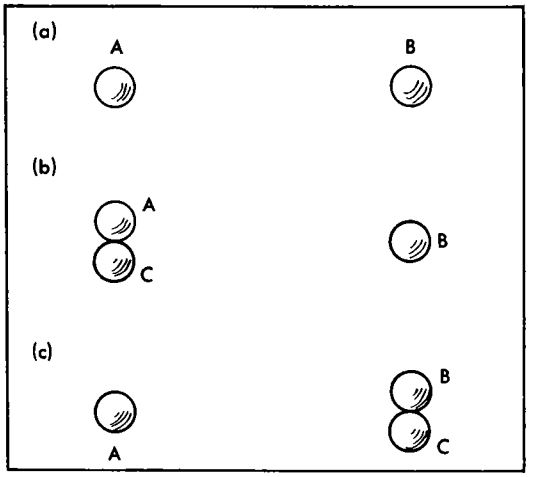
(b) From the symmetry of the figure, one can see that the resultant force on each charge will be of the same magnitude and that each resultant will be directed toward the center of the square. When all are released at the same instant, they will move toward the center of the square.

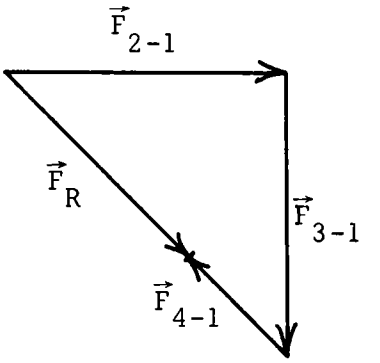
PROBLEM 5

Two equally charged identical conducting spheres A and B repel each other with a force of 2.0×10^{-5} newton [Fig. 27-28(a)]. Another identical uncharged sphere C is touched to A [Fig. 27-28(b)] and then moved over right next to B [Fig. 27-29(c)].

(a) What is the electric force on A now?

(b) What is the net electric force on C (after it has touched A) when it is halfway between A and B?





This is a basic problem which touches essentially on all the points covered in this chapter about macroscopic charges and their interactions. We use the following facts:

(1) Force between charges goes as $1/\text{distance}^2$.

(2) Force is proportional to both charges.

(3) When charge is redistributed the total charge is conserved, i.e., constant.

(4) Charges on conducting bodies move freely.

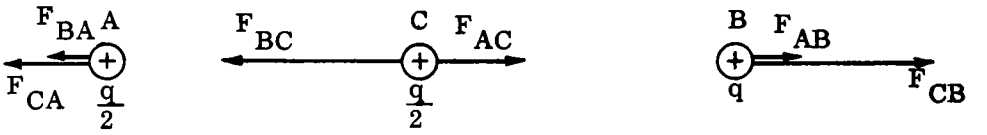
We can summarize the solution by using sketches of the situation at different stages in the problem.

Initially the force F is given as 2.0×10^{-5} newton.

(a) If an identical conducting sphere C is placed in contact with A, A and C will share equally the original charge on A, provided no other charged bodies are nearby. After C has been moved over to B we have, as shown below. The force on A can be calculated using fact (2) above:

$$\frac{F_A\ (\text{now})}{2.0 \times 10^{-5}} = \frac{(\frac{1}{2}q)(\frac{3}{2}q)}{(q)(q)} = \frac{3}{4}.$$

$\vec{F}_A(\text{now}) = \underline{1.5 \times 10^{-5}\ \text{newton to the left}}$.



(b) If C has touched only A but not B, the charge on it is $q/2$ and we have the above situation when C is halfway between A and B.

Since the distance between A and C is half that between A and B, then the force exerted by sphere A on sphere C is:

$$\frac{F_{AC}}{2.0 \times 10^{-5}\ \text{nt}} = \frac{(q/2)(q/2)/(\frac{1}{2}\overline{AB})^2}{(q)(q)/(\overline{AB})^2} = 1.$$

$$\vec{F}_{AC} = 2.0 \times 10^{-5} \text{ newton to the right.}$$

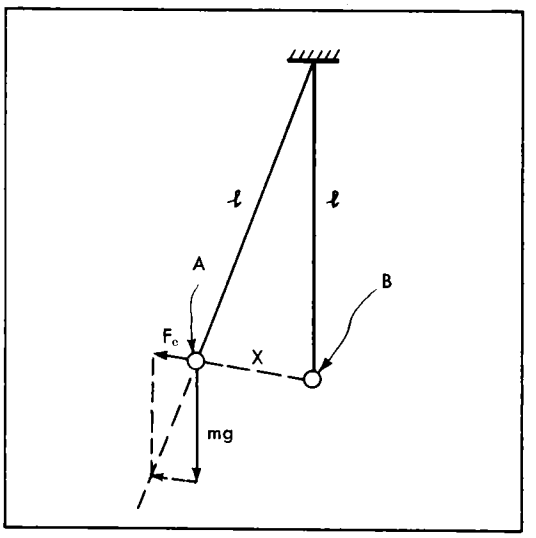
Sphere B is the same distance from C as A is, but has twice the charge. Hence, the force it exerts on C is

$$\vec{F}_{BC} = 4.0 \times 10^{-5} \text{ newton to the left.}$$

The net force on sphere C due to spheres A and B is then

$$\vec{F}_C = \underline{2.0 \times 10^{-5} \text{ newton to the left}}.$$

Some students may interpret the problem as meaning that C has first touched A, then touched B, and has then been moved halfway between A and B. In this case, C has shared charge twice, and the charge on it is $3q/4$, which will result in a different answer.



PROBLEM 6

Two charged balls A and B each with mass m are placed as shown in Fig. 27-29. The ball A is free to move; B is fixed in position.

(a) How does the force F_e between them depend upon X?

(b) Find the relationship between the electric force F_e and the weight mg in terms of X and l.

(c) If the charge remains the same and the separation X is cut in half, how must the weight have changed?

Ball B is fixed in its position, and A is assumed to be a pendulum bob in static equilibrium.

(a) Since X is the distance of separation and F_e is the electric force of repulsion,

$$\underline{F_e = K/X^2},$$

where K depends on the charges of the balls.

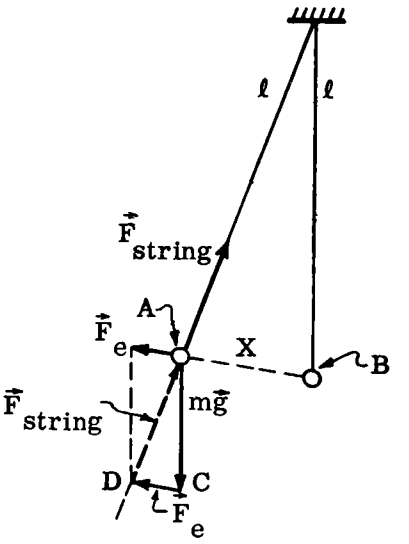
(b) There are three forces acting on A: the electric force $\vec{F}_e$; gravity; and the tension in the string, $\vec{F}_{string}$. In order for A to be in equilibrium, the net force acting on it must be zero:

$$\vec{F}_e + m\vec{g} + \vec{F}_{string} = 0.$$

Since $\vec{F}_{string}$ is a tension, it must act along the string, and must have a magnitude such that it closes the vector triangle ACD. In other words, the vector sum of $\vec{F}_e + m\vec{g}$ must act in the direction of the string.

Then by similar triangles:

$$\underline{\frac{F_e}{mg} = \frac{X}{l}}.$$



(c) If the X distance, corresponding to equilibrium, is cut in half, F_e will increase fourfold. We can write for the new equilibrium:

$$\frac{4F_e}{m'g} = \frac{X}{2l}, \quad \text{or} \quad \frac{F_e}{m'g} = \frac{X}{8l}.$$

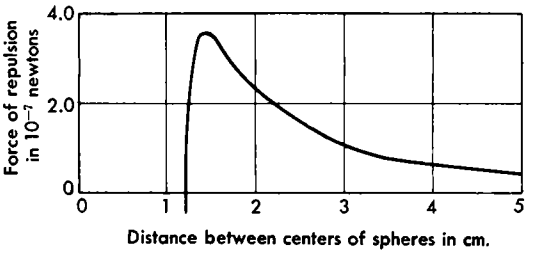
Since $F_e/mg = X/l$, we find $m'g = 8\,mg$; the weight must have increased by a factor of 8.

PROBLEM 7

The graph in Fig. 27–30 shows the electric force of repulsion on a tiny charged conducting sphere as a function of its separation from a large conducting sphere. The large sphere has a radius of 1 cm, and has 10 times the charge of the small sphere.

(a) How is the force changing as the separation changes from 5 cm to 3 cm?

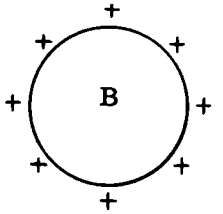
(b) Explain the behavior of the force between separations 2 cm and 1 cm.



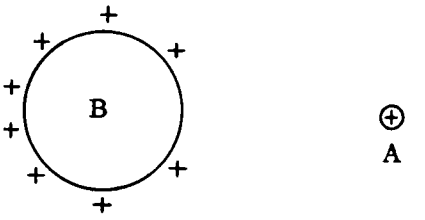
This is an interesting problem which brings out the effects of induction in a conductor. When the tiny sphere (A) is far away from the large sphere (B), the charge distribution on the latter is spherically symmetric, i.e., uniform. The force between the spheres is then the same as between point charges. (This is a consequence of the $1/r^2$ nature of the force.)

As the spheres are brought closer together, however, the charge on the large sphere will be redistributed. In fact, for small enough separation, the part of the large sphere which is nearest to the small sphere will become charged by induction with charge of sign opposite to that originally on the spheres, and attraction will result. The small sphere is considered so small that even if its charge moves around a bit none of the forces is changed — its charge simply can't move very far from its center. It is considered in this discussion to be a point charge. Three special cases are shown below.

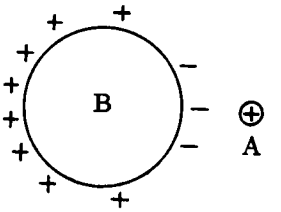
(1) Large separation: repulsion the same as between point charges.



(2) Intermediate separation: repulsion less than between point charges, because the charge on the large sphere has been effectively pushed farther away.



(3) Very close: a negative charge has been induced on the side of the large sphere. Compared to the positive charge, this negative charge is so close to the small sphere that attraction results.



(a) At 5 cm and 3 cm we measure, on Figure 27–30, ordinates of 0.25 cm and 0.67 cm respectively. Thus the ratio

$$\frac{F\text{ (at 3 cm)}}{F\text{ (at 5 cm)}} = \frac{0.67}{0.25} = \underline{2.7}.$$

The force increases by a factor of 2.7 when the separation is decreased from 5 cm to 3 cm. Had the force varied as the inverse square we would have expected a ratio

$$\frac{F\text{ (at 3 cm)}}{F\text{ (at 5 cm)}} = \frac{5^2}{3^2} = 2.8.$$

In this region, then, the force is increasing much as it would if the charges were point charges.

(b) As the spheres move closer, at 2 cm the force of repulsion between them is rising. At about 1.4 cm the force of repulsion begins to fall rapidly as a result of the induction described in (2) and (3) above. At a separation of 1.2 cm

the attraction due to inductive effects equals the repulsion of the two net charges. At this separation, the net force is zero, as shown on the graph.

The graph in the text ends at a separation of 1.2 cm. As the spheres approach still more closely, the force reverses sign and becomes more strongly one of attraction. In the idealization that A is a point charge, the attractive force would become infinite. The actual attracting force would stop rising as the spheres touch. After they touch, charge will flow between them, and the character of the force changes radically. The students need only understand the role of the two effects: the repulsion of the net charges vs. the attraction due to induction.

PROBLEM 8* (See Short Answers.)
PROBLEM 9* (See Short Answers.)
PROBLEM 10* (See Short Answers.)
PROBLEM 11* (See Short Answers.)
PROBLEM 12

If you continue the experiment shown in Fig. 27-18, what sphere velocities will you expect to get in the situations in Fig. 27-31?

The terminal velocity of the sphere in each case is proportional to the driving force (the vector sum of the electrical and gravitational forces) on the sphere.

PROBLEM 13

Suppose we measure the speed of the plastic spheres between the plates of a micro-micro-balance and find it is three halves of their speed under gravity alone. The weight of a sphere is 2.8×10^{-14} newton.

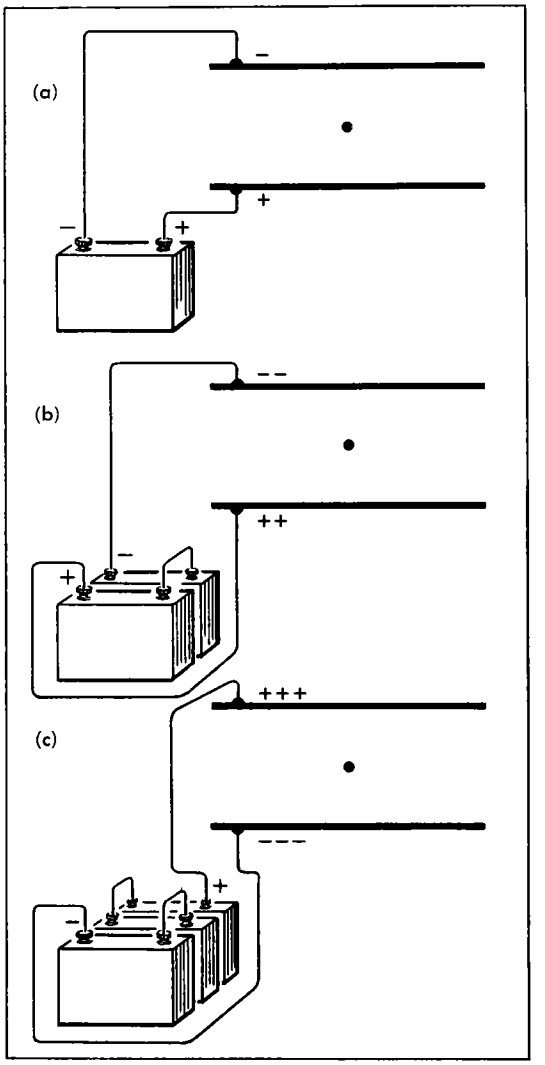
(a) What is the electrical force on them if they are moving upward?

(b) If they are moving downward?

This problem reviews the method of measuring electrical forces used in the Millikan experiment. The driving force resulting from the electrical force and the gravitational force causes the plastic sphere to acquire a velocity in the direction of the driving force and proportional to it.

(a) If the sphere moves upward with a speed $\frac{3}{2}$ that of its fall under gravity alone, the driving force is upward and of magnitude $\frac{3}{2} \times$ weight $= 4.2 \times 10^{-14}$ newton. This requires that the electrical force be *upward*, and of magnitude 4.2×10^{-14} newton $+ 2.8 \times 10^{-14}$ newton $= \underline{7.0 \times 10^{-14} \text{ newton}}$.

(b) If the sphere moves downward at a speed $\frac{3}{2}$ that of its fall under gravity, the added electrical force must be *downward*, and of magnitude $(\frac{3}{2} - 1)$ (weight) $= \frac{1}{2}(2.8 \times 10^{-14}$ nt) $= \underline{1.4 \times 10^{-14} \text{ newton}}$.



PROBLEM 14

There is a relationship between the charge on two parallel plates and the force they exert on a charged object between them. There is also a relationship between the number of batteries connected and the force on a charged object between the plates. What is the relationship between the number of batteries and the amount of charge the batteries put on the plates?

The force exerted on a charged object between two parallel plates is directly proportional to the charge on the plates. It is also directly proportional to the number of identical batteries connected in series to the plates. Hence, the charge deposited by the batteries must be directly proportional to the number of batteries.

PROBLEM 15

How would increasing the size of the plastic spheres used in a Millikan experiment affect their terminal velocity?

We assume that the size is increased by making a larger sphere out of the same material, so that the mass is increased at the same time. The students should realize that the tiny spheres in the Millikan experiment fall much more slowly than does a macroscopic plastic sphere.

Also, the clusters of Millikan spheres fall more quickly than do single spheres. Hence, terminal velocity seems to increase with size.

PROBLEM 16* (See Short Answers.)

PROBLEM 17

Refer to Fig. 27-22(b).

(a) Without moving the plates, how can we increase the force on the elementary charge between the plates which are a distance $2d$ apart so that it will equal the force on the plates which are a distance d apart?

(b) The plates which are a distance d apart are insulated so that when they are disconnected from their battery the number of charges on the plates remains constant. When the plates are disconnected and then separated to a distance $3d$, it is found that the force on the elementary charge has not changed. If they are reconnected to batteries, how many batteries would have to be connected in series with these plates so that force will not change?

(a) Connecting two batteries in series to the plates which are $2d$ apart will make the force the same as on the plates which are d apart.

(b) When the plates are connected to batteries, three batteries are needed at separation $3d$ to maintain the same force that one battery maintains at separation d.

The explanation of why the force exerted on the elementary charge does not change with distance when the plates are insulated and disconnected from the battery is given in the box on page 491.

PROBLEM 18

Only one of several identical metal spheres is charged. The charged sphere A experiences a force $F = 1.0 \times 10^{-4}$ newton when placed midway between certain charged parallel plates.

(a) Sphere A is then touched to one of the uncharged spheres (B). What force does A now experience when placed between the same plates?

(b) A is touched to another of the uncharged spheres (C). What force does A now experience?

(c) What force is exerted on sphere B when it is between the plates? On sphere C?

This problem requires an application of two basic ideas.

1. The force which acts on a charged object placed between charged parallel plates is proportional to the charge on the object.

2. Charge is conserved, so that if a charged sphere is connected to an identical uncharged sphere, each will end up with one half of the initial charge (in the absence of induction effects by other charged bodies which could make the sharing unequal).

(a) When the charged conductor A is touched to an identical uncharged conductor B, each takes one half the initial charge on A. The force on A between the charted plates is half as large as initially, i.e., 5.0×10^{-5} newton.

(b) If A is touched to another identical uncharged sphere C, its charge is again divided in half, and the force is again reduced in half to $\frac{1}{4} \times 1.0 \times 10^{-4}$ newton $= 2.5 \times 10^{-5}$ newton.

(c) Since sphere B has the same charge as sphere A [in part (a)], it will be subject to the same force, 5.0×10^{-5} newton. Sphere C has the same charge as sphere A [in part (b)]. Hence it will be subject to a force 2.5×10^{-5} newton.

PROBLEM 19

Two large metal plates are separated by 0.10 m. They are connected to the terminals of a battery. A small charged ball halfway between them experiences an electric force of 3×10^{-4} newton. The plates are now moved apart until the separation is 0.15 m. The same battery is connected to them.

(a) What force now acts on the ball?

(b) If we add two more identical batteries in series at the new separation, what is the force on the ball?

The electric force field between parallel plates varies directly with the number of batteries (in series) and inversely with the separation.

(a) The separation is increased by a factor of $\frac{3}{2}$ so that the force is reduced by a factor of $\frac{2}{3}$:

$$\tfrac{2}{3} \times 3 \times 10^{-4} = 2 \times 10^{-4} \text{ newton}.$$

(b) The number of batteries is tripled, resulting in a threefold increase in the force to $3 \times 2 \times 10^{-4} = 6 \times 10^{-4}$ newton.

PROBLEM 20* (See Short Answers.)

PROBLEM 21

A plastic sphere of mass 3.06×10^{-15} kg is in an electric force field which exerts an upward force of 1.00×10^{-14} newton on each positive electric particle. The electric force exerted on the sphere is sufficient to balance the force of gravity on it.

(a) What is the excess of electric particles on the sphere?

Suppose that each molecule has a mass of 3.00×10^{-26} kg.

(b) How many molecules are there in the sphere?

(c) What fraction of the molecules in the sphere have lost or gained an elementary charge?

This problem emphasizes that in macroscopic charged bodies only an exceedingly small fraction of the atoms or molecules have lost or paired a charge.

(a) Since the sphere is balanced, the electric force must be equal in magnitude to the force of gravity ($mg = 3.06 \times 10^{-15}$ kg $\times$ 9.8 m/sec^2 $= 3.0 \times 10^{-14}$ nt). The force on each elementary charge is 1.00×10^{-14} newton, so there is an excess of 3 positive electric particles.

(b) The sphere, even though quite small compared to familiar objects, still contains an enormous number of molecules:

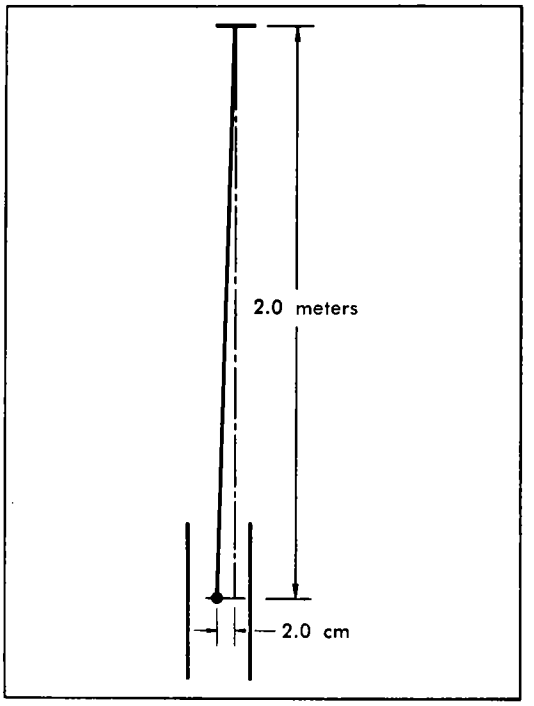
mass of sphere = (no. of molecules) $\times$ (mass of one molecule)

3.06×10^{-15} kg = (no. of molecules) $\times$ $(3.00 \times 10^{-26}$ kg).

Number of molecules = 1.02×10^{11}.

(c) The fraction of molecules which have gained or lost an elementary charge is

$$\frac{3}{1.02 \times 10^{11}} = 2.94 \times 10^{-11},$$ a rather small fraction!



PROBLEM 22

A sphere of mass 4.5×10^{-3} kg is hanging on a string 2.0 meters long between two oppositely charged parallel plates as shown in Fig. 27-32. At equilibrium the ball has been pulled 2.0 cm from its original position.

(a) What is the magnitude of the electric force on the sphere?

The electric force on each elementary charge between the plates is 3.0×10^{-14} newton.

(b) What is the excess of electric particles on the ball?

This problem involves the quantitative evaluation of a moderately large charge on an electrified object by measurement of the electrical force exerted on it by a set of fixed charges, i.e., the charges on parallel plates, the force on a unit charge being known. The electrical force on the body is measured by comparing it with its weight in a kind of balance.

$AB = 0.020$ m

$AC = 2.0$ m

$BC = \sqrt{(2.0)^2 - (0.02)^2} \approx 2.0$ m, as shown in the figure.

(a) By similar triangles

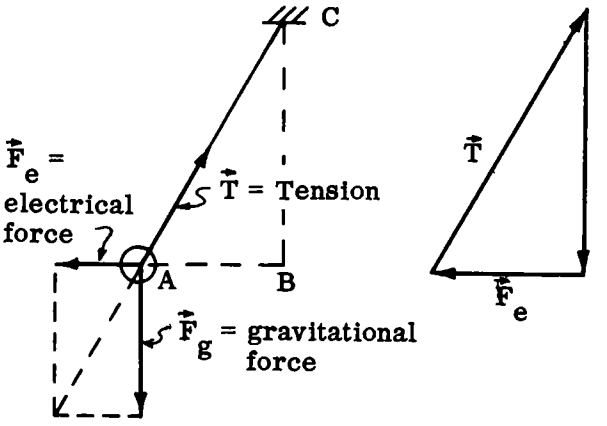
$$\frac{F_e}{F_g} = \frac{AB}{BC} = \frac{0.020}{2.0} = 0.01.$$

$F_g = mg = 4.5 \times 10^{-3}$ kg $\times$ 9.8 m/sec^2

$= 4.4 \times 10^{-2}$ newton.

$F_e = 0.01(4.4 \times 10^{-2}$ nt) = 4.4×10^{-4} nt.

The use of a pendulum balance is a simple and neat way of measuring a force which is rather small in comparison to available weights.

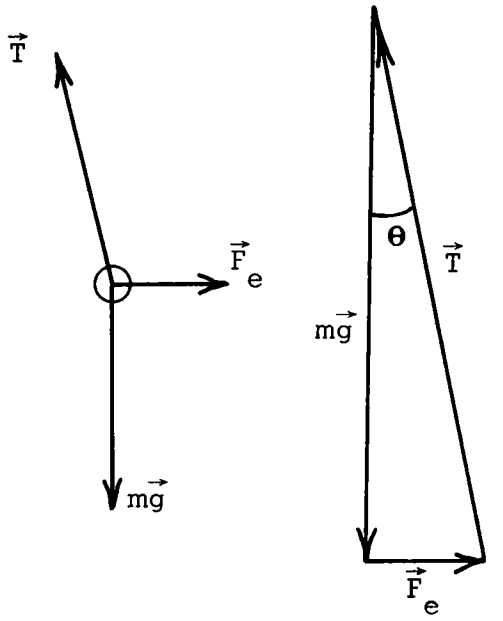


(b) Given that the electrical force on an elementary charge is 3.0×10^{-14} newton, the observed force here is $4.4 \times 10^{-4}/3.0 \times 10^{-14}$ $= 1.5 \times 10^{10}$ times larger. Hence, there is an excess of about 1.5×10^{10} elementary charges on the ball.

PROBLEM 23

In an experiment two small conducting spheres (120 milligrams each) were suspended on two fine insulating fibers 82 cm long. When equal charges were placed on the two spheres, they separated and came to rest with a distance of 10 cm between centers. What was the number of elementary charges on each sphere?

When each sphere remains at rest, the net force on it is zero. The vectors representing the three forces — gravity, tension, and electric repulsion — must form a closed triangle.



The number of elementary charges, q, on each sphere is given by

$$F_e = \frac{kq^2}{r^2}, \quad \text{or} \quad q = r\sqrt{F_e/k}.$$

From the force diagram, $F_e/mg = \tan\theta$ and $F_e = mg \tan\theta$. From the displacement diagram, $\sin\theta = \frac{5}{82} = 0.061$. We see that the angle must be very small and $\sin\theta \approx \theta$ (in radians) $\approx \tan\theta$. Thus,

$$q = r\sqrt{mg \tan\theta/k}$$

$$= 0.10\ \text{m}\sqrt{\frac{(1.2 \times 10^{-4}\ \text{gm})(9.8\ \text{m/sec}^2)}{(6.1 \times 10^{-2})} \Big/ 2.3 \times 10^{-28}\ \text{nt-m}^2/(\text{elem ch})^2}$$

$$= \underline{5.6 \times 10^{10}\ \text{elementary charges.}}$$

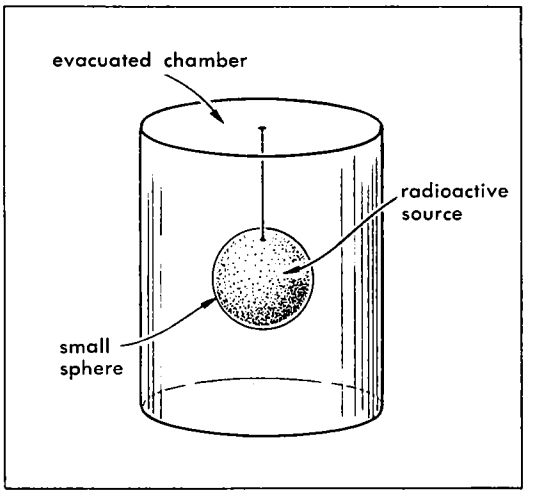
PROBLEM 24

A thin layer of radioactive material emitting beta particles is painted on a small conducting sphere. The sphere is then hung on an insulating thread in an evacuated chamber (Fig. 27-33).

After 50 days we measure the charge on the sphere and find it to be 4.32×10^{10} positive elementary charges. (The sphere has been kept in an evacuated chamber, so very few charges have leaked off.) In a separate experiment with a Geiger counter we find beta particles shooting out at the average rate of 1.00×10^4 per second.

(a) What is the charge of one beta particle?

(b) What do you think beta particles may be?



As the sphere is charged positively, the beta particles must carry a negative charge.

(a) The number of beta particles

$$= \text{rate of emission} \times \text{time}$$

$$= 1.00 \times 10^4 \frac{\text{beta particles}}{\text{sec}} \times \left(50\ \text{days} \times \frac{24\ \text{hours}}{\text{day}} \times \frac{3600\ \text{sec}}{\text{hour}}\right)$$

$$= 4.32 \times 10^{10}\ \text{beta particles emitted in 50 days.}$$

The beta particles carried off a total of 4.32×10^{10} negative elementary charges. Therefore, each beta particle must carry <u>one negative elementary charge</u>, just as an electron does.

(b) Before we jump to a conclusion identifying beta particles with electrons, other properties (e.g., mass) will have to be compared, but this experiment gives us one reason to entertain this hypothesis.

chapter
28

Energy and Motion of Charges in Electric Fields

Chapter 28 considers the kinetic and potential energies of individual charged particles in electric fields, and introduces the concepts of current, EMF, and potential difference. Throughout, the major concern is with the motion of elementary charges.

Batteries in this chapter are essentially "black boxes" which furnish a constant amount of energy per unit charge. Our atomistic approach tells us something about what must be happening in the way of charge-exchange reactions within the cell, but we are not specifically concerned with the chemistry of the cells or with the design of practical batteries.

CHAPTER SUMMARY

SECTION 1. When a charged particle moves in a constant electric force field, the work done by the field causes an equivalent change in the kinetic energy of the particle. The mass of the particles can be determined by measuring the speed achieved by particles of known charge traveling a fixed distance in a constant field. This method is used to determine the mass of the proton (1.7×10^{-27} kg) and the electron (0.91×10^{-30} kg). A simple planetary model of the hydrogen atom is considered (with the proton for the sun, and the electron for the planet).

SECTIONS 2 AND 3. The electric current in a circuit is seen to be comprised of the total number of positive charges passing per unit time in one direction plus the total number of negative charges passing per unit time in the opposite direction. The total steady current must be the same at any point in a closed circuit. The number of elementary charges flowing can be determined by measurement of the mass transported by ions in a solution.

SECTIONS 4 AND 5. Measurements of the amount of energy transformed into heat when moving charges collide with a barrier lead to verification of the previously assumed fact that the force exerted by a constant electric field on a charged particle is independent of the speed of the particle. Batteries are then seen to be devices which supply a definite amount of energy per elementary charge, generally called the EMF of the battery. (A 270-volt battery supplies 4.34×10^{-17} joule per elementary charge.) If batteries are connected in series, the total EMF is the sum of the individual EMF's. A battery of EMF $\mathcal{E}$, connected to a circuit in which a current I is flowing, supplies energy to the circuit at a rate $I\mathcal{E}$ joules/sec (watts).

SECTION 6. The quantitative definition of the electric field at some point in space is introduced as the force per elementary charge acting on a single positive elementary charge placed at that point. The electrical potential energy of two charges separated by a distance r is shown, by analogy with the gravitational field, to be

$$U_e = k\frac{q_1 q_2}{r}.$$

The electric potential drop between two points A and B is defined as the energy per elementary charge that the field transfers to one positive elementary charge as it moves from A to B. This potential drop is shown to be independent of the path followed by the charge from A to B.

SECTION 7. The EMF of a battery derives from the chemical energy released in chemical reactions between the constituents of the cell. "Practical" units of EMF, charge and current, are introduced: 1 volt = 1.6×10^{-19} joule/elem chg; 1 coulomb = 6.25×10^{18} elem chg; and 1 ampere = 1 coulomb/sec. Units of energy are: 1 volt × elem chg = 1.6×10^{-19} joule = 1 electron-volt (ev); 1 volt × 1 coulomb = 1 joule.

SECTION 8 provides a short, qualitative summary and indicates some of the questions that will be considered in the work ahead.

SCHEDULING CHAPTER 28

This chapter presents very basic material and will require full development through class discussion and problem solving.

If you are pressed for time you should consult the scheduling suggestions in the Introduction to Part IV of the Guide.

Chapter 28	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTIONS	CLASS PERIODS	LAB PERIODS	EXPERIMENTS	CLASS PERIODS	LAB PERIODS	EXPERIMENTS
1	1	0	—	2	0	—
2, [3]	1	0	—	1	1	IV-7
4, [5], 6 [7], [8]	2	0	—	2	0	—

The table above suggests possible schedules for presenting this chapter, consistent with the schedules outlined in the Introduction to Part IV. Sections which may be deemphasized without loss in continuity are enclosed with brackets [ ].

RELATED MATERIALS FOR CHAPTER 28

LABORATORY. Experiment IV-7, *The Charge Carried by Ions in Solution*, involves a quantitative electrolysis of dilute sulfuric acid, using copper electrodes. This experiment should follow Section 3. See the Lab Notes for suggestions.

HOME, DESK, AND LAB. See HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILMS. "Counting Electrical Charges in Motion," by James S. Strickland of PSSC, shows how an electrolysis experiment enables one to compute the number of elementary charges passing through a circuit in a given time. An ammeter in the circuit is calibrated. The random nature of the motion of elementary charges is demonstrated using very weak currents of only a few charges flowing per second. In contrast to the electrolysis experiment, charges are counted directly one by one as they flow through a circuit. Running time: 23 minutes.

"Elementary Charges and Transfer of Kinetic Energy," by Francis L. Friedman of M.I.T. Electrons are accelerated in a vacuum through a known distance in a known electric field between two charged plates. The separation of the plates and the strength of the electric field are identical to the plate separation and the electric field used in the film, "Millikan Experiment." The experiment checks the predicted energy acquired by the electrons as they traverse the distance between the plates. This is done by measuring the rise in temperature of the anode as a result of a known current flowing for a specific length of time. This predicted total dissipation of energy, when produced by a mechanical source, gives the same temperature rise in an identical anode. The elementary charges in an electrolysis experiment are shown to be identical to those in a Millikan experiment. Running time: $33\frac{1}{2}$ minutes.

"E M F," by Nathaniel Frank of M.I.T., shows that the energy transfers demonstrated in the preceding film are independent of the geometry of the electrodes in the diode. The energy per elementary charge delivered by a battery depends only on the chemical constitution of the battery. Running time: 20 minutes.

28-1 Determining the Mass of the Electron and of the Proton

PURPOSE. To show that, since we can predict the change in kinetic energy (speed) of a charged particle on passing through an electric force field, the mass of a charged particle can be determined by measuring the speed achieved by the particle in traversing a known distance in a known field.

CONTENT. (a) If a particle moves a distance d in the direction of a constant force F, the work done, Fd, is a measure of the change in kinetic energy, $\frac{1}{2}mv^2$, of the particle. If the particle starts from rest, $Fd = \frac{1}{2}mv^2$.

(b) If F and d are known, and we can measure the speed of the particle, we can directly determine the mass of the particle.

(c) The speed of elementary charged particles can be measured by an oscilloscopic technique which determines the time taken for them to traverse a known distance ("time of flight").

(d) Accepting other evidence which shows that each proton and each electron carries exactly one elementary charge, time-of-flight measurements show the proton mass to be 1.7×10^{-27} kg (more accurately 1.67×10^{-27} kg) and the electron mass 0.9×10^{-30} kg (more accurately 0.911×10^{-30} kg).

(e) By comparing these masses with each other and with the known mass of the hydrogen atom, we can construct a planetary model of the hydrogen atom.

EMPHASIS. These ideas are important, but should go over rather easily. The principle of the measurement should be stressed, not the details. A detailed discussion of the planetary model of the hydrogen atom is not necessary at this point; this subject will be considered in Chapters 32 and 33.

CAUTION. At this initial stage, in order to avoid obscuring the central points, several assumptions which underlie the development given in Section 1 are not stated explicitly. For clarity at this early stage they should not be taken up with the class. Some of them come up a bit later in the course. We mention them here so that you may be fore-armed if questions arise.

(1) We assume that the electric force acting on a charged particle which is moving between two flat metal plates is constant, *independent of the speed of the particle.* This fact is not obvious, but can be verified by experiment. One definitive test is described in Section 4, where the increase in thermal energy of a barrier struck by moving charges is shown to be equal to the predicted kinetic energy of the charges. This is also shown in the film "Elementary Charges and the Transfer of Kinetic Energy."

(2) We assume that energy conservation holds on a microscopic scale, so that the work done on the charged particle does in fact only go into an increase in the particle's kinetic energy. This is justified on the basis of its consistency with *all* the known facts.

(3) We assume that the kinetic energy of the particle is given by the expression $\frac{1}{2}mv^2$. This holds exactly only for particles moving with speeds small compared to the speed of light.

We can see the difficulty by considering the motion of an electron across a larger potential gradient. Suppose that, instead of using a 90-volt battery in this experiment, we had used a 900,000-volt supply. What would be the final speed achieved by the electron? As before, we would write

$$Fd = \tfrac{1}{2}mv^2.$$

Now F would be 10,000 times larger than before, so $Fd = 1.44 \times 10^{-13}$ joule/elem chg. If $m = 0.91 \times 10^{-30}$ kg, we would predict a measured speed of

$$v = \sqrt{2Fd/m} = \sqrt{\frac{2.88 \times 10^{-13}}{0.91 \times 10^{30}}}$$

$$= \sqrt{3.1 \times 10^{17}} = 5.6 \times 10^{8} \text{ m/sec},$$

almost twice the speed of light. Since such a result is inconsistent with the theory of relativity, something must be wrong. In fact, measurements of the speed show that it is a bit less than that of light.

The difficulty lies in our expression for the kinetic energy. According to the theory of relativity, the kinetic energy is not simply $\frac{1}{2}mv^2$, but is $c^2(m - m_0)$, where c is the speed of light (3.00×10^8 m/sec), m_0 is the "rest mass" of the particle (here 0.911×10^{-30} kg), and m is the relativistic mass $(m_0/\sqrt{1 - v^2/c^2})$. Solving for v, we find $v = 2.8 \times 10^8$ m/sec, close to, but still less than, the speed of light. (At low speeds, of course, the relativistic expression for the relation of kinetic energy and speed agrees with the Newtonian expression.)

DEVELOPMENT. The time-of-flight experiment to measure the masses of elementary particles is completely analogous to the methods used in Chapters 23–25, where we were concerned with the energies of macroscopic objects. If we know the force acting on a body and the distance the body moves in the direction of the force, we know the work done on the body. If energy is conserved, this work must produce an equal change in the energy of the body — in this case, in the body's kinetic energy, since there is no other place for it to go. The situation is exactly the same as that for a Dry Ice puck or any other object accelerated by a constant force. If we know the force, the distance, and the mass, we can deduce the final speed; if we know the force, the distance, and the final speed, we can deduce the mass.

DEMONSTRATION: A TIME-OF-FLIGHT MEASUREMENT. A demonstration similar to this described in Section 28–1. The cathode-ray oscilloscope is set at approximately 1 sweep per second. (On a 5-inch tube the spot will move about 10 cm per second.) A steel ball is then rolled down an inclined trough made of two meter sticks and passes over three pairs of aluminum-foil strips, making electric contact between the two halves of each pair [Fig. (a)]. When the first gap is closed, the oscilloscope sweep is triggered. Shortly after the sweep has started, the rolling ball closes a second gap which momentarily connects a voltage to the vertical deflection plates of the oscilloscope, giving a sudden vertical deflection of the spot as it sweeps across the scope. This marks the instant of passage of the ball across the second gap. The ball closes a third gap at a later instant, again deflecting the spot.

From a knowledge of the distance between the second and third gaps, and the speed of the spot, the average speed of the ball can be computed from the distance between the vertical deflections or spikes on the screen.

The capacitor-resistor combination across the oscilloscope deflection circuit input gives the vertical spike a steep side on the left and a sloping side on the right, like the spikes shown in Fig. 28–7. When the ball initially closes the gap, the capacitor charges almost instantly from the battery, and the oscilloscope spot shoots upward very steeply. As the ball leaves the gap, the charging voltage to the capacitor ceases, whereupon the capacitor discharges through the resistor, causing the oscilloscope spot to move downward in a sloping curve to the base line, rather than dropping instantly as it would if there were no capacitor in the circuit.

The time constant for the capacitor-resistor combination is about 20 milliseconds. This will make the base of the spike appear to be about 1 centimeter long since, after about 5 time constants, the voltage falls

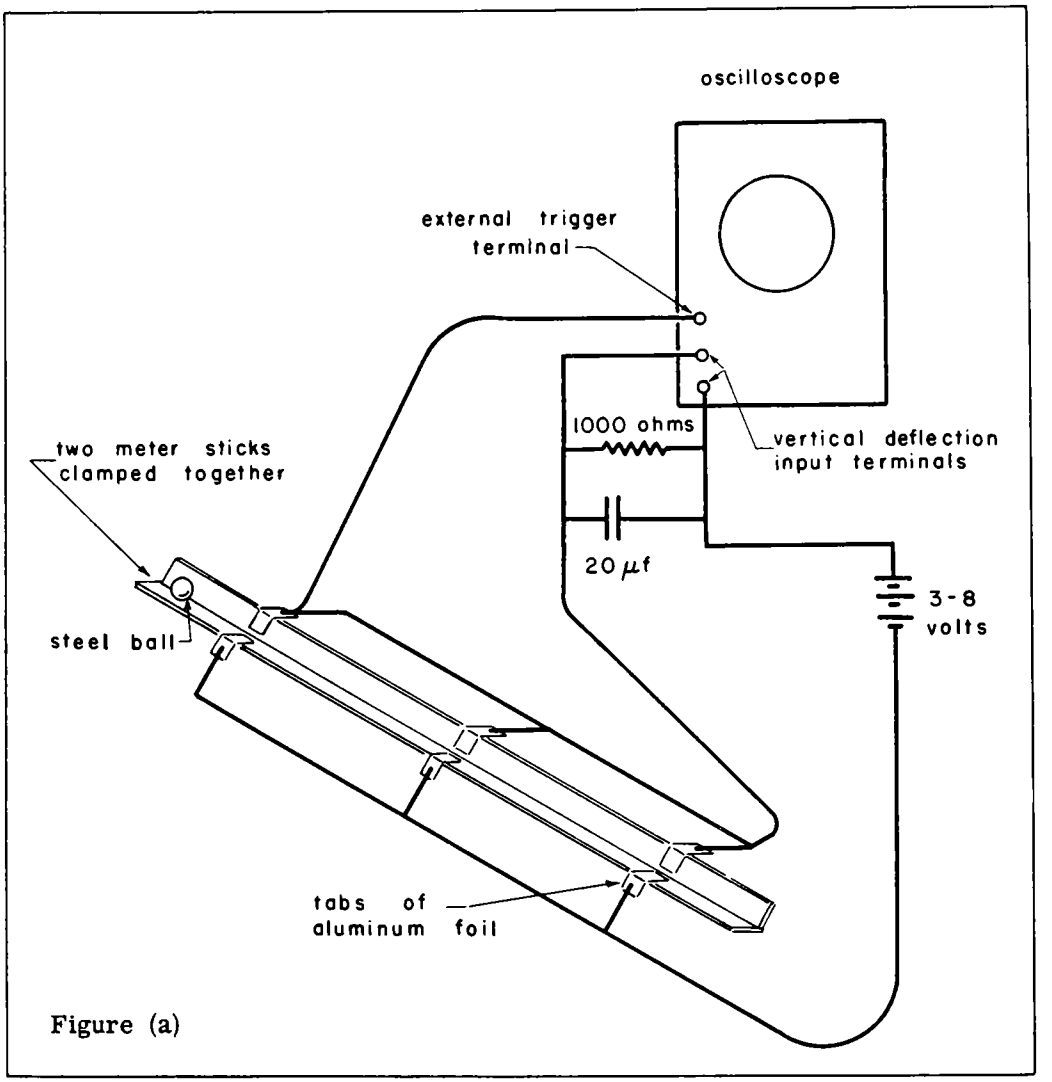


Figure (a)

nearly to zero. The spike can be made sharper by reducing the time constant. Use either a smaller capacitor, a smaller resistor, or both. If the spike is made too sharp, the spot will rise and fall so rapidly that it will leave a very faint trace which is not visible to the class.

COMMENT. The potential difference used to accelerate the ions or electrons determines the kinetic energy per elementary charge; it follows that what we measure is the mass per elementary charge. In fact, this is all that is determined in any experiment in which the motion of charged particles in an electric field (or in a magnetic field or both) is observed. All such experiments effectively measure the acceleration in a situation where we know the force per elementary charge but not the force on the particle. To find the mass itself we need to know the charge from an entirely different kind of measurement. For particles as abundant as electrons and protons this can be done directly by counting the particles and measuring the charge transported by a beam of particles (Chapter 27). For rarer particles, or for individual particles, one must resort to less direct methods. For instance, the densities of the bubbles in a bubble chamber track and of developed grains in the track in a photographic emulsion depend on the charge of the particle which makes the track (the density depends also on the velocity). Indirect arguments such as charge conservation can also be used. For example, if we observe a hydrogen atom to break up into an electron and a proton, we infer that the electron and proton have equal and opposite charge.

The results of the mass measurement on the electron, the proton, and the hydrogen atom suggest that a hydrogen atom is made up of a proton and an electron. The analogy to a sun and a much lighter planet is obvious, particularly since there is a similar, central attractive force. This model should be easily grasped and requires no discussion at this point.

The measurement of the velocity of an atomic or subatomic particle by a time-of-flight method was preceded historically by methods involving the effect of a magnetic field. This is discussed in Chapter 30 and

should not be discussed now, whether or not you plan to use Chapter 30. It may be interesting to know that time-of-flight measurements are used frequently in nuclear physics. In the discovery of negative protons, for instance, these particles were distinguished from other negative particles, and their mass determined, by observing the time of flight over a flight path of about 30 feet. This determination of speed is identical to the one we have used. Another measurement (this happened to be deflection in a magnetic field) had to be made in order to determine the mass per unit charge, and it was necessary to establish that the particles in question carried one and only one elementary charge.

QUIZ PROBLEMS

(1) A body is accelerated from rest by a constant force of 2.0 newtons, acting for a distance of 2.5 meters.

(a) If the body has a mass of 3.0 kg, what is its final speed? (1.8 m/sec.)

(b) How long does it take for the body to cover the 2.5-meter distance? (2.8 sec.)

(c) If another body, accelerated by the same force for the same distance, achieves a final speed of 120 m/sec, what is the mass of this second body? (6.9×10^{-4} kg.)

(2) In the experiment described in the text in Figure 28–1, the distance between the plates is changed from 9.3 mm to 18.6 mm. The 90-volt battery is still connected. What is the final speed achieved by a proton in crossing between the plates? (1.31×10^5 m/sec.); by an electron? (5.6×10^6 m/sec.); by a singly-charged oxygen ion (mass 16 amu)? (0.33×10^5 m/sec.); by a doubly-charged oxygen ion? (0.47×10^5 m/sec.)

28–2 Electric Current

28–3 Electrolytic Measurement of Electric Currents

PURPOSE. To provide a quantitative definition of an electric current in terms of moving elementary charges, and to show how currents can be measured by electrolysis.

CONTENT. (a) Electric currents consist of elementary charges in motion. The direction of the current is defined as the direction of motion of positive charges.

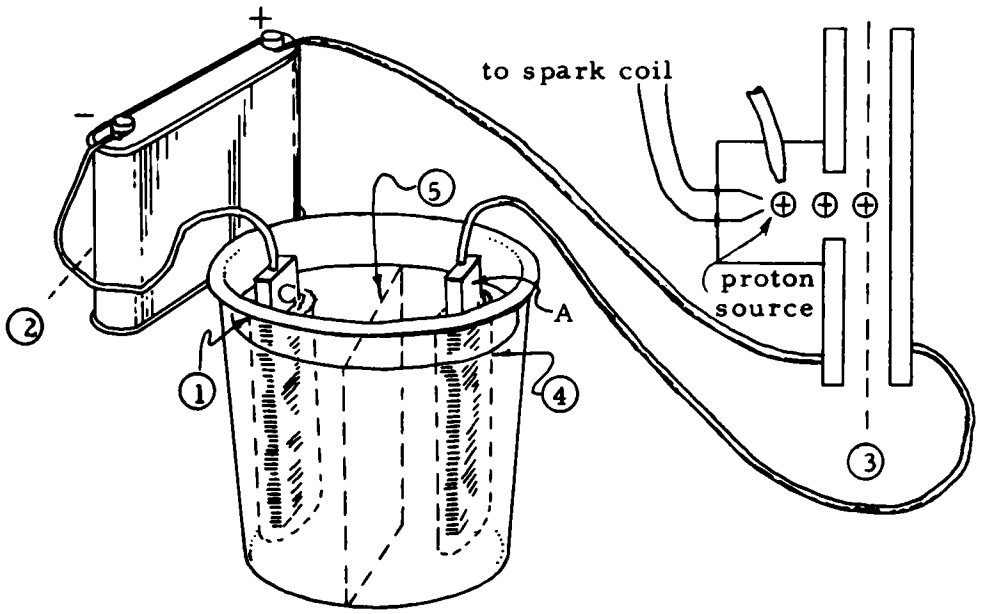
(b) The total current is the sum of the number of positive charges passing per second in one direction plus the number of negative charges passing per second in the opposite direction.

(c) In a closed circuit under steady state conditions, no net charge accumulates at any point in the circuit. This implies that the total current must be the same at any point in the circuit.

(d) The mass of an element deposited in an electrolytic cell is shown to be directly proportional to the amount of charge transported across the cell. Quantitative measurements show that the charges carried by ions in solution are small whole-number multiples of the elementary charge. Thus a measurement of the mass deposited per second in an electrolytic cell determines the current in the cell.

EMPHASIS. Students should grasp the concept of an electric current as comprised of moving elementary charges. However, unless this course is integrated with one in chemistry, the details of the electrolytic reactions need not be stressed. If time is short, Section 28–3 can be skipped or covered entirely in conjunction with Experiment IV–7.

DEVELOPMENT. It probably will be useful to go through a blackboard discussion of the relation between the motion of charges and the current in various



parts of a circuit. In doing this it will be wise to use a circuit in which the current is carried by different charges in different parts of the circuit. For example, one can insert the apparatus of Figure 28–1 into the circuit of Figure 28–8, as illustrated in the sketch. The polarity of the battery in our sketch is reversed from that in Figure 28–8. (Note that no charge is injected into the circuit by the spark coil.) Assuming that the ions in the electrolytic cells are all singly charged, and assuming that a current of, say, 10^6 elementary charges per second flows around the circuit clockwise, at what rate do various charged particles cross surfaces 1, 2, 3, 4, and 5? The answers are:

(1) 10^6 positive ions, each carrying one elementary charge, cross from the liquid to the cathode each second. The current flows from right to left.

(2) 10^6 electrons per second cross from left to right, opposite to the direction of the current.

(3) 10^6 protons per second cross from the left to the right, in the same direction as the current.

(4) 10^6 negative ions cross this surface per second from the liquid to the anode, opposite to the direction of the current.

(5) Negative ions cross the surface from left to right and positive ions from right to left. All that can be said is that the rate at which negative ions cross and the rate at which positive ions cross *add* to 10^6 per second. This could be, for example, 400,000 negative ions and 600,000 positive ions per second or 700,000 and 300,000 or any other combination. The direction of the current coincides with the direction of motion of the positive ions. (In the above, we have assumed that the ions present are singly charged.)

* * *

Someone usually raises the question of the sign convention in currents. For better or worse, a long time ago it was decided to call the charge of a proton positive and that of an electron negative. The only reasonable convention regarding currents is one that a current flowing into a region shall be defined as positive if it increases the net positive charge (or decreases the net negative charge). This implies that a current carried by negative charges (such as electrons) is opposite to the motion of these charges.

Nature has apparently provided an equal number of positive and negative electric charges in the universe. Under some conditions (e.g., ionic conduction in solid salts), electric currents are carried predominantly by positive charges. Under other conditions (e.g., electronic conduction in metals), the current is carried by negative charges. In some cases (e.g., in p-type semiconductors), the current is actually carried by negative charges, but it is more convenient to picture the conduction process in terms of positive charge carriers ("holes"). In any event, with the spectrum of electrical phenomena which must be described, there is nothing to be gained by reversing our present arbitrary definitions of + and −, although there have been frequent proposals to do just this (usually from those who are concerned principally with electronic conduction in metals and vacuum tubes).

COMMENT. The quantitative study of electrolytic reactions such as that in Figure 28–11 enables us to determine the charge on ions in solution, and to show that all charges are integral multiples of the same elementary charge (see Problem 18). This is the main point to be made in discussing electrolysis. It is not important in this course that students know, for example, that silver exists in solution as Ag^+ and copper as Cu^{++}. They should know that, by reference to a standard electrode, the charge carried by any ion can be determined. If this course is integrated with one in chemistry, you will undoubtedly wish to expand considerably on this material, correlating the results of such experiments with known chemical valences.

QUIZ PROBLEM

An electric current is passed for 1 minute through hydrochloric acid (HCl). At the negative electrode, a total of 20 milligrams of gas is collected during this time. What is the average current which flowed through the cell? (2×10^{20} elementary charges/sec.)

28-4 Experimental Checks on Energy Transfer, Electric Forces, and Elementary Charges

PURPOSE. To show that the assumptions made previously can be verified (e.g., that all elementary charges are the same, that the force on a moving charge is independent of its speed) by directly measuring the kinetic energy carried by a beam of moving charges.

EMPHASIS. While it is important to see that these assumptions have an experimental base, extensive class discussion should not be necessary.

DEVELOPMENT. This section can be discussed in conjunction with Problem 18 in HDL. It is only by a quantitative comparison of the results of electrolytic measurements and measurements on accelerated charges that we can assert that all elementary charges are in fact indistinguishable except for sign.

Be sure to consider the similarity between this heating experiment and that in Section 5. See the Guide for the next section.

28-5 EMF and the Energy Supplied by a Battery

PURPOSE. To define the EMF of a battery as the energy supplied per elementary charge.

CONTENT. (a) A battery is basically a device for converting stored chemical energy into electrical energy.

(b) A battery supplies a fixed amount of energy per elementary charge to a circuit; for example, a 270-volt battery supplies 4.34×10^{-17} joule/elementary charge.

(c) The energy supplied by a battery per elementary charge (defined as the EMF of the battery) in an ion beam accelerated from one parallel metal plate to another, is independent of the distance between the plates, the area of the plates, and the amount of current flowing.

(d) If batteries are connected in series, the total EMF available to the circuit is the sum of the EMF's of the individual batteries.

(e) A battery of EMF, $\mathcal{E}$, through which a charge q flows, supplies a total energy $q\mathcal{E}$; if the current (charge per second) flowing is I, the battery supplies power (energy per second) at a rate $I\mathcal{E}$. If I is measured in elem chgs/sec, and $\mathcal{E}$ in joules/elem chg, $I\mathcal{E}$ is in joules/sec (watts).

EMPHASIS. These ideas are important, but should go rather easily. The section can be treated entirely as a reading assignment. Unless this course is integrated with chemistry, it will be wise not to discuss the detailed chemical reactions which occur in actual batteries. It is sufficient for our present purposes to consider batteries simply as "black boxes" which supply a fixed EMF.

It will be helpful to point out the similarity between the basic experiment of Section 4 and that shown in Figure 28–12 of this section. In the first, electrons are allowed to gain all the kinetic energy supplied them by a battery hooked between two plates, and then allowed to smash into a metal plate, and the energy delivered is measured by the temperature rise in the plate. In Figure 28–12 the battery again supplies energy to electrons, but this time instead of accelerating across a vacuum and hitting a plate, they move through a metal resistor, transferring any energy gained to the metal atoms in collisions (see Section 29–4 C). This leads to a temperature rise which is measured. In both cases the battery was the ultimate energy source; the energy was given to the electrons; they were slowed down by a material which warmed up. The difference was only in the behavior of the electrons — and the logical use to which the results were put.

QUIZ PROBLEMS

(1) One 90-volt and two 45-volt batteries are connected in series. What are the maximum and minimum values of the EMF of the combinations? (2.9×10^{-17} joule/elem chg; 0.)

(2) A 270-volt battery supplies a constant current of 10^{17} elementary charges/second to a circuit for 1 hour. By how much has the chemical energy of the battery changed? (Decreased by 1.56×10^{4} joules.) This neglects heat absorption and release which always accompanies such a process.

28-6 Electric Field and Electric Potential Difference

PURPOSE. To introduce the concepts of electric field and electric potential difference in terms of interactions between elementary charged particles.

CONTENT. (a) The electric field vector $\vec{\mathbf{E}}$ is defined as the force per elementary positive charge; the electric force $\vec{\mathbf{F}}$ on a body carrying q elementary charges is $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$.

(b) From Coulomb's law, the electric field at a distance x from q_1 elementary charges grouped at a point is $\vec{\mathbf{E}} = kq_1/x^2$, pointing radially away from q_1.

(c) Two point charges, q_1 and q_2, separated by a distance r, have an electric potential energy $V_e = kq_1q_2/r$.

(d) The electric potential energy per elementary charge is defined as the electric potential V; for a point charge q_1, the potential $V = kq_1/r$. This is equal to the kinetic energy which would be gained by an elementary charge in going from r to infinity.

(e) The change in electric potential energy when an elementary charge is moved between two points is called the potential difference between those points.

(f) The potential difference depends only on the initial and final positions of the elementary charge, and is independent of the path taken by the charge in moving between the two points.

EMPHASIS. The concept of electric potential energy is extremely important and should be developed fully through class discussion and homework. The electric field is just a specific case of the electric force vector (now simply referred to a unit charge) previously considered, and need not be given special stress at this point. The three-dimensioned representation of the electric potential (Figure 28–14) is important, and will be used in Chapter 32.

DEVELOPMENT. *Electric Field* The idea that the electric force on a charged body is directly proportional to the charge on it has been used before, in the microbalance experiment, in Section 1 of this chapter, and elsewhere. All that should be required here is to recast the situations in terms of the definition of the electric field. For example:

(1) What is the electric field between the accelerating plates in the apparatus shown in Figure 28–1? (1.55×10^{-15} newton per elementary charge, directed from the + to − plate.)

(2) What is the electric field, in the apparatus of Figure 28–2, in the region to the right of the deflecting plates? (0, since the force on an elementary charge is zero in that region.)

(3) What is the electric field at a distance of 10^{-10} m from a proton? (2.3×10^{-8} nt/elem chg, directed away from the proton.) What is the electric field at a distance of 10^{-10} m from an electron? (2.3×11^{-8} nt/elem chg, directed toward the electron.)

Electrical Potential Energy of Two Charges

The idea of electric potential energy should be developed fully. Here, as elsewhere, it is necessary to talk in concrete terms, illustrating the situation in terms of actual motions, pushes and pulls. If the subject is discussed in the abstract, without reference to concrete examples, many students will fail to get a real understanding of the nature and uses of the idea of potential energy.

Two objects with like charges repel each other. If we push them together, we have to do work. What happens to the energy supplied? Is it lost? Suppose one of the objects is held in a fixed position, and the other is projected with some given initial velocity directly toward the first. Assuming no other forces act, the electric force of repulsion slows the moving object down. What happens to the kinetic energy which is lost as the body slows down? Can we get it back? It should be possible for students to supply their own answers, based on their experience with similar situations in which forces depend only on the separation.

From this kind of discussion we come to the conclusion that, for like charges, there is an electric potential energy which increases as the two bodies move closer. How do we calculate the magnitude of the change in potential energy? We must find the work done by the electric force. If the force between the two bodies was constant as they moved closer, we would simply multiply the force by the displacement along the force (i.e., the displacement along the line between the two charges; this is just the change in the distance between the charges). But the force is not constant. How do we find the work when the force is not constant? How did we do this for the force of a spring? We must find the area under the curve of the force $\left(F = k\frac{q_1q_2}{r^2}\right)$ as a function of the distance, r, between the initial and final separations r_1 and r_2. You may not need to elaborate on this procedure if it has already been discussed thoroughly in Part III. In that case, it should be sufficient here to state the result.

If you need to, you can take a specific example, say $q_1 = q_2 = 1$ elementary charge, $r_1 = 1$ meter, $r_2 = 2$ meters; and draw, or ask students to draw the force-distance graph and find an approximate answer for U_e to be compared with the result given by the formula.

Using the above expression, we can compute the force at several separations, and draw the graph of force-distance for this example.

$$\text{At } r = 1 \text{ meter}, F = k\frac{1 \times 1}{1^2} = 1.0\,k.$$

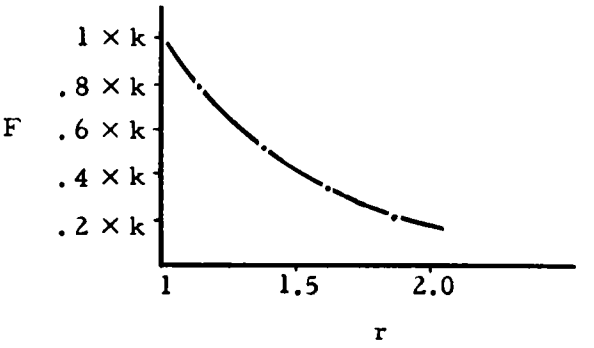
$$\text{At } r = 1.125 \text{ meters}, F = k\frac{1 \times 1}{1.125^2} = 0.79\,k.$$

$$\text{At } r = 1.375 \text{ meters}, F = k\frac{1 \times 1}{1.375^2} = 0.529\,k.$$

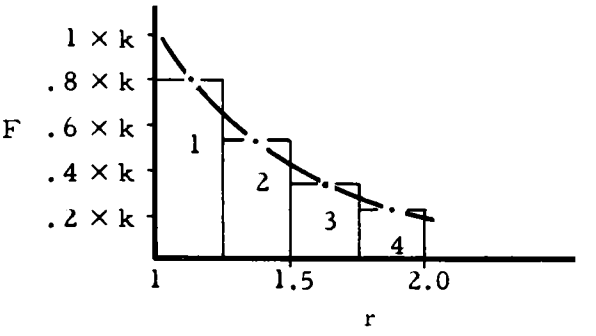
$$\text{At } r = 1.625 \text{ meters}, F = k\frac{1 \times 1}{1.625^2} = 0.378\,k.$$

$$\text{At } r = 1.875 \text{ meters}, F = k\frac{1 \times 1}{1.875^2} = 0.284\,k.$$

$$\text{At } r = 2.0 \text{ meters}, F = k\frac{1 \times 1}{2^2} = 0.25\,k.$$



We can divide the area under the curve into rectangles whose total area approximates the area under the curve. In this example, it is convenient to divide the area into four rectangles.



Now we can compute the work done in each interval and find the total work (the sum of the area of all four rectangles). In each case $W = Fd$.

For rectangle 1, $\Delta W_1 = F\,\Delta r = .79\,k \times .25 = .198\,k.$
For rectangle 2, $\Delta W_2 = F\,\Delta r = .529\,k \times .25 = .132\,k.$
For rectangle 3, $\Delta W_3 = F\,\Delta r = .378\,k \times .25 = .094\,k.$
For rectangle 4, $\Delta W_4 = F\,\Delta r = .284\,k \times .25 = .071\,k.$
Adding $W = .495\,k.$

Using the equation for potential energy with this example, we get

$$\Delta U_e = k\frac{q_1q_2}{r_1} - k\frac{q_1q_2}{r_2}$$

$$= k\frac{1 \times 1}{1} - k\frac{1 \times 1}{2} = 0.500\,k.$$

If the area were divided into smaller and smaller rectangles, the sum of their areas would come closer and closer to 0.500.

We can now answer quantitatively all of the questions we put at the beginning of the discussion, and it is worthwhile to do this explicitly with numerical examples. We can calculate the work required to push the two charges together; it is equal to the increase in electric potential energy. We can calculate the gain in kinetic energy as they are allowed to fly apart; it is equal to the decrease in potential energy, and so on.

One should next go briefly over at least one example involving a plus charge and a minus charge, in which the force between the charges is attractive instead of repulsive. If we go over the same arguments we find that, since work has to be done by outside forces to pull the charges apart, the electric potential energy must increase with increasing separation. However, the algebraic expression for the potential energy decreases in absolute value as the separation becomes greater. Thus, for the case of charges of opposite sign, the potential energy must be *negative* for finite distances of separation.

If the idea of a negative potential energy is hard to swallow, one can point out that it is only changes in potential energy that are important; the value of the potential energy at a given point, by itself, has no physical significance. This can be illustrated with a simple example. We drop a 2-kg object from a height of 10 meters above the ground. What is its kinetic energy just before it hits the ground? We would probably choose ground level as the zero of potential energy. The initial potential energy is then: $mgh_1 = 2 \times 9.0 \times 10 = 196$ joules.* The final potential is zero, so the kinetic energy gained is 196 joules. If we now dig a hole 10 meters deep and drop the object into it from ground level, we have: initial potential energy = 0, final potential energy = $mgh_2 = 2 \times 9.8 \times -10 = -196$ joules. The kinetic energy gained equals the decrease in potential energy. This is $0 - (-196 \text{ joules}) = 196$ joules, as before. The only significance of saying that the potential energy at the bottom of the hole is -196 joules is the fact that this is 196 joules lower than at ground level. It would take 196 joules of work to raise the 2-kg mass to the arbitrarily chosen zero potential energy level.

We can finally note that the equation $U_e = k\frac{q_1q_2}{r}$ gives the correct sign for either like or unlike charges if we write the sign as well as the magnitude of q_1 and q_2. It is better however for the student to reason out the sign by a simple physical argument: is energy put in or taken out when the charges are brought in closer, starting at infinite separation?

QUIZ PROBLEMS

(1) The electric field at a distance of 10^{-10} meters from a point charge q is 6.9×10^{-8} newton per elementary charge, directed toward q.

(a) What is the value of q? (-3 elementary charges.)

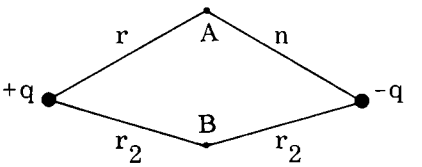
(b) What is the electric field at a distance of 3×10^{-10} m from q? ($\frac{1}{9} \times 6.9 \times 10^{-8}$ nt/elem chg toward q.)

(c) What is the electrical potential at a distance of 10^{-10} m from q? (-6.9×10^{-18} joule/elem chg.)

(d) What is the work which would have to be done to bring a unit elementary *negative* charge from infinity to within 10^{-10} m of q? (6.9×10^{-18} joule.)

(2) Using the information in (1), how much work would be required to move a charge of 3 elementary positive charges from a distance of 10^{-10} m from q to a distance of 3×10^{-10} m from q? (1.38×10^{-17} joule.)

(3)



Points A and B are equidistant from two equal and opposite charges. What is the electric potential at A? (0) How much work would be required to move a charge q from A to B? (0)

28-7 Batteries, Volts, and Amperes

PURPOSE. To introduce the volt, coulomb, and ampere as "practical" electrical units.

CONTENT. (a) In batteries, chemical energy is transformed into electric potential energy; a cell gives a fixed energy per elementary charge, as long as its

---

* Note that in the case of objects moving up and down near the surface, we use $U_g = mgh$ instead of $U_g = -G\frac{mm_{earth}}{r}$.

This is a satisfactory approximation because the change in potential energy between two levels, h_1 and h_2, measured from a point near the earth's surface (or r_1 and r_2 measured from the earth's center) would be:

$$\Delta U_g = mgh_2 - mgh_1, \text{ or}$$

$$\Delta U_g = -G\frac{mm_{earth}}{r_2} + G\frac{mm_{earth}}{r_1}$$

$$= Gmm_{earth}\frac{1}{r_1} - \frac{1}{r_2}$$

$$= Gmm_{earth}\frac{r_2 - r_1}{r_1 r_2}.$$

But $r_2 - r_1 = h_2 - h_1$, and when $r_2 - r_1$ is much smaller than r_1 (i.e., $r_1 \approx r_2$), we can write $r_2 r_1 \approx r_1^2$. Then

$$U_g = Gmm_{earth}\frac{h_2 - h_1}{r_1}.$$

Since

$$g = \frac{Gm_{earth}}{r_1^2}, \quad \Delta U_g = mg(h_2 - h_1).$$

chemical composition and temperature remains constant.

(b) The EMF of batteries is conveniently measured in volts; 1 volt = 1.6×10^{-19} joule/elem chg.

(c) Charges may be measured in coulombs; 1 coulomb = 6.25×10^{18} elem chg.

(d) Current is conveniently measured in amperes; 1 ampere = 1 coulomb/sec.

(e) EMF (volts) × charge (coulombs) = Energy (joules): EMF (volts) × current (amperes) = Power (watts).

(f) The electron-volt (1.6×10^{-19} joule) is a convenient measure of energy on an atomic scale.

EMPHASIS. Students should be familiar with the practical units, but very little class time should be required. This section can be handled as a reading assignment.

DEVELOPMENT. It may be helpful to review some familiar facts. The microbalance experiments show that the energy transferred to an elementary charge which crosses between the plates depends only on the battery connected to the plates, not on the plate separation. We could perform an even more direct measurement by measuring the kinetic energy acquired by the electrons, and their charge. This gives the potential difference directly. If we were to change the separation of the plates or their size and shape, we would always find that the electrons acquired a kinetic energy of 1.44×10^{-17} joule per elementary charge, when a 90-volt battery is used. Each metal electrode is at the same potential as the battery terminal; a 90-volt battery maintains a potential difference of 1.44×10^{-17} joule per elementary charge.

The statement above is an experimental fact. From this it follows that when one positive elementary charge is transferred inside the battery from the negative to the positive terminal it gains an amount of electric potential energy equal to the EMF. A battery supplies electric energy (qV when q elementary charges are transferred). If we connect several batteries in series, the potential differences add. We understand this by thinking of one, or several, elementary charges as being transferred through each battery in turn, each increasing the electric potential energy by an amount which is characteristic of the battery. We get the total energy gain by adding each in turn.

Most students are familiar with the fact that car lights dim when the starter switch is pressed. Apparently, connecting the starter to the car battery changes the potential difference between the battery terminals. Crudely, the EMF of the cell is not changed, but the energy/elem chg dissipated in the outside circuit is no longer such a large fraction of $\mathcal{E}$. The starter is almost a short circuit, and for an ideal short circuit (with + and − terminals connected together) all the energy supplied would have to be dissipated inside the cell (heating it). Then the potential difference between the terminals (parts of one good conductor) would be practically zero. If we connect devices which draw only a moderate current (spark coil, lights, radio), the battery potential is almost equal to the EMF. This difference between $\mathcal{E}$ and the potential difference between the battery terminals is easy to understand after completing the second half of the chapter. It is not an essential point.

The text hints at what goes on in a battery from an atomic point of view. This explains, in the atomic model, why a battery does maintain a definite $\mathcal{E}$. It is strongly urged that you do not elaborate on this point, omitting all detailed chemistry.

28-8 Status Report

PURPOSE. To point out that the results of the previous sections do not describe the forces between two charged particles when both are moving. To summarize and lay out what is to come.

EMPHASIS. Treat as a reading assignment.

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The following table classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1.* 5.6×10^{-17} joule.
2. (a) 5.8×10^{-17} joule; (b) 2.9×10^{-17} joule.
3.* 7.64×10^{-6} sec.
4. 3.0 m/sec.
5. (a) 2 amu; (b) 4 amu; (c) doubly charged helium ions.
6.* $I = 2 \times 10^{16}$ elementary charges/second.
7.* $I = 3.6 \times 10^{16}$ elementary charges/second.
8. (a) Left to right;
   (b) 10^{16} elementary charges/second;
   (c) 0 inside, 10^{16} elementary charges/second outside;
   (d) $(5 \times 10^{15})/n$, where n is the number of elementary charges/ion.
9. Connect meters in series.
10. See discussion on page 28–14.
11.* 4.50×10^{22} atoms of copper.
12. (a) 5.94×10^{18} elementary charges/second.
    (b) See discussion on page 28–14.
13.* 2.9×10^{-17} joule.
14.* $m = 2.7 \times 10^{-26}$ kg.
15. (a) 1.6×10^{-17} joule;
    (b) 1.6×10^{-15} newton/elementary charge;
    (c) See discussion on page 28–15.
16. 1.8×10^{16} ions.
17. One joule.
18. See discussion on page 28–16.
19.* $F = 4.2 \times 10^{-14}$ newton.
20.* 1.7 joules/sec = 1.7 watts.
21.* The potential energy at a distance r is $U = kq$ (1 elementary charge)$/r$. All this will be transferred to the moving charge as kinetic energy at infinity.
22.* $\vec{E} = 9 \times 10^{-18}$ newton/elementary charge, pointing toward the object.
    $V = -5 \times 10^{-18}$ joule/elementary charge.
23. (a) Proton: 1.6×10^{-26} newton, gravitational force;
    1.6×10^{-14} newton, electrical force;
    Electron: 8.9×10^{-30} newton, gravitational force;
    1.6×10^{-14} newton, electrical force.
    (b) The forces remain constant.
    (c) Proton: $+1.6 \times 10^{-28}$ joule, gravitational force;
    $+1.6 \times 10^{-16}$ joule, electrical force;
    Electron: $+8.9 \times 10^{-32}$ joule, gravitational force;
    -1.6×10^{-16} joule, electrical force.
24. See discussion on page 28–17.
25. (a) 2.3×10^{-18} joule total, $\frac{1}{2}$ for each;
    (b) 2.3×10^{-18} joule for one, 0 for the other.
26. See discussion on page 28–17.
27. See discussion on page 28–18.
28.* EMF = 1.9×10^{-18} joule/elementary charge = 12 volts.
29.* $I = 5.00$ amp.
30.* 14 ev.

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1	1, 3	2#, 4	5#		5#
2	6, 7	9	8#, 10#		8#, 10#
3	11	12			
4–5	13, 14, 19, 20	16	17#, 18	27	18
6	21, 22		25, 26		26
7	28, 29, 30	15, 24	23		15

PROBLEM 1* (See Short Answers.)

PROBLEM 2

Two large parallel metal plates, 6.2 cm apart in a vacuum tube, are connected to the terminals of two 90-volt batteries in series. A doubly charged oxygen ion starts from rest at the surface of one plate and is accelerated across to the other plate.

(a) With what kinetic energy $\frac{1}{2}mv^2$ does the oxygen ion crash into the other plate?

(b) If the ion starts halfway between the plates, with what kinetic energy does it hit the negative plate?

This straightforward problem illustrates the application of conservation of energy to the motion of charged particles in a uniform electric field. One approach is the following.

(a) We first calculate the force on an oxygen ion, using the results of the Millikan experiment discussed in Section 27–7. There it was found that with three 90-volt batteries in series connected to plates 0.31 cm apart, the force on an elementary charge was 1.4×10^{-14} newton. Since the force exerted by charged parallel plates on an ion between them varies directly with the number of batteries in series and with the charge of the ion, and inversely with the separation of the plates, the force on a doubly charged oxygen ion exerted by plates 6.2 cm apart connected to two 90-volt batteries is

$$F = (1.4 \times 10^{-14}\ \text{newton}) \times \tfrac{2}{3} \times 2 \times \left(\frac{0.31\ \text{cm}}{6.2\ \text{cm}}\right) = 9.3 \times 10^{-16}\ \text{newton}.$$

If an oxygen ion moves the full distance of 6.2 cm between plates, the work done by the electrical force will be

$$9.3 \times 10^{-16}\ \text{newton} \times 0.062\ \text{meter} = \underline{5.8 \times 10^{-17}\ \text{joule}}.$$

Since the ion starts with no kinetic energy, this will be its final kinetic energy as it reaches the plate.

(b) If the ion starts from rest halfway between the plates, it travels only one half the distance, i.e., 3.1 cm. The work done by the electric force, and therefore the kinetic energy acquired by the ion will be

$$9.3 \times 10^{-16}\ \text{newton} \times 0.031\ \text{meter} = \underline{2.9 \times 10^{-17}\ \text{joule}}.$$

Note: After the chapter has been completed, the problem may be done by the following alternate method.

With a potential difference of 180 volts between the two plates, one elementary charge would acquire a kinetic energy of 180 electron volts in traveling from one plate to the other.

(a) Thus a doubly charged oxygen ion will gain 360 electron volts in going from one plate to the other.

(b) Since the electric field is uniform between two such plates, the potential difference will be uniformly divided. Thus in traveling one half the total distance, the doubly charged oxygen ion will gain 180 electron volts of energy.

PROBLEM 3* (See Short Answers.)

PROBLEM 4

In a time-of-flight experiment like that described in this chapter, you have adjusted the sweep circuit of the oscilloscope so that the beam moves across the tube in 0.5 sec. The total length of the trace made in that time is 10 cm. The contact points on the track for the rolling ball are 1.2 m apart. If the spikes on the oscilloscope trace are 8.0 cm apart, what is the speed of the ball?

On our oscilloscope scale, 10 cm = 0.5 sec, or 1 cm = 0.05 sec. If the spikes on the trace are 8.0 cm apart, the time interval between the spikes must be (8.0 cm)(0.05 sec/cm) = 0.4 sec. Since the ball traveled 1.2 m during this interval, its speed is

$$v = 1.2\ \text{m}/0.4\ \text{sec} = \underline{3.0\ \text{m/sec}}.$$

PROBLEM 5

In the apparatus used to measure the mass of the positive hydrogen ion (proton) of Section 28–1, you use a source of heavy hydrogen and observe a spike which corresponds to ions arriving with a time of flight 1.4 times that of the usual positive hydrogen ions.

(a) What is their mass if they are singly charged?

(b) If doubly charged?

(c) Could they be singly charged helium ions? Doubly charged helium ions?

Remember that the mass of a helium atom is 4 atomic mass units, and the mass of a hydrogen atom 1 atomic mass unit.

(a) If the ions in question are singly charged they acquire the same kinetic energy as the protons, i.e.,

$$1/2mv^2 = 1/2m_p v_p{}^2.$$

Since the time of flight is larger by a factor

of 1.4 than for protons, $v/v_p = 1/1.4$. We would conclude

$$\frac{m}{m_p} = \left(\frac{v_p}{v}\right)^2 = 2.0.$$

Hence, if singly charged, the mass of heavy hydrogen would be about 2 amu.

(b) If the ions were doubly charged, they would acquire twice the kinetic energy of the protons:

$$\tfrac{1}{2}mv^2 = 2 \times \tfrac{1}{2}m_p v_p^2.$$

Therefore,

$$\frac{m}{m_p} = 2 \times (v_p/v)^2 \approx 4.$$

In this case the mass of the ion would be 4 amu.

(c) The experimental result is therefore consistent with doubly (but not singly) charged helium ions since the mass of He is 4 amu. It is also consistent with the assumption of the existence of a hydrogen ion having twice the normal mass. This isotope of hydrogen is called deuterium (by analogy to the protons, these ions are called deuterons).

PROBLEM 6* (See Short Answers.)

PROBLEM 7* (See Short Answers.)

PROBLEM 8

You ionize some gas between parallel metal plates, making equal numbers of negative and positive ions in every small volume between the plates. There is a battery and a current meter connected in series with the plates, and a steady current is flowing through the circuit. The positive plate is on the left, and the negative on the right.

(a) In what direction do you expect the electric current to go between the plates?

(b) If the current meter measures 10^{16} elementary charges per second, how many positive elementary charges are brought per second by the ions coming to the negative plate?

(c) How many negative elementary charges are coming per second to the negative plate?

(d) In one second, how many positive ions cross to the right through a plane halfway between the plates?

This problem illustrates that the total current at any point in a circuit is given by the sum of the flow of positive charges (in the direction of the current) and the negative charges (flowing in the opposite direction) per unit of time.

In this problem we are concerned only with the steady-state behavior of the circuit. In the steady state, positive and negative ions are created in equal numbers per unit time by the X-ray beam. The positive ions will move toward the negative plate on the right, and the negative ions toward the positive plate on the left. If there were no recombination, all of the charges would contribute to the current, which would then be equal to the number of ion pairs produced per second. With recombination, the total current will be less, but the number of positive charges reaching the right-hand plate must still equal the number of negative charges reaching the left-hand plate per second. The current will in any event be given by the number of charges striking either plate per second.

(a) The direction of the current is the direction of motion of the positive charges, hence from left to right.

(b) The current must be equal to the number of positive charges striking the negative plate per second, and must be the same everywhere in the circuit. Hence the number of positive charges striking the negative plate must be 10^{16} elementary charges/second.

(c) There are *no* negative charges coming to the negative plate *from within the tube*, but the external circuit must supply negative charges equal to the positive charges reaching the plate per second. Hence, the flow of negative charge to the negative plate, *from the external circuit*, is 10^{16} elementary charges/second.

(d) At a plane halfway between the plates, the current must consist of equal numbers of positive and negative charges flowing in opposite directions. Thus the number of positive charges crossing to the right at this point must be half the current, or 5×10^{15} elementary charges/second. If each ion carries n elementary charges, the positive-ion current must be $(5 \times 10^{15})/n$ per second.

PROBLEM 9

You are given some metal wire, batteries, and a calibrated current meter. How can you calibrate a second current meter?

When two meters are connected in series in a circuit, the same current must flow through both. The current on the calibrated meter can be read, and the scale of the other meter can be marked accordingly. By varying the number of batteries connected in series to the circuit, one can vary the current, so a number of values can be marked on the second meter. If the scale of the second meter appears to be linear, the other values can easily be interpolated.

In this problem we are not concerned with the possibility that if the wires are good conductors, the current might exceed the capacity of the meter. We can assume that the meters operate within the range of the batteries and wires we are given.

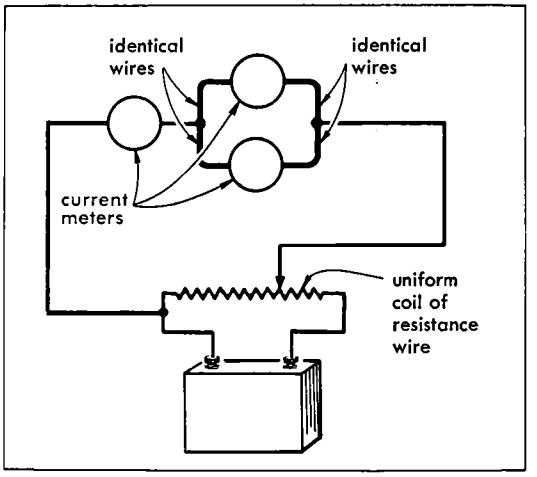
PROBLEM 10

You are given a half dozen identical current meters, one of which has a single calibration point — that is, a mark correctly indicating when a 1-ampere current (a current of 6.25×10^{11} elementary charges per second) is passing through it. You also have a few batteries and a large number of identical metal wires. Answer the following questions to complete the calibration of the current meter.

(a) Assume you have marked a number of current meters to read 1 ampere. Then you connect three as in Fig. 28-16. Explain how you can get a 2-ampere calibration point on one of them.

(b) Make sketches showing how to connect the current meters in order to mark one scale at 3 and 4 amperes.

(c) How would you connect the current meters so as to calibrate one of them to read $\frac{1}{2}$ and $\frac{1}{3}$ amperes? (Use sketches.)



(a) With the arrangement of Figure 28-16, each of the two meters connected in parallel should carry one half of the current in the third meter. The latter will carry 2 amperes when each of the former carries 1 ampere. We adjust the resistor so that each of the two meters in parallel reads 1 ampere, and mark the deflection of the third meter as 2 amperes.

(b) When the divider is set so that meters 1, 2, and 3 all read 1 ampere, meter 4 must be indicating the total current, 3 amperes. Similarly, this arrangement with 1 ampere in each of the four parallel meters gives 4 amperes in the other meter. Alternatively, one meter in series with two meters in parallel could be used. Adjusting the current in each of the two parallel meters to 2 amperes [as per calibration in part (a)] will give 4 amperes through the series meter.

(c) When the current is adjusted to give a 1 ampere reading on meter 3, meters 1 and 2 must each be carrying (and therefore indicating) $\frac{1}{2}$ ampere.

With meter 4 indicating 1 ampere, the other meters must each be indicating $\frac{1}{3}$ ampere.

PROBLEM 11* (See Short Answers.)

PROBLEM 12

A current meter has been built and you want to calibrate its scale. It is connected in series with an electrolytic cell containing a solution of silver nitrate ($Ag^+NO_3^-$) and a battery. A steady current which deflects the meter to the upper part of the scale flows for half an hour. At the end of this period, it is found by weighing that the negative electrode has 1.92 gm of silver deposited on it.

(a) How many elementary electric charges per second passed through the circuit? (*Note:* A mole of silver has a mass of 108 grams.)

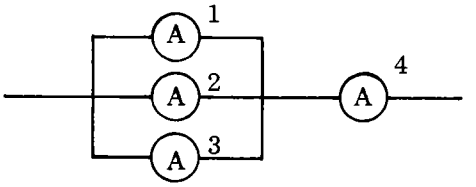
(b) What would you do to complete the calibration of the instrument?

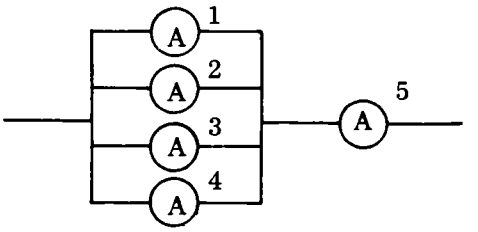
(a) Each silver ion carries one elementary charge. To find the number of elementary charges per second passing through the circuit, we divide the total number of silver atoms deposited by the time in which they were deposited:

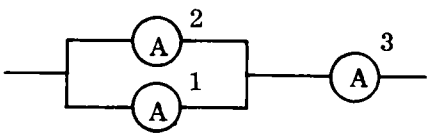
$$\frac{1.92 \text{ gm}}{108 \text{ gm/mole}} \times \frac{6.02 \times 10^{23} \text{ atoms/mole}}{1800 \text{ sec}} = 5.94 \times 10^{18} \text{ atoms/sec},$$

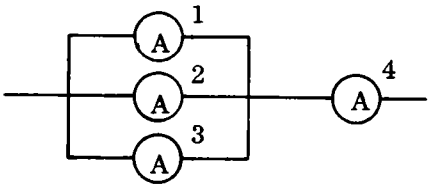
or 5.94×10^{18} elementary charges/second.

(b) The zero point of the meter must be marked when there is no flow of charge, and several other points within its range must be established by the above method to determine whether the meter movement is linear. If it is,









the scale can be made with equal divisions up to the full range of the meter, using the calibrated points to determine the size of each division.

PROBLEM 13* (See Short Answers.)

PROBLEM 14* (See Short Answers.)

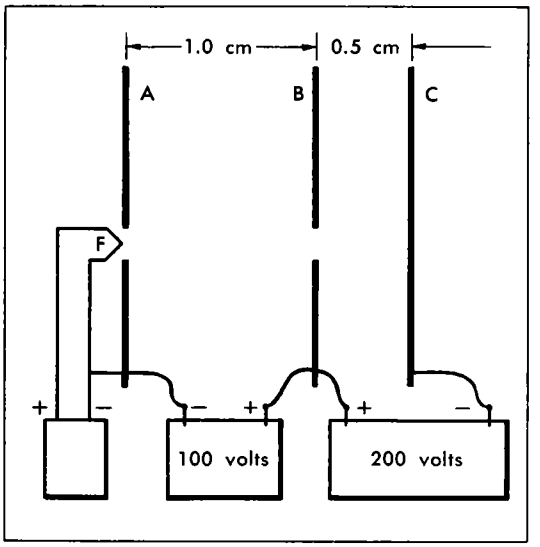
PROBLEM 15

In Fig. 28-17, electrons emitted at F are accelerated toward B.

(a) How much energy will the electrons have which hit plate B?

(b) What is the electric field between A and B and between B and C?

(c) Some will travel through the hole in B. Where will they go?



(a) The energy supplied to an electron moving through a potential difference of 100 volts is

$$\begin{aligned} q\,\Delta V &= (1 \text{ elem.ch.})(100 \text{ volts}) \\ &= 100 \text{ electron volts} \\ &= \underline{1.6 \times 10^{-17} \text{ joule}}. \end{aligned}$$

(b) The electric field is the force per elementary charge, or F/q. Since the energy acquired by a charge equals the force F on it times the distance d it travels, $q\,\Delta V = Fd$, the electric field can be expressed as

$$E = \frac{F}{q} = \frac{(q\,\Delta V/d)}{q} = \frac{\Delta V}{d}.$$

The electric field in the region between A and B is

$$E_{AB} = \frac{100 \text{ volts}}{1.0 \times 10^{-2} \text{ m}} \times \left(\frac{1.6 \times 10^{-19} \text{ joule/elementary charge}}{1 \text{ volt}}\right) = \underline{1.6 \times 10^{-15} \text{ newton/elementary charge}}.$$

The direction of the field is from the positive plate, B, to the negative plate, A. The field between B and C is

$$E_{BC} = \frac{200 \text{ volts}}{5 \times 10^{-3} \text{ m}} \times \left(\frac{1.6 \times 10^{-19} \text{ joule/elementary charge}}{1 \text{ volt}}\right) = \underline{6.4 \times 10^{-15} \text{ newton/elementary charge}}.$$

The direction of the field is from plate B to plate C.

(c) Since the electric field is uniform everywhere between the parallel plates, the potential halfway between B and C is 100 volts less than at B. When the electrons reach the halfway mark, they will have given up the entire 100 electron volts of kinetic energy they acquired between A and B, and they will be attracted back to B. If they hit B, on this trip or a later one, they will give up as heat the 100 ev they will have regained.

PROBLEM 16

With the same apparatus as in Problem 2, how many doubly charged oxygen ions would have to go from the positive plate to the negative plate before that plate is heated up by one joule by the dissipation of kinetic energy of the ions?

Since one 90-volt battery supplies 1.4×10^{-17} joule to each elementary charge, the energy supplied by two 90-volt batteries to a doubly charged ion is

$$2(2)(1.4 \times 10^{-17} \text{ joule}) = 5.6 \times 10^{-17} \text{ joule}.$$

(This is approximately the same answer we calculated by a different method in Problem 2.) If each ion transfers its entire energy to the plate when it crashes into it, the number of ions necessary to supply one joule of energy is

$$N = \frac{1 \text{ joule}}{5.6 \times 10^{-17} \text{ joule/ion}} = \underline{1.8 \times 10^{16} \text{ ions}}.$$

The most common oxygen ions are negatively charged, but from their direction of travel these must be positively charged.

PROBLEM 17

Suppose the doubly charged ions in the last problem were made by ripping electrons off oxygen atoms located halfway between the plates. Then the oxygen ions are accelerated in one direction and the electrons

are accelerated in the opposite direction from the midpoint between the plates. How much energy will the electrons have dissipated heating the positive plate in the time that one joule is dissipated at the negative plate?

The magnitude of the force on each elementary charge depends only on the setup of the plates and battery, and the distance traveled in the direction of the force is the same for the electrons and ions. Since the same number of elementary charges go in each direction, the energy they bring to each plate is also the same.

In greater detail, the force on the electrons (toward the positive plate) is equal in magnitude to half the force on the ions [which is toward the negative plate (we are dealing with positive oxygen ions, not the more common negative ions)] since the electrons have one elementary charge and the ions have two. Starting halfway between plates, electrons and ions move the same distance, but because the force on the electrons is half that on the ions, the electrons acquire one half as much kinetic energy as the ions. However, since two electrons are produced when one ion is formed, the kinetic energy acquired by the two electrons together is equal to the kinetic energy acquired by the one oxygen ion. It follows that the amount of energy transferred to the positive plates by the electrons is equal to that transferred to the negative plate by the positive ions. In this particular instance both equal 1 joule.

PROBLEM 18

What evidence do you have that all elementary charges are the same? Be prepared to discuss it in class.

This question provides a good review of Sections 1–4.

All of the evidence — from the Millikan experiment, from time-of-flight measurements, and from electrolytic cells — shows that the force acting on any charged particle in a given electric field is an exact integral multiple of some least value. This can only be true if all elementary charges are identical, regardless of the mass or any other property of the charged particle, and without regard to the way in which the charge is produced.

Other sources of evidence include the neutrality of atoms and molecules, and the argument of Section 28–4 which can also be done with just a wire heater, a battery calibrated by a Millikan experiment and a current meter calibrated electrolytically.

PROBLEM 19* (See Short Answers.)
PROBLEM 20* (See Short Answers.)
PROBLEM 21* (See Short Answers.)
PROBLEM 22* (See Short Answers.)
PROBLEM 23

(a) Find the electron and the gravitational forces on an electron and on a proton between two large parallel metal plates, 20 cm apart. The potential difference between the plates is maintained by 20,000 volts' worth of battery. The plates are horizontal; consider the upper plate positive.

(b) Move the particles up 1 cm. What are the forces now?

(c) What are the changes in gravitational potential energy and in electric potential energy in this motion?

(a) The gravitational forces on both electron and proton are directed downward and given by mass × gravitational field strength.

For the proton: gravitational force
$= (1.67 \times 10^{-27}\ \text{kg}) \times (9.8\ \text{newtons/kg})$
$= \underline{1.6 \times 10^{-26}\ \text{newton}}.$

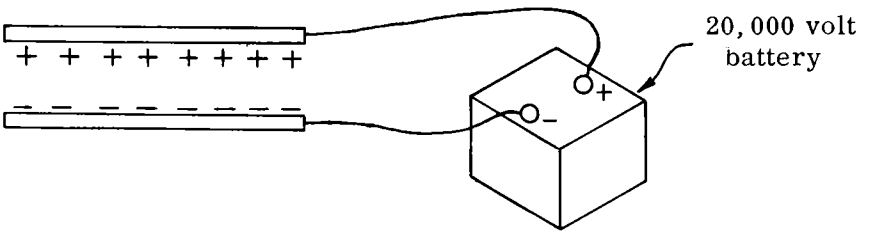
For the electron: gravitational force
$= (9.1 \times 10^{-31}\ \text{kg}) \times (9.8\ \text{newtons/kg})$
$= \underline{8.9 \times 10^{-30}\ \text{newton}}.$

Since the charge of both electron and proton is one elementary charge, the magnitude of the electrical forces is the same for both, but is directed up for the negatively charged electron and down for the positively charged proton. The work that would be done on a charge q moving through a potential difference ΔV equals the drop in potential energy of the system: $W = Fd = q\,\Delta V$.

Hence,

$$F = \frac{q\,\Delta V}{d} = \frac{(1\ \text{elem. ch.})(2 \times 10^{4}\ \text{volts})}{(1.6 \times 10^{-19}\ \text{joule/elem. ch.})(0.20\ \text{meter})(1\ \text{volt})}$$

$$= \underline{1.6 \times 10^{-14}\ \text{newton}}.$$



This is enormous compared to the gravitational forces.

(b) The electrical force field between parallel plates is independent of position; the gravitational force field of the earth is also essentially unchanged when one moves an additional distance of only 1 cm from the center of the earth. This means that both electrical and gravitational forces remain constant.

(c) This part of the problem probably can be discussed most usefully by leaning on the analogy between the uniform gravitational field and the uniform electrical field.

To move the proton up 0.01 meter against the downward gravitational force of 1.6×10^{-26} newton requires that work be done by an applied upward force in the amount of 1.6×10^{-26} newton $\times$ 0.01 meter $= 1.6 \times 10^{-28}$ joule. Similarly, to move the proton up 0.01 meter against the downward electrical force of 1.6×10^{-14} newton requires that work be done by an applied upward force in the amount of 1.6×10^{-14} newton $\times 10^{-2}$ meter $= 1.6 \times 10^{-16}$ joule. Therefore corresponding energy changes for the proton are an increase of 1.6×10^{-28} joule in the gravitational potential energy, and an increase of 1.6×10^{-16} joule in the electrical potential energy.

The energy changes for the electron can be obtained in the same way. The gravitational potential energy is increased by 0.89×10^{-29} newton $\times 10^{-2}$ meter, or 8.9×10^{-32} joule, while the electrical potential energy is *decreased* by 1.6×10^{-16} joule. This is a decrease because the electron moved in the same direction as the electrical force.

PROBLEM 24

In nuclear fission, the nucleus of an atom of uranium 238 sometimes breaks into just two fragments, owing to the Coulomb force of repulsion between its protons. Imagine that the split has just happened, and the two fragments are still touching each other (roughly 10^{-14} m apart, center-to-center) and at rest. Find their total kinetic energy when they are very far apart, if one has $Z_1 = 42$ elementary charges and the other has $Z_2 = 50$ elementary charges. Give your answer in joules and in millions of electron volts (Mev).

This problem and the following call for the application of conservation of energy in cases involving electrical potential energy. This problem presents a simplified picture of nuclear fission which is useful only for an order-of-magnitude calculation.

Since the only forces acting on the two fragments are the electrical forces of repulsion exerted by each fragment on the other, the electrical potential energy of interaction of the two fragments plus the sum of their energies remains constant. As the fragments fly apart, the potential energy decreases with a corresponding increase in the sum of their kinetic energies. The initial potential energy is

$$U_{\text{coul}} = \frac{kZ_1Z_2}{r} = \frac{1.2 \times 10^{-28} \times 42 \times 50}{10^{-14}}$$
$$\approx 5 \times 10^{-11} \text{ joule} = 300 \text{ Mev.}$$

This will all be converted into kinetic energy of the fragments if we take the simple model proposed. Actually, some of it goes into excitation of the fragments and a very small fraction into extracting a few free neutrons, which also are liberated in most fission reactions.

PROBLEM 25

(a) Two protons are released from rest 10^{-10} m apart. What will their kinetic energy be at large separation?

(b) Suppose one of the protons is nailed down. What will the kinetic energy be at large separation?

(a) The initial potential energy of the protons due to the Coulomb force is

$$k\frac{q_1q_2}{r} = 2.3 \times 10^{-28} \times \frac{(\text{newton-meter})^2(1 \text{ elem.ch.})^2}{(\text{elem.ch.})^2(10^{-10} \text{ meter})}$$
$$= 2.3 \times 10^{-18} \text{ joule,}$$

where the zero of potential energy has been taken at large (infinite) separation. The final potential energy is zero. Since the initial kinetic energy is zero, the final kinetic energy is 2.3×10^{-18} joule.

Each proton has one half of the total kinetic energy (by conservation of momentum) so that the kinetic energy of each proton is 1.2×10^{-18} joule.

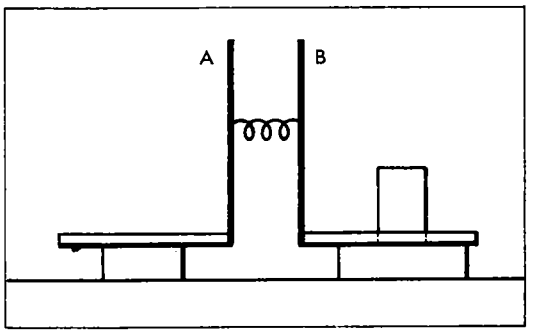
Energies of elementary particles are often given in electron-volts. Since 1 ev $= 1.6 \times 10^{-19}$ joule, we have:

$$\text{Sum of kinetic energies} = \frac{2.3 \times 10^{-18}}{1.6 \times 10^{-19}}$$
$$= 14.4 \text{ ev.}$$

(b) Because no energy can be stored or dissipated, the movable proton must carry all the kinetic energy, 2.3×10^{-18} joule. The fixed proton does not move, and has zero kinetic energy.

PROBLEM 26

Two vertical metal plates stand parallel to each other (Fig. 28-18). Both are mounted on insulating bases,



but plate A is bolted to the table and plate B is set on a Dry Ice puck so that it can move freely. They are connected by an insulating spring which can be either stretched or compressed.

(a) Plate B is held fast in such a position that there is no force exerted by the spring, and the two plates are charged by connecting them to a battery. The battery is disconnected and plate B is let go. Describe the motion qualitatively but in detail.

(b) Can you think of a purely mechanical system which behaves in the same way?

(a) The battery puts opposite charges on the two plates, so they attract each other, and B moves in toward A. The spring is compressed and exerts a backward force on B, but at the beginning the electrical attraction is greater than the force of the spring and B accelerates. When the force of the spring equals the electrical attraction, there is no net force on B and B moves for an instant with constant speed. Beyond this point, the force of the spring exceeds the electrical attraction; B is slowed down, brought to rest, and pushed back. It is brought to rest again at its original starting point, once more is drawn in, and so continues in vibratory motion.

If you wish to take the discussion further, suppose that the plates are large and close together, and use the argument in the box on page 491 to say that the electric field at plate B produced by the charges on plate A is independent of the distance between them. Then the electrical force on B is constant. The elastic force of the spring is proportional to the distance that B moves from its starting place. Plate B moves under the combined action of a constant electrical force and a linear elastic force. This gives it simple harmonic motion.

In terms of energy, the electrical potential energy of the plates is proportional to their separation and grows less as they come closer together. The elastic potential energy of the spring is proportional to the square of the distance plate B has moved from its starting point. Initially, the electrical potential energy decreases faster than the elastic potential energy increases; the difference appears as the kinetic energy of B. As B approaches A, the elastic potential energy increases more and more rapidly, the kinetic energy begins to decrease, and B comes to rest when the net potential energy is once more as great as it was at the start.

(b) If a weight is hooked to the end of a vertical spring without stretching it and then let go, its motion will be like the motion of plate B. As the extended discussion shows, the likeness is exact; the weight moves under the combined action of the constant force of gravity and the elastic force of the spring.

PROBLEM 27

Two identical charges q are nailed in place on the y axis, equal distances a above and below the origin of coordinates

(a) Find an expression for the potential energy due to the interaction of an elementary charge on the x axis, a distance x from the origin, with one of the nailed-down charges; with both of the nailed-down charges.

(*Note:* Measure the potential energy with the zero chosen when the elementary charge is at infinity. Express your answer in terms of the constant k in Coulomb's law, the distances a and x, and the charge q.)

(b) What would be the potential energy due to the interaction of the elementary charge if the two nailed-down charges were equal and opposite?

(a) The potential energy due to the Coulomb interaction of the elementary charge on the x-axis with only one of the nailed-down charges on the y-axis is found from the expression $U = k(q_1q_2/r)$.

In this case $q_1 = q$

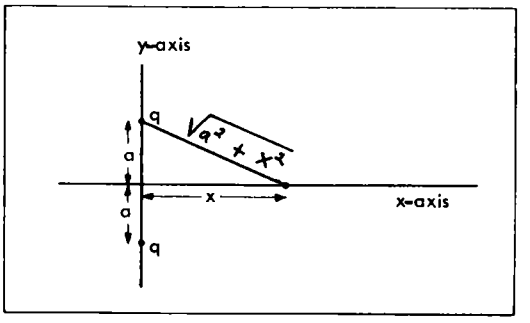
$$q_2 = 1 \text{ elem.ch.}$$

and

$$r = \sqrt{a^2 + x^2}.$$

So

$$U = k\frac{q(1 \text{ elem.ch.})}{\sqrt{a^2 + x^2}}.$$



The potential energy due to the Coulomb interaction of the elementary charge with both nailed-down charges is equal to the amount of work done against the net Coulomb forces when the elementary charge is brought from infinity to its position on the x-axis. But the work done against the net force equals the (algebraic) sum of the work done by each force (Section 23–9), or the sum of the potential energies due to the interaction with each nailed-down charge. Hence, for the interaction with both nailed-down charges:

$$U_{\text{tot}} = 2k \frac{q(1 \text{ elem.ch.})}{\sqrt{a^2 + x^2}}.$$

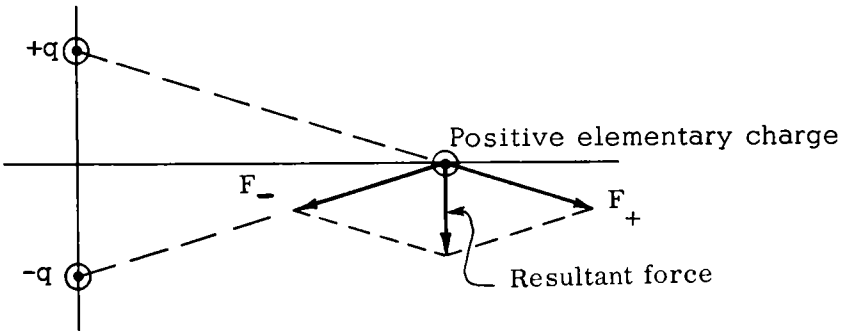
Note that this expression does not include a term due to the interaction of the nailed-down charges with each other. This is because we are choosing our zero of energy with the movable charge at infinity, but with the nailed-down charges in place on the y-axis.

Since the electric potential is just the electric potential energy per elementary charge, potential also can be calculated by taking the algebraic sum of the potential contributions from the individual charges that create the total electric field.

(b) If the nailed-down charges were equal and opposite, the total potential energy would be

$$U_{\text{tot}} = k \frac{(+q)(1 \text{ elem.ch.})}{\sqrt{a^2 + x^2}} + k \frac{(-q)(1 \text{ elem.ch.})}{\sqrt{a^2 + x^2}} = 0.$$

The last result, which may appear strange at first sight, comes about from the fact that the net force acting on the elementary charge is perpendicular to the x-axis so that no work is required to move the charge anywhere along the x-axis, even as far as infinity where, of course, the potential energy has been taken as zero (by definition).



PROBLEM 28* (See Short Answers.)
PROBLEM 29* (See Short Answers.)
PROBLEM 30* (See Short Answers.)

chapter
29

Electric Circuits

Chapter 29 turns to a more detailed consideration of energy flow in electric circuits, both ohmic and non-ohmic. Throughout, the major concern is with the motion of elementary charges rather than with circuits per se. While the text provides a sturdy base for the analysis of electrical devices, a detailed treatment of Ohm's law and associated problems is aside from the main purpose of the course.

As noted in the text, Ohm's law describes current flow only in a very special (but technologically very important) case — that of electronic conduction in metals at constant temperature. It would be a serious error to consider the development of this chapter as "leading up" to Ohm's law as a climax. Instead, Ohm's law is properly considered as a by-product, having first thoroughly treated the effect of electric fields on individual elementary charges.

We are not concerned primarily with an analysis of what we ordinarily call "D.C. circuits." We are interested in currents as a manifestation of the fact that elementary charges move in a predictable manner under the action of an electric field. We shall need this concept of currents and moving charges to discuss magnetic interactions in the next chapter. Otherwise, this subject is not particularly important to the continuity of the volume.

CHAPTER SUMMARY

SECTIONS 1 THROUGH 3. When a battery is connected into an open circuit of any sort, charges move under the action of the electric field until the potential difference between the positive and negative conductors exactly equals the EMF of the battery. There is then no electric field inside any conductor. If the circuit is closed, so that a steady current flows, electric fields do not vanish inside the conductors, but depend on the sizes and shapes of the conductors. The sum of the potential differences around the circuit is still equal to the EMF of the battery. Potential differences can be measured with an electrometer or a high-resistance voltmeter. According to our previously developed ideas, the kinetic energy acquired by a charged particle moving in a vacuum depends only on the charge and the potential difference across the region through which the charge moves. This conclusion can be checked experimentally.

SECTION 4. With an increase in the potential difference between two metal plates separated by an ionized gas, the current increases, then saturates, then increases in a complicated manner. The current flowing between two metal plates which are separated by a vacuum depends on the temperature of the plates and on the potential difference between them. The current flowing in a metal is directly proportional to the potential difference. $V = RI$, the proportionality constant being defined as the resistance, which is determined by the dimensions and various physical properties of the conductor.

SECTION 5. Various electric circuits are discussed. In any circuit, no matter how complex, the total energy supplied by the battery must be accounted for by the energy dissipated in various circuit elements. This shows that the sum of the potential differences around the circuit is always equal to the applied EMF.

SCHEDULING CHAPTER 29

If you are pressed for time you should consult the scheduling suggestions in the Introduction to Part IV of the Guide. You may wish to omit this chapter and the next one and proceed directly to Section 9 of Chapter 31, thus omitting electric circuits and magnetism. While such topics are of undisputed importance and interest, they may be deferred until subsequent courses if you are short of time and prefer to emphasize atomic and nuclear physics.

The following table suggests possible schedules for presenting this chapter, consistent with the schedules outlined in the Introduction to Part IV. As noted before, if time is very short, this chapter may be omitted.

Chapter 29	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTIONS	CLASS PERIODS	LAB PERIODS	EXPERIMENT	CLASS PERIODS	LAB PERIODS	EXPERIMENTS
1	1	0	—	2	0	—
2, 3	1	0	—	1	0	—
4, 5	1	1	IV–9	2	2	IV–8 IV–9

RELATED MATERIALS FOR CHAPTER 29

LABORATORY. Experiment IV–8, *The Charge on a Capacitor*, is intended to follow Section 29–1. It deals with the relationship of the potential difference between the plates of a capacitor to the quantity of charge on the plates.

Experiment IV–9, *Energy Transferred by an Electric Motor*, is intended to be used at the start of Section 5. It gives a quantitative view of energy transfer in an electric circuit.

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILMS. "Electric Potential Energy and Potential Difference," by Nathaniel Frank of M.I.T., is a two-part film in which the mechanism by which a battery establishes an electric field is analyzed, the electric potential energy in the system is measured and discussed, and the energy transformations in a steady current-carrying circuit are explained. Running time for Parts I and II: 54 minutes.

29–1 Conductors, Batteries, and Potential Difference

PURPOSE. To apply the concepts of electric field and electric potential difference to open and closed circuits.

CONTENT. (a) When conductors are connected to a battery, charge flows very quickly, then stops when the conductors connected to one terminal are all at the same potential and differ in potential from the conductors connected to the other terminal by the EMF of the battery.

(b) If a single conductor is connected to both terminals of a battery, a steady current flow ensues. The flow of charge is maintained by an electric field inside the conductor. If the conductor is a wire, the field inside is along the direction of the wire.

(1) If the wire is uniform, the field times its length equals the EMF of the battery: $E\ell = \mathcal{E}$.

(2) If the field varies and is E_1 for a distance ℓ_1 and E_2 for ℓ_2, then $E_1\ell_1 + E_2\ell_2 = \mathcal{E}$.

(c) If a charge q moves through a potential difference V, then the electric energy of the system must change by $U_e = qV$. This is true, however, only if V remains constant during the motion of q. V does remain constant if a battery is connected across the points between which q moves. V remains approximately constant if q is much smaller than other charges involved. However, the case of two charged metal plates is considered where, initially, the plates are uncharged but acquire a charge $+q$ on one and $-q$ on the other. Since the potential difference is initially at zero and ends up at its final value V, it is clear that the potential difference is changing. Then the total electric energy of the system is $\frac{1}{2}qV$, not qV as might naively have been expected.

EMPHASIS. These ideas are centrally important to advanced work in electricity, but are not crucial to the continuity in later parts of this course. If this section is treated, it should be done well — one or two days on this section alone — but it may be treated as reading assignment only, or omitted.

DEVELOPMENT. If you cover this section in class, it will be wise to stress the various conclusions in terms of the motion (or lack of it) of individual elementary charges.

(1) That a single conductor forms an equipotential should be easy to show the class. Simply be sure the logic is definite and clear:

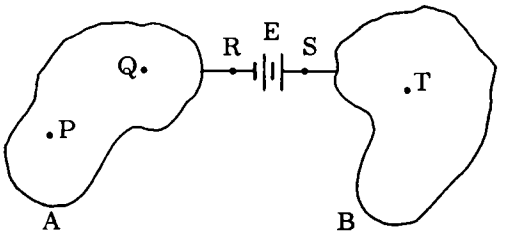
(a) If the situation is a static one — settled down — no charges are moving.

(b) A conductor contains charges which can move if a force is applied. If the charges are not moving, no force is being applied to them and the electric field is zero everywhere inside the conductor (and perpendicular to its surface just outside the conductor).

(c) A single elementary charge can therefore be moved from place to place on a conductor without doing work on it. Therefore every point on the conductor is at the same potential.

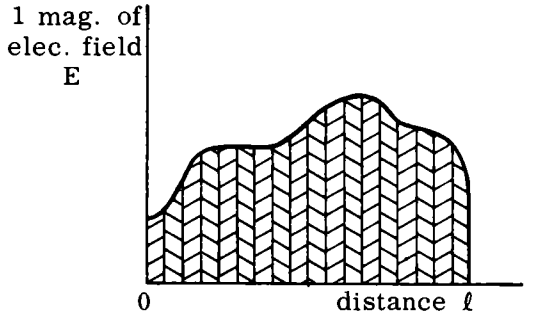
Sometimes students will ask how good a conductor we are talking about. The question is not how good a conductor, but how long we are willing to wait for the charges to "settle down." Consider some magnets resting on corks which are floating in water. They will quickly arrange themselves into a proper pattern. If we use syrup instead of water, the arrangement will

take longer, but they will eventually arrive at the same arrangement. If we wait long enough, even a rubbed piece of plastic will eventually become an equipotential, though it may take months or years.



You may wish to draw two odd-shaped conductors A and B attached to a battery of EMF $\mathcal{E}$. Consider how much work is done in moving an elementary charge from P to Q (none) Q to R (none). Why? Because this is a single conductor — an equipotential. How much work is required to get a charge through the battery, i.e., from R to S? Answer: $\mathcal{E}$ (or $-\mathcal{E}$ depending on the sign), because that is what an EMF means. S to T? (none). What is the potential difference between P and T? ($\mathcal{E}$); R and S? ($\mathcal{E}$); Q and S? (ϵ); etc. The idea to develop is that a battery simply maintains, in an open circuit, a constant potential difference ϵ between conductors attached to its two terminals.

(2) If students understand the situation when current does not flow, then they should readily understand what happens when current does flow. If no current flows when there is no electric field inside the conductor, then when current does flow, there must be an electric field inside the conductor. The work done in moving a unit charge a distance $\Delta\ell$ under the action of an electric field $\vec{\mathbf{E}}$, in the direction of $\vec{\mathbf{E}}$, is just $E\,\Delta\ell$. Thus we can easily find the total work done on a unit charge if we know how $\vec{\mathbf{E}}$ varies with distance.

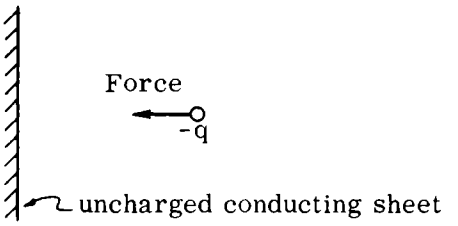


We simply add all the little $E\Delta\ell$'s — this gives us the total work done per unit charge, or the energy per unit charge in moving a distance ℓ. This is just the potential difference between 0 and ℓ; so $V_0 - V_\ell =$ area under graph of E vs. distance from 0 to ℓ. We can break any circuit up into an arbitrary number of series intervals, and in this way find the energy per unit charge in each interval. Finally in our circuit we must come to a battery, which supplies an EMF ϵ. The EMF is the energy per elementary charge supplied to the whole circuit. Thus, it must just equal the sum of the potential differences across the various series intervals in the circuit. So

ϵ = sum of V's around a circuit
= area under graph of E vs. distance for circuit between one battery terminal and the other.

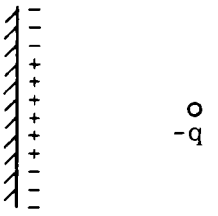
What we have stated, of course, is simply a somewhat formal declaration of the conservation of energy. Thus these considerations must hold for any circuit, no matter how complicated.

COMMENT. Students frequently do not realize that Coulomb's law holds only for *point* charges. If we are dealing with extended conductors, we must also consider the effect of induced charges. This is required, for example, to explain why a point charge would be attracted to an uncharged conducting plane.



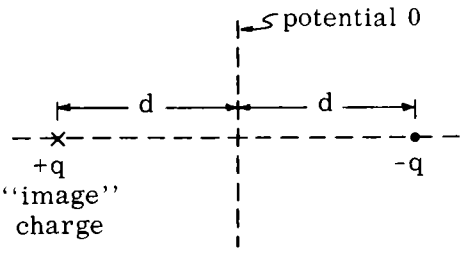
There are various ways of looking at this problem. From Figure 29–5, we can see that the charge must rearrange itself in the plane as shown below.

Since the induced plus charges are closer to the charge $-q$ than the induced minus charges, $-q$ will be attracted to the plate, even though the net charge on the plate is zero.



We can also look at this situation quantitatively, by considering the fact that the conductor must be an equipotential; in fact, if uncharged, it must be at 0 potential.

This would be true if we took the plane away and put an equal and opposite charge $+q$ an equal distance behind the surface. We would then have a plane



of zero potential at the position of the front surface of the conductor. So, we can replace the effect of the conductor by considering the effect of this "image" charge. The force on $-q$ is obviously $\frac{-kq^2}{4d^2}$, the minus sign telling us that the force is toward the plate.

29-2 Measurement of Potential Difference

PURPOSE. To describe methods of measuring potential difference.

CONTENT. (a) Electrometers measure potential difference by measuring the force between charged bodies.

(b) Voltmeters measure potential difference by determination of the amount of current flowing through a large resistance.

(c) The current flowing through a voltmeter may affect the potential difference being measured.

EMPHASIS. This section is not central with reference to later work and need not be stressed in class discussion. It can be handled as a reading assignment. It is easier to discuss voltmeters after Ohm's law has been covered. See Problem 27 of the HDL, for example.

29-3 A Further Check on Potential Difference and Energy

PURPOSE. To emphasize the fact that the amount of kinetic energy acquired by charged particles moving through a potential difference depends only on the potential difference and the charge.

CONTENT. The change in kinetic energy of a charged body moving in an electric field is equal to the potential difference through which it moves times the number of elementary charges it carries. A knowledge of the actual electric field along the path is not required.

EMPHASIS. The point is very important, but should be obvious from previous considerations. It results in a great simplification in the treatment of many problems. If you have the time to treat the material on electricity thoroughly, the illustrations provided here are very worthwhile. If you are pressed for time, the ideas presented here will not be required in later material.

DEVELOPMENT. The reasoning follows from conservation of energy and is therefore not new. It is probably appropriate to develop it by discussion and suggestions by the students. They should be able to arrive at it independently, perhaps through a mechanical analogy. A frictionless roller coaster is a suitable simple mechanical analog.

A roller coaster of mass m starts from rest at A. What is its kinetic energy as it passes B (and beyond)? To find the details of the motion we would need to know the resultant force at each point along the path, but to predict the kinetic energy at B we need to apply only conservation of energy. Therefore we need only to know the change in gravitational potential energy. Conservation of energy tells us that, starting from rest at A, the roller coaster will have, at B, an amount of kinetic energy equal to the change (i.e., decrease) in gravitational potential energy between A and B:

Kinetic Energy at B = (Potential Energy at A) − (Potential Energy at B) = $mgh_A - mgh_B$.

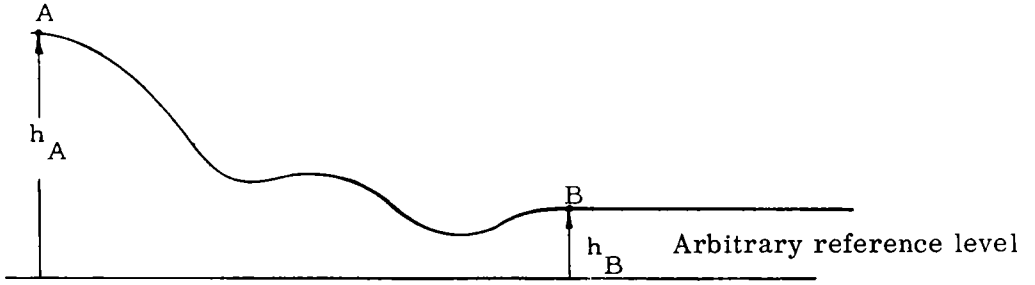
We note that as long as the roller coaster does get from A to B we need not know anything about the potential energy or the force at intervening points.

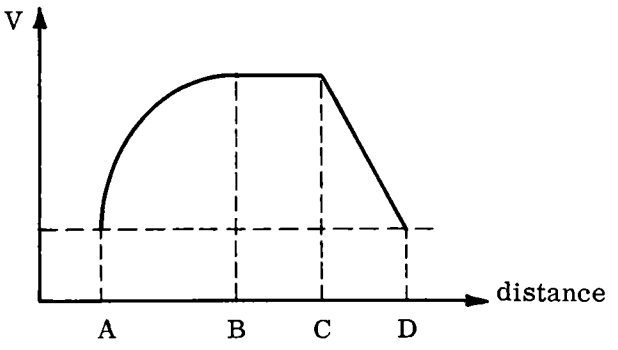
We note also that it does not matter what we take as the zero of potential energy; only the difference counts.

We can make the analogy even more complete by talking about "gravitational potential difference," i.e., change in potential energy per unit mass.

The gravitational potential difference in our example is $gh_A - gh_B$, and we have:

(Kinetic energy at B) = mass × (potential difference) = $m(gh_A - gh_B)$.





It may be interesting to let the student sketch, or help you sketch, the potential along the path of the electrons in Figure 29–8, with the uniform retarding field adjusted to just stop the electron current. The sketch should look something like the one above.

Why is this roller coaster upside down? (The electrons have negative charge, the roller coaster has positive inertial mass.)

29–4 Current versus Potential Difference

CONTENT. A qualitative discussion of the relationship between current and potential difference in ionized gases, in a vacuum diode, and in a metal. The reasons for the observed behavior are discussed in terms of the underlying phenomena on an atomic scale.

EMPHASIS. This material is not required in subsequent sections and can be omitted without loss of continuity. If covered, it should be handled as a reading assignment. It is a valuable presentation because it is one of the few treatments of this subject which put Ohm's law in proper perspective.

COMMENT. Many students will be interested in reading the material presented. There is, however, little to be gained by extensive class discussion, since a deeper understanding would require a background in atomic physics which is lacking at this stage. The following brief comments may help you in answering questions by interested students.

* * *

The device shown in Fig. 29–9 (minus the X-ray tube) is an ionization chamber. It can be used to detect ionizing radiation such as is produced, for instance, by radioactive substances. The passage of a fast charged particle through the gas leaves a trail of electron-ion pairs. The chamber can be used with a current measuring device to measure the average rate at which such charged particles pass through the chamber, or the change in potential difference following the passage of a single particle can be amplified and used to detect the passage of individual particles. With a sufficiently high potential difference, the electrons produce secondary ion pairs and this can result in breakdown of the gas and an appreciable pulse of current. This is a Geiger tube.

* * *

The region where the current is limited by space charge is just the beginning of the curves in Figure 29–11, where the current vs. potential difference curve is concave upward. Crystal diodes exhibit curves which are in some respects different from those for a vacuum diode. Since crystal diodes operate at an absolute temperature lower by a factor of $\frac{1}{3}$ to $\frac{1}{6}$ than vacuum tube diodes, the current falls off much more abruptly when a reversed potential difference is applied. This is because the electrons in the crystal diodes have thermal energies smaller by a factor of $\frac{1}{3}$ to $\frac{1}{6}$ and therefore can be stopped by a retarding potential smaller by approximately this amount. Crystal diodes also do not exhibit saturation, since there is an enormous supply of electrons available.

* * *

A quantitative statement of the derivation of Ohm's law from the simple model discussed is not difficult to obtain. We suppose electrons to be making collisions with the positive ions and to start off with a random velocity after each collision. Suppose the time interval between collisions is t. Between collisions each electron will gain an additional velocity equal to the impulse (force $\times$ time $= qEt$) divided by the mass. The average added velocity due to the electric field is then $\frac{1}{2}(qEt/m)$ which is clearly proportional to the electric field.*

---

* Actually it is not the collisions with positive ions of the lattice which cause the electrical resistance. In a perfect solid all atoms are equally spaced from one another (at the absolute zero of temperature). The collisions are then specular (not diffuse), and do not cause any resistance. Resistance results from imperfections in the lattice. At normal temperatures, these arise mostly from the thermal vibrations of the lattice atoms. When the atoms vibrate, they are not exactly equally spaced, and there is some diffuse scattering. Thus the resistance generally increases with increasing temperature. At very low temperatures, the resistance arises mostly from other lattice imperfections, particularly impurity atoms. The actual treatment of electrical resistance is quite complicated, since only *some* of the electrons in a metal are "free" in the sense that they can move under an electric field. In simple metals, such as copper, about one electron per atom is "free." In some metals, such as germanium, only a very tiny fraction of the electrons are free, and this number decreases sharply with decreasing temperature. This effect far over-rides the effect of temperature on the thermal scattering, so that the resistance of germanium increases sharply with decreasing temperature, and is infinite at absolute zero.

Because of these possible complications, it is best to specify Ohm's law at constant temperature, i.e., that the current is directly proportional to the potential difference *if the temperature of the metal is kept constant.* This law is obeyed over a wide range of voltage and currents for a large number of conductors.

PURPOSE. To examine electric circuits from the viewpoint of the motion of individual charges, where both energy and charge must be accounted for.

CONTENT. (a) A circuit consists of a battery which causes charge to move through a number of devices or elements.

(b) All of the energy supplied by the battery to the circuit must be accountable in some form, by heat, by mechanical work, etc.

(c) The EMF of the battery is divided into potential differences across each of the elements connected in series in the circuit. The actual fraction of $\mathcal{E}$ across a particular element depends on the physical nature of the element and (generally) on the current.

(d) If two circuit elements are connected in parallel, the potential difference across both must be the same; if they are connected in series, the current flowing through both must be the same.

(e) The law of conservation of energy shows that the sum of the potential differences around the various series elements of a circuit must exactly equal the applied EMF for any circuit, no matter how complex.

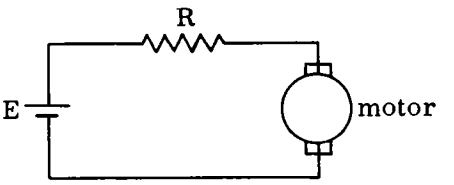
EMPHASIS. While this section lays the groundwork for a powerful approach to an understanding of electric circuits, these ideas are not required in subsequent chapters. Therefore the section can be omitted without loss of continuity.

DEVELOPMENT. If you discuss this section the best approach is probably through a quantitative example such as Problem 18. The approach in terms of energy exchanges is quite useful even in more complicated circuits, for instance in circuits in which non-steady currents flow, and energy can be stored in capacitors and inductors. Laboratory Experiments IV-8 and IV-9 will be of great help in getting across the essential points of this section.

COMMENT. The very general description of a circuit presented in the text is highly preferable to one which may be simpler conceptually, but is of limited applicability, such as a circuit containing only a battery and a number of resistors. Note that the text's description encompasses all electrical circuits, as long as we are careful to include all sources and sinks of energy as parts of the circuit. For example, if our circuit contains an electric motor, we must also include as a part of the circuit the mechanical system on which the motor is doing work and thereby storing or dissipating energy. If our circuit contains a radio transmitter, we must also include in the circuit all energy radiated into space which is not dissipated in a receiver. Thus we see that our "circuit" is essentially only a closed system (possibly as big as the whole universe in the case of a radio transmitter) in which the total energy is conserved.

QUIZ PROBLEMS.

1. A battery of EMF $\mathcal{E} = 100$ volts is connected to a motor through a resistor $R = 10$ ohms. A current of 5 amperes flows through the circuit.



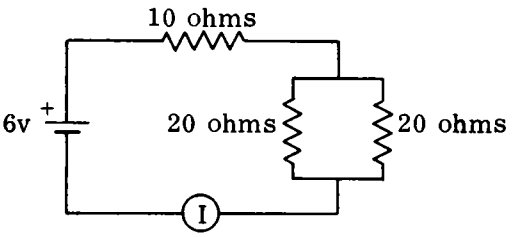
(a) What is the potential drop across the resistor? (50 volts.)

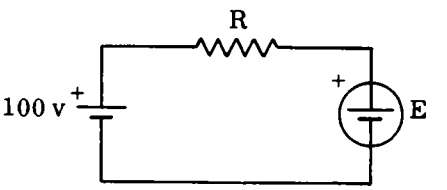
(b) What is the potential drop across the motor? (50 volts.)

(c) If there is no heat generated in the motor, how far can it lift a 10-kg mass in one second? (50 volts $\times$ 5 amperes $\times$ 1 second = 250 joules = 9.8 $\times$ 10 newtons $\times$ d meters; d = 2.55 m.)

2. (a) In the circuit shown, what is the current indicated by the meter I? (0.3 amp.)

(b) What is the rate at which heat is generated in the 10-ohm resistor? (0.9 watt.) In one of the 20-ohm resistors? (0.45 watt.)





3. A large 100-volt supply is to be used to charge a battery of EMF $\mathcal{E} = 12$ v at a charging rate of 1 ampere for a total time of 10 hours.

(a) How large should the resistor R be? (88 ohms.)

(b) What is the total change in chemical potential energy of the battery being charged? (4.3×10^5 joules.)

(c) What is the total energy supplied to the circuit? (3.6×10^6 joules.)

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The following table classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1.* The electric field is also in the direction of the wire.
2.* E = 3.0 volts/meter = 3.0 newtons/coulomb.
3.* The electric field in the first wire will be the same as before, since the difference in potential from one end of the wire to the other still equals the EMF of the battery, no matter what else has been connected in parallel.
4.* V = 18 volts.
5. (a) Series + to −; (b) 91.5 volts; (c) 88.5 volts.
6. (a) and (b). See discussion and graphs on pages 29–8 and 29–9.
7. (a) 1.7×10^{-6} joule; (b) 8.6×10^{-7} joule; (c) Converted to thermal energy by the resistance of the circuit.
8. (a) 4.32×10^{-17} joule; (b) 3.24×10^{-17} joule; (c) 8.6×10^{-7} joule.
9.* If it removes much charge, it will change the potential difference it is supposed to measure.
10.* The current in the voltmeter must be small compared with the current in the circuit.
11. See discussion on page 29–12.
12. (a) 2.0 volts; (b) 3.0×10^{7} m/sec; (c) 2.0×10^{-9} sec; (d) 3.5×10^{5} m/sec; (e) 3.5×10^{-4} m, or 0.35 millimeter.
13. (a) $s = E\ell^2/4V$; (b) Independent; (c) Independent.
14.* 45 electron volts = 7.6×10^{-18} joule.
15.* If the negative terminal is connected to the right-hand plate, as in Figure 29–8, the electron will give up 30 electron volts of its kinetic energy in order to do work against the repulsive force. This energy is transferred to the 30-volt battery. When the electron hits the plate, it will give up the other 15 electron volts as heat.
16.* $R = 4.4 \times 10^{4}$ ohms. The potential difference across the resistor is (90–37) volts.
17. (a) 2.00 amperes; (b) Connect in series with coil.
18. (a) Same; (b) Same; (c) Less; (d) B.
19. (a) 1.05 ohms; (b) I^2R = 4.20 joules/sec.
20. 1.17×10^{5} m/sec.
21. (a) $nAv\,\Delta t$; (b) $qnAv$; (c) 8.5×10^{22} electrons/cm^3; (d) 4.4×10^{-2} cm/sec.
22.* V = 4.9 volts.
23. (a) 2:1; (b) R proportional to length.
24. (a) 1:1; (b) 2:1; (c) Twice as much; (d) R proportional to $1/A$.
25. (a) Resistor 140 volts; diode 200 volts.
    (b) Diode 2×10^{-3} ampere; resistor 3×10^{-3} ampere.
    (c) 5×10^{-3} ampere.
    (d) 2.9×10^{4} ohms.
26. (a) 7×10^{-3} ampere; (b) 200 volts.
27. (a) 90 volts; (b) 180 volts; (c) 170 volts; (d) 84 volts.

COMMENTS AND SOLUTIONS

PROBLEM 1* (See Short Answers.)
PROBLEM 2* (See Short Answers.)
PROBLEM 3* (See Short Answers.)
PROBLEM 4* (See Short Answers.)

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1	1, 2, 3, 4	5	7#	6, 8	7#
2	9, 10	11			
3	14, 15			12#, 13	12#
4–5	16, 22	19, 20, 24	17, 23, 25	18, 21, 26, 27#	27#

PROBLEM 5

(a) How would you connect a 90-volt "B" battery and a 1.5-volt flashlight cell to obtain the maximum potential difference?

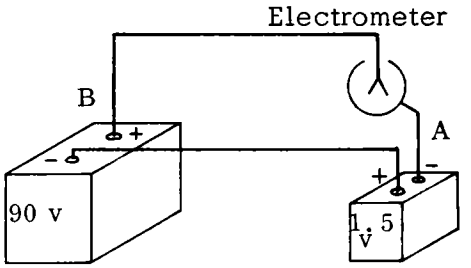
(b) How big is that potential difference?

(c) Suppose you disconnect the 90-volt battery, turn it end for end, and then reconnect it. What potential difference do you get?

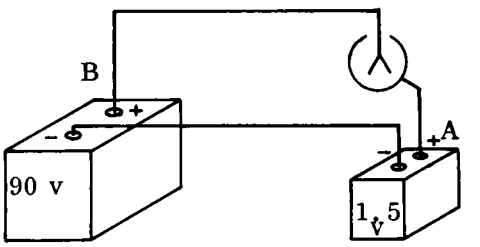
(a) and (b) If the batteries are connected *in series*, $+$ *to* $-$, the potential difference between A and B is the sum of the potential differences supplied by each battery. The increase in potential energy for a positive elementary charge going from A to B through both batteries (or a negative charge going from B to A) would be

$$(1.6 \times 10^{-19})(90 + 1.5) = 91.5 \times 1.6 \times 10^{-19} \text{ joule},$$

which corresponds to a 91.5-volt potential difference.



(c) If the connections to either of the batteries are reversed, the potential difference from A to B would be less. The increase in potential energy for a positive elementary charge going from A to B would still be $90(1.6 \times 10^{-19})$ joule when it went through the 90-volt battery, but there would be a decrease of $1.5(1.6 \times 10^{-19})$ joule when it went through the 1.5-volt battery. The net gain is $(90 - 1.5)(1.6 \times 10^{-19})$ joule, so that the potential difference is only $88.5 \times 1.6 \times 10^{-19}$ joule/elem. ch. (or 88.5 volts).



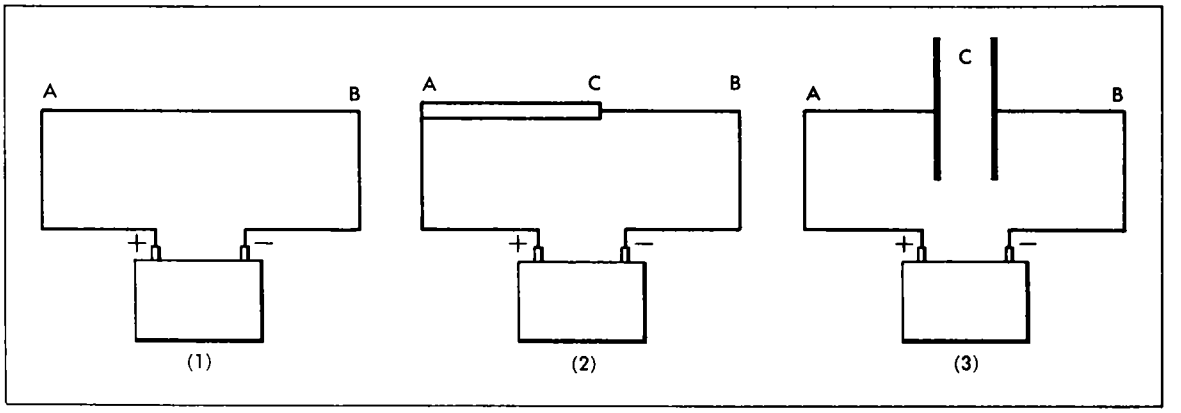
PROBLEM 6

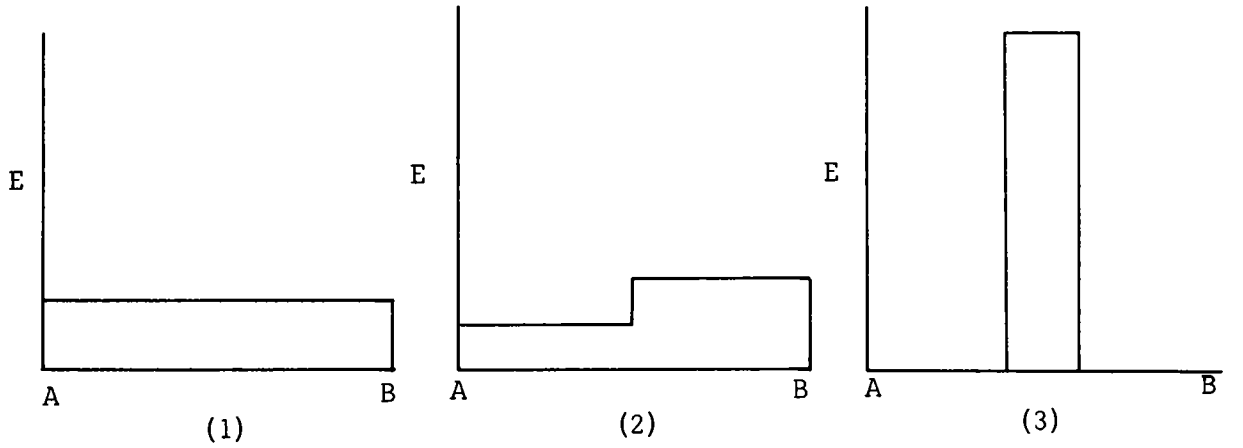
(a) Plot the electric field intensity against position along AB for the cases shown in Fig. 29–17.

The circuits in (2) and (3) are the same as in (1), except that in (2) AC consists of two parallel lengths of the same uniform wire and in (3) there are two large parallel plates at C whose separation is $\frac{1}{5}$ the distance AB. The EMF of the battery is the same in every case.

(b) Plot the difference in potential between B and each intermediate point against position along AB for the three cases.

(a) The electric field is the same everywhere inside a uniform straight wire, so E must be a constant for case (1). In case (2), the sum of the currents through the two parallel wires is the same as the current passing from C to B, since no charge accumulates at C. Hence, the current through each of the parallel wires is only half as great as the current from C to B, and the electric field inside the parallel wires must be only half as great as the electric field between C and B. In case (3), the electric field between the parallel charged plates is the same at every point. When the plates are fully charged, there is no electric field between the battery and the plates: if there were, charges would be moving onto the plates or off the plates, contradicting our hypothesis that the plates are fully charged. Notice that the total area under the curve is the same in every case.





If we had plotted the force on a positive elementary charge against position, the area under the curve would have been the work done by the electric field in moving it there, and the work done represents potential energy spent:

force × distance = work = change in potential energy.

Then,

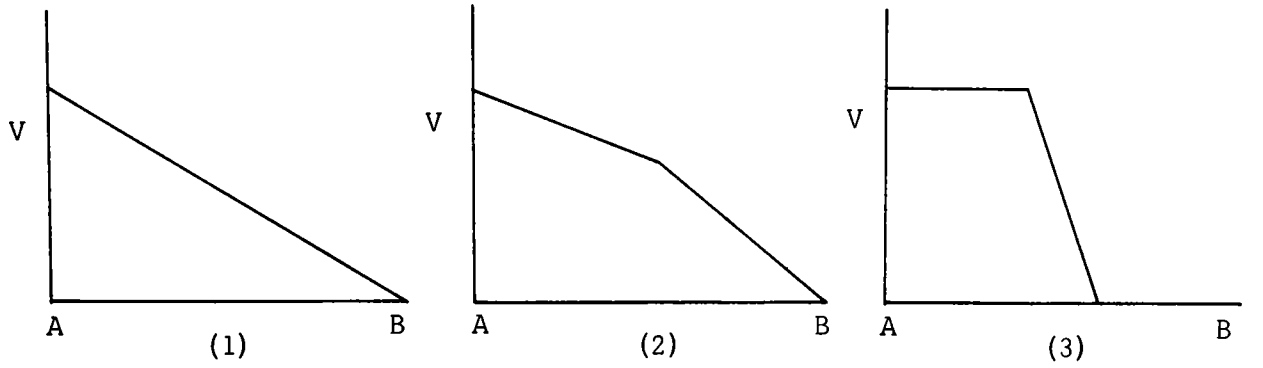
$$\frac{\text{force}}{\text{elementary charge}} \times \text{distance} = \frac{\text{change in potential energy}}{\text{elementary charge}},$$

or

electric field intensity × distance = change in potential,

and the difference in potential between A and B is the same in every case.

(b) Since the difference in potential between any point and B is just the electric field intensity at that point times its distance from B, we can plot the area under the graphs in part (a) from B to each point in order to obtain the graphs of potential difference with respect to B as a function of position.



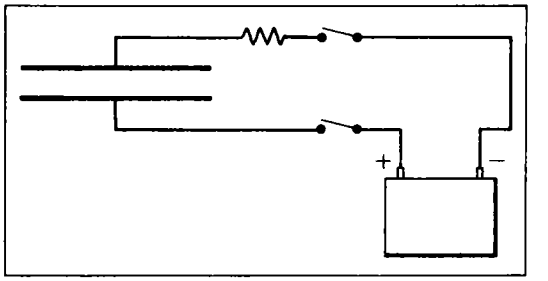
PROBLEM 7

Two uncharged parallel plates are connected as shown in Fig. 29–18. When the switches are closed, connecting the plates to a 270-volt battery, charges flow until each plate has 6.4×10^{-9} coulomb.

(a) How much work does the battery do in charging the plates?

(b) How much energy is stored in the charged plates?

(c) Where does the rest of the energy go?



(a) The energy the battery supplies per unit charge passing through it is the EMF. Therefore, the total energy supplied will be

$$\mathcal{E}Q = (270 \text{ joules/coulomb})(6.4 \times 10^{-9} \text{ coul})$$
$$= \underline{1.7 \times 10^{-6} \text{ joule}}.$$

(b) There is no electric field to work against when the first elementary charge is moved from one plate to the other. The work against the electric field required to move each succeeding elementary charge increases in direct proportion to the charge on the plates. The last elementary charge must be placed on the plates against a potential difference on the plates almost exactly equal to the EMF. Hence, the average work done by the battery in moving each charge onto the plates is one half the work done in moving the last charge. The total work required to move all the charges onto the plates is the total charge times $\frac{1}{2}\mathcal{E}$, the average work per charge:

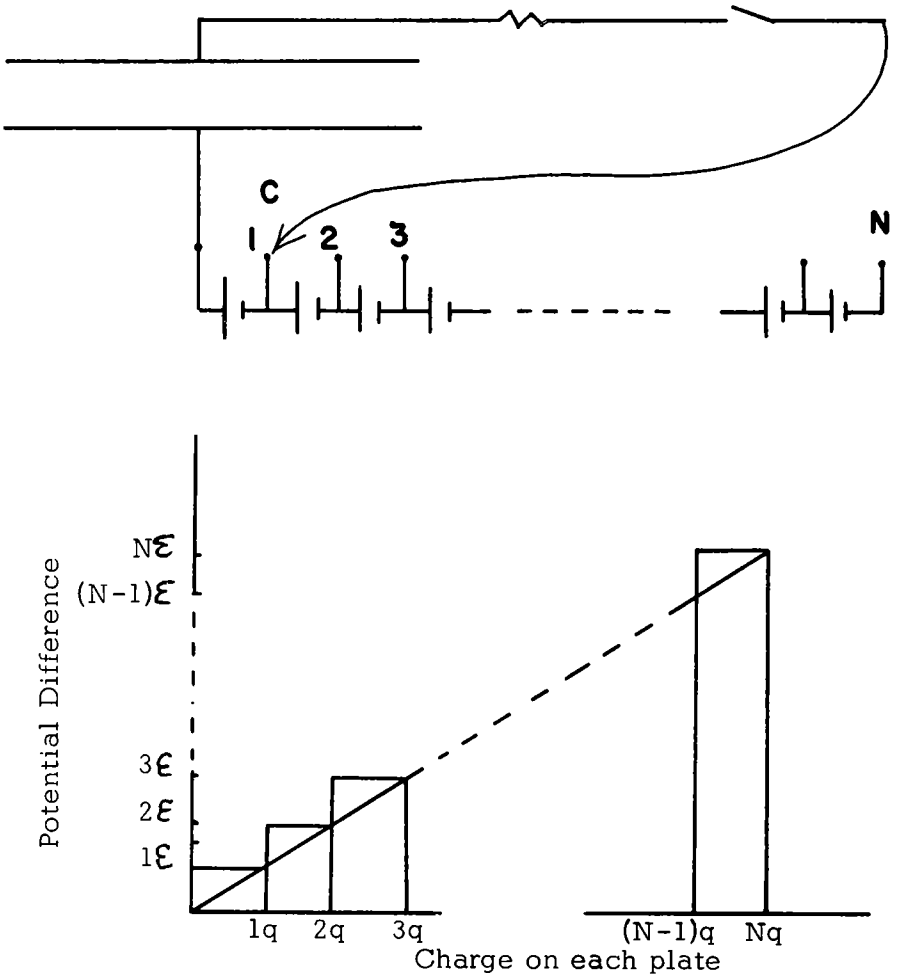
$$\tfrac{1}{2}\mathcal{E}Q = (135 \text{ joules/coul})(6.4 \times 10^{-9} \text{ coul})$$
$$= \underline{8.6 \times 10^{-7} \text{ joule}}.$$

This is the amount of energy that would be released if the plates discharged, and is therefore the amount of energy stored in them.

(c) The remainder of the energy supplied by the battery was converted to thermal energy by the resistance of the circuit. This is one half the work done by the battery.

For Further Discussion

Suppose we arrange a switch so that the battery can be connected to the plates one cell at a time. Call the EMF of each cell $\mathcal{E}$. Since they are all alike, each cell will add the same charge q to the plates. When the first cell is switched in, the work it does is $\mathcal{E}q$, represented by the first rectangle on the graph. The potential energy given to the charged plates is represented by the triangular area below the diagonal. When all the cells have been switched in, the work



they have done is represented by the sum of the areas of all the rectangles, $(1 + 2 + 3 + \cdots + N)\mathcal{E}q = N/2(1 + N)\mathcal{E}q = (N^2 + N/2)\mathcal{E}q$. As before, the potential energy given to the plates is $\frac{1}{2}(N\mathcal{E})(Nq) = N^2/2(\mathcal{E}q)$.

The ratio

$$\frac{\text{Work done by battery}}{\text{Potential energy stored}} = \frac{N^2 + N}{N^2} = \frac{N + 1}{N} = 1 + \frac{1}{N}.$$

When $N = 1$, as in the assigned problem, the ratio is 2, but if N can be large, the ratio is not much greater than 1.

If we could invent a "battery" whose EMF could be changed continuously (or in an infinite number of steps), the work it did could be made as close as we pleased to the potential energy stored.

If we have a storage battery, we can use the switch to discharge the plates through it and recover most of their potential energy. By the same kind of argument as in the charging process, the energy delivered to the battery is

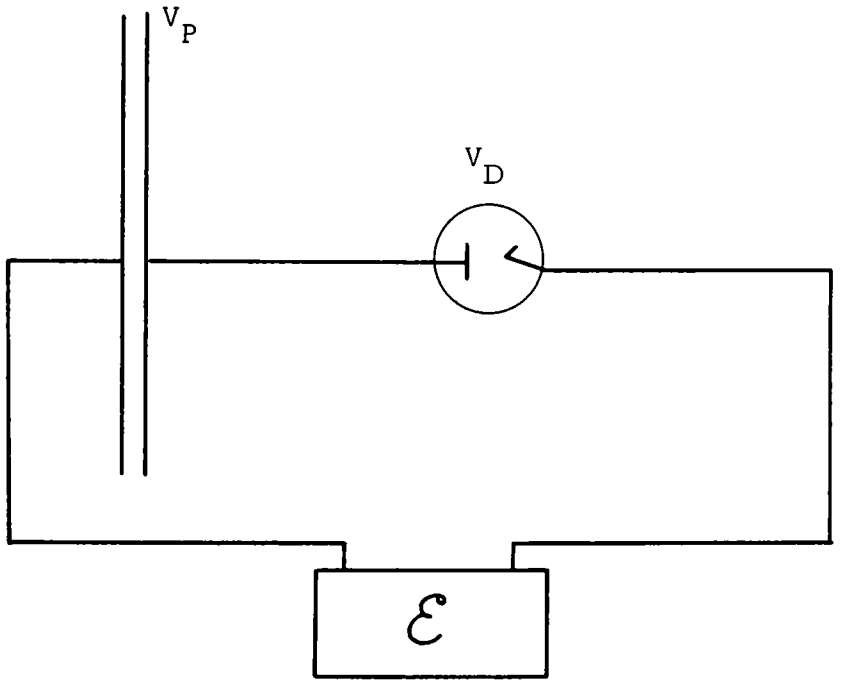
$$\frac{N - 1}{2}[(N - 1) + 1]\mathcal{E}q = \frac{N^2 - N}{2}\mathcal{E}q.$$

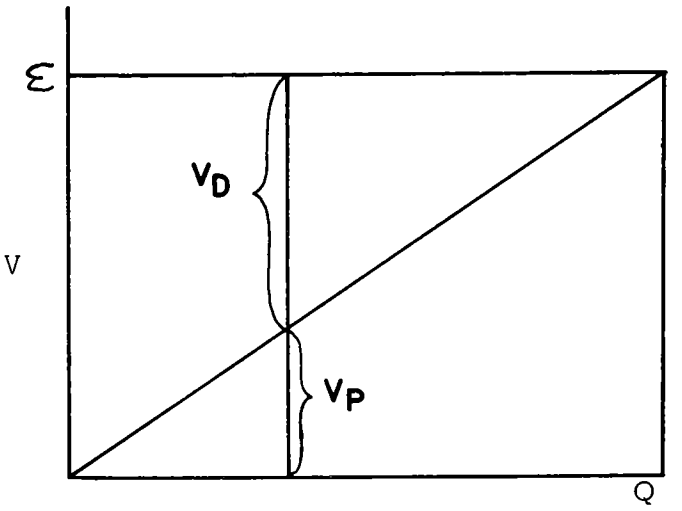
Then the ratio

$$\frac{\text{Energy restored to battery}}{\text{Energy taken from battery}} = \frac{N^2 - N}{N^2 + N} = \frac{N - 1}{N + 1}.$$

This ratio can be made very nearly equal to 1 by taking a large number of small steps in charging and discharging, and equal to 1 if the EMF of the battery can be varied continuously. When all of the energy taken from the battery can be restored to it, we say that the process is *reversible*.

To see how the energy divides between the charged plates and the resistance of the circuit, consider a charging circuit in which the resistor has been replaced by a diode. The battery crowds electrons onto the cathode or the diode; those that are boiled off in thermionic emission are strongly repelled. They acquire large speeds, and when they hit the anode, their kinetic energy is converted into internal (thermal) energy, raising the temperature of the anode. As electrons accumulate on the anode, the net accelerating force on the electrons crossing the diode becomes less, the velocities they pick up become smaller, and they transfer less kinetic energy to the anode. The battery gives the same potential energy to each electron; as less is converted into internal energy of the anode, more remains as potential energy for the parallel plates.





In terms of energies in the circuit, the potential difference between the parallel plates V_P is proportional to the charge on them. The EMF of the battery is fixed. A small positive charge q acquires energy $\mathcal{E}q$ from the battery and transfers $V_P q$ as potential energy to the parallel plates. The difference goes to kinetic energy in the diode and from that to internal energy. A potential difference V_D appears across the diode. On the graph, the whole rectangular area represents the work done by the battery. The area of the lower triangle represents the potential energy of the parallel plates. The area of the upper triangle is the kinetic energy of the electrons crossing the diode which is transformed to thermal energy of the anode.

On this model, we can think of a resistor as a long series of tiny diodes, in each of which electrons are accelerated and transfer their kinetic energy to thermal energy of the whole body. The V_D of the graph is then the sum of all these energies transferred per unit charge transported.

PROBLEM 8

The two plates of Problem 7 are charged with the 270-volt battery so that each plate has 6.4×10^{-9} coulomb or 4.0×10^{10} elementary charges. The battery is disconnected and the negative plate is heated until electrons are emitted from its surface and accelerated toward the positive plate.

(a) What is the kinetic energy in joules of an electron arriving at the positive plate when this process has just begun?

(b) What is the kinetic energy of one of these electrons when 3.0×10^{10} elementary charges are left on the negative plate?

(c) What is the total kinetic energy which these electrons bring to the positive plate if the process continues until the plates are completely discharged?

(a) $E_K = (270 \text{ volts}) \times (1.60 \times 10^{-19} \text{ coulomb})$
$= \underline{4.32 \times 10^{-17} \text{ joule}}.$

(b) The potential difference between the plates is proportional to the charge on them. Hence,

$$E_K = \left(\frac{3 \times 10^{10} \text{ elem.ch.}}{4 \times 10^{10} \text{ elem.ch.}} \times 270 \text{ volts}\right) \times (1.60 \times 10^{-19} \text{ coulomb})$$
$$= \underline{3.24 \times 10^{-17} \text{ joule}}.$$

(c) The total kinetic energy is the total charge times the average potential difference between the plates:

$$E_{tot} = Q(\tfrac{1}{2}V) = \tfrac{1}{2}(6.4 \times 10^{-9} \text{ coulomb})(270 \text{ volts})$$
$$= \underline{8.6 \times 10^{-7} \text{ joule}}.$$

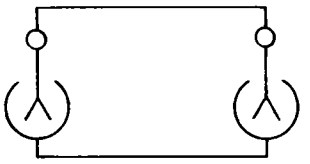
PROBLEM 9* (See Short Answers.)

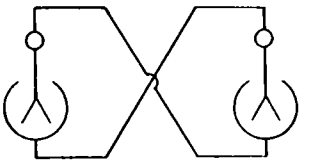
PROBLEM 10* (See Short Answers.)

PROBLEM 11

Suppose you were given two electrometers which measure potential differences in the 10,000-volt range. One of them is calibrated but the other is not. You are also given a glass rod and a piece of cloth, but you are forbidden to buy any batteries. Can you calibrate the second electrometer by connecting it to the first?

One way to calibrate the second electrometer is to connect the two electrometers in parallel, i.e., stem to stem and case to case (or stem to case and case to stem). If the glass rod is rubbed and touched to either stem or to the wire connecting the stems, we can charge the electrometers. The charge will divide in some unknown way between the two; however, the division of charge is such that the potential difference from case to stem is the same for both electrometers. If the potential difference across the two connected electroscopes were not identical, then there would be a potential difference along the connecting wires, and a current would flow until equilibrium was reached.





The calibrated electrometer indicates what this potential difference is, and the scale on the other electrometer can be marked accordingly. We repeat this a few times adding (or removing) charges each time, and the job is done.

A much simpler procedure would be to charge our rod and instead of touching it to the electrometer stems, just bring it near. The amount of induced charge (and therefore the potential difference) can be adjusted simply by changing the position of the glass rod.

PROBLEM 12

An electron gun shoots a stream of electrons between two deflecting plates of length $l = 6.0$ cm (Fig. 29-19). The electron mass is 0.91×10^{-30} kg. The potential difference between the filament and plate of the gun is 2500 volts ($2.5 \times 10^3 \times 1.6 \times 10^{-19}$ joule per elementary charge). The electric field E between the deflecting plates is 1000 newtons per coulomb (1000 newtons per 6.25×10^{18} elementary charges).

(a) If the deflecting plates are separated by 0.20 cm, what is the potential difference from one deflecting plate to the other?

(b) What is the speed of the electrons leaving the gun?

(c) How long do the electrons take to pass through the length l?

(d) As an electron passes between the deflecting plates, it picks up a sidewise velocity because of the electric force exerted on it. How big is this sidewise velocity when the electron leaves the deflecting plates?

(e) How far is the electron pushed sidewise in passing between the plates?

(a) Since the energy per charge = force per charge $\times$ distance, then the potential drop from one plate to the other is

$$\begin{aligned} V &= Ed \\ &= (1000 \text{ nt/coul})(2.0 \times 10^{-3} \text{ m}) \\ &= 2.0 \text{ joules/coul} = \underline{2.0 \text{ volts}}. \end{aligned}$$

(b) The accelerating potential of 2500 volts between filament and plate gives each electron a kinetic energy of 2500 electron volts, or

$$\begin{aligned} \tfrac{1}{2}mv^2 &= E_K \\ &= (2500 \text{ ev})(1.60 \times 10^{-19} \text{ joule/ev}) \\ &= 4.00 \times 10^{-16} \text{ joule}. \end{aligned}$$

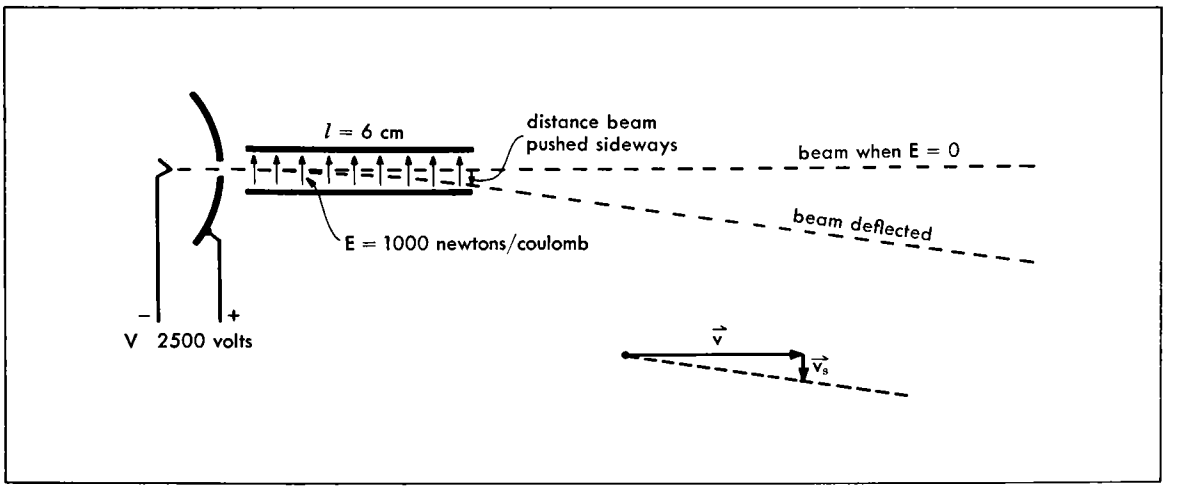
Hence,

$$v^2 = 2E_K/m,$$

and

$$\begin{aligned} v &= \sqrt{\frac{2(4.00 \times 10^{-16} \text{ joule})}{0.91 \times 10^{-30} \text{ kg}}} \\ &= \underline{3.0 \times 10^7 \text{ m/sec}}. \end{aligned}$$

(c) The experimental setup presumably has been arranged so that there is no electric field component outside the gun in the direction of the undeflected beam. (Call this direction x.)



This will be the case if the accelerating plate of the gun is at a potential intermediate between that of the two deflecting plates. If so, the x-component of velocity all the way across the deflecting plates is the same as when the electron left the gun, i.e., 3.0×10^7 meters/second. The time required for the electrons to traverse the distance $\ell = 0.060$ meter will be:

$$t = \frac{\ell}{v_x} = \frac{0.060 \text{ meter}}{3.0 \times 10^7 \text{ meters/sec}}$$
$$= \underline{2.0 \times 10^{-9} \text{ second.}}$$

(d) The electric field between the plates exerts a sidewise force $F_s = qE$ on any charge q between the plates. This force, acting for a time interval t, gives the charged particle a momentum mv_s:

$$F_s t = mv_s.$$

Hence, $$v_s = F_s t/m = qEt/m.$$

For the electron,

$$v_s = \frac{(1.60 \times 10^{-19} \text{ coul})(1.00 \times 10^3 \text{ nt/coul})(2.0 \times 10^{-9} \text{ sec})}{9.1 \times 10^{-31} \text{ kg}}$$
$$= \underline{3.5 \times 10^5 \text{ m/sec.}}$$

(e) Since the acceleration is constant, and the initial sidewise velocity is zero, the average sidewise velocity is half the final sidewise velocity. The sidewise displacement s is

$$s = v_{s_{ave}} t$$
$$= (2.0 \times 10^{-9} \text{ sec}) \left(\frac{3.5 \times 10^5 \text{ m/sec}}{2} \right)$$
$$= \underline{3.5 \times 10^{-4} \text{ m,}} \quad \text{or} \quad 0.35 \text{ millimeter.}$$

PROBLEM 13

(a) For the apparatus used in Problem 12, find a formula for the sidewise displacement of any ion accelerated through the same potential difference V in the gun and deflected by the same deflecting field E. Write the formula in terms of E, V, and ℓ, the length of the deflecting field.

(b) What is the influence of the mass of the ion?

(c) What is the influence of the charge of the ion?

(a) We repeat steps (b), (c), (d), and (e) in Problem 12.

Step (b): $qV = \frac{1}{2}mv_x^2$, hence $v_x^2 = \frac{2qV}{m}$.

Step (c): $t = \frac{\ell}{v_x}$.

Step (d): $mv_s = F_s t = qEt$, hence $v_s = qEt/m$.

Step (e): $s = \frac{1}{2} v_s t = \frac{1}{2} \frac{qEt^2}{m}$. Since $t = \frac{\ell}{v_x}$, we have $s = \frac{1}{2} \frac{qE}{m} \times \frac{\ell^2}{v_x^2}$, and from (b), $s = \frac{1}{2} \frac{qE}{m} \times \frac{\ell^2}{\frac{2qV}{m}} = E\ell^2/4V$.

(b) and (c) From part (a) the deflection is seen to be independent of the mass and charge of the ion. Of course, the ions have to be negative. If they were positive they would never leave the gun, but reversing all potential differences would restore the status quo. The trajectories of ions of one sign of charge moving under the action of a given electric field are the same for all ions regardless of mass or charge (provided they all start from rest at the same point).

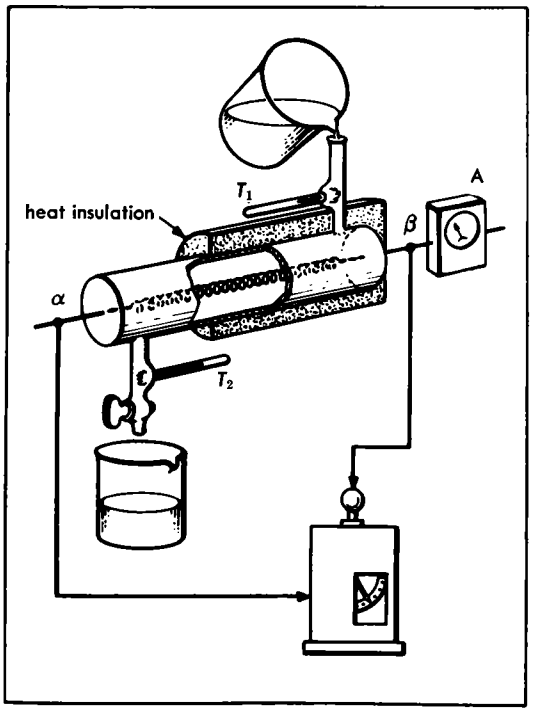
PROBLEM 14* (See Short Answers.)

PROBLEM 15* (See Short Answers.)

PROBLEM 16* (See Short Answers.)

PROBLEM 17

Figure 29–20 shows a device for calibrating the ammeter A. It is an insulated can with a coil of wire in it. Water flows in at the top of the diagram and passes thermometer T_1. Then, after flowing past the coil of wire, it flows out at the bottom of the diagram past thermometer T_2. The difference of potential across the coil in the calorimeter is measured, using a calibrated electrometer, so that we know the potential difference between the points α and β. In an experi-



ment (i) the flow of water is 1 gram per second, (ii) thermometer T_2 reads a temperature 1°C higher than thermometer T_1 and (iii) the electrometer reads a potential difference of 2.09 volts.

(a) What is the current through the ammeter?

(b) How would you calibrate an ammeter with this device?

As the electrons flow through the coil they gain energy in an amount equal to the work done by the electric force. However, they almost immediately transfer the energy acquired to the atoms of the metal. When the current is first turned on, the wire heats up. The wire soon becomes hotter than the surrounding water and loses heat to the water. This exchange quite rapidly reaches a steady state in which the temperature of the wire is constant and the energy transferred to the electrons by the electric field gives rise to an equivalent amount of heat transferred to the water. Consider what happens in one second (after a steady state has been reached).

(a) In one second, 1 gram of water passes through. The temperature of the gram of water is increased by 1°C, so the energy transferred to the water is 1 calorie = 4.18 joules (see page 455). The work done by the electric field on a charge q is qV = 4.18 joules to the charge q passing in one second must have been:

$$q = \frac{4.18 \text{ joules}}{2.09 \text{ volts}} = 2.00 \text{ coulombs.}$$

A flow of 2.00 coulombs in 1 second is a current of 2.00 amperes.

(b) To calibrate an ammeter we connect it in series as shown in the figure. The ammeter will carry the same current as the coil. By varying the applied EMF (not shown) which causes the current to flow, we can vary the potential difference between α and β. For each V we measure with the electrometer, we can calculate the current.

PROBLEM 18

In the circuit shown in Fig. 29–21, two resistors, a diode, an electric motor M, and an electrolytic cell are connected in series with a battery. The difference of potential across each of these circuit elements is measured and it is found that they are all the same. A certain amount of heat is dissipated per second in the first resistor. Compare this with the amount of heat dissipated per second:

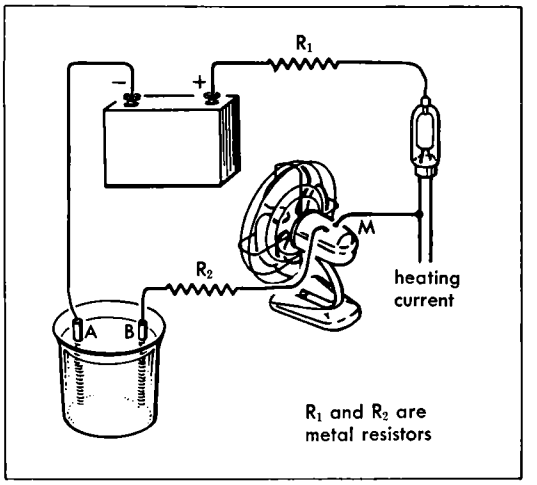
(a) In the second resistor.

(b) In the diode.

(c) In the motor which runs a fan.

(d) If there is actually less heat dissipated in the electrolytic cell than in resistor R_1, which terminal, A or B, would be positive when the cell is used as a source of potential difference.

The principle of energy conservation enables us to make useful qualitative statements about the behavior of each of the elements of the circuit. The same current must flow through each of the elements, since they are connected in series, and we are told that the drop in electrical *potential* is the same across each element. We can conclude that the same amount of *energy* is supplied to each element per unit time. For each element, the amount of heat dissipated will then depend on what else is done with the energy supplied.



(a) The resistors are simply long metal wires. They can do nothing with the energy they receive except dissipate it as heat. Since they both receive the same energy per second, they must dissipate the *same* amount of heat per second.

(b) In the diode, as in part (a), the only place the electrical energy can go is into heat. All of it will be converted into heat, and consequently the same amount of heat is dissipated as in the resistors. The conversion of electrical energy into heat is a two-step process: the potential across the diode accelerates the electrons from the filament to the plate, changing electrical potential energy into translational kinetic energy of the electron. This kinetic energy is then converted to heat when the electrons collide with the plate.

In the above, we have assumed that the diode requires no other source of energy to perform its function. This would be true for a crystal diode, for example. For a vacuum tube diode such as is shown in the figure, however, an auxiliary source of current is required to heat the filament so that the electrons can escape more easily and travel to the plate. Heat will be dissipated at the filament due to

the passage of this auxiliary current, but the extra heat due to the current in the main circuit will still be the same as that dissipated in the resistors.

(c) The motor is not an isolated system; it is turning a fan which moves air, and this requires energy. If the motor is efficient, only a small part of the energy supplied to it will be dissipated as heat within the motor itself. Most of the energy will go into increasing the kinetic energy of the air molecules, thus warming up the air.

(d) The passage of a current in the electrolytic cell causes chemical changes in it, i.e., the *chemical energy* of the cell changes. It can increase or decrease, depending on which way the cell is connected in the circuit. If it *increases*, then some of the electrical energy provided by the battery is used in accomplishing this change, so that only a part will be dissipated in heat. In this case *less* heat will be dissipated than in the resistor. On the other hand, if the chemical energy *decreases*, then this amount of energy plus the energy from the battery must all be dissipated in heat. Then the amount will be *greater* than in the resistors.

Since, in this problem, we are told that the cell dissipates less heat than the resistor, the cell is connected so as to increase its chemical energy. This means that passing current from terminal B to terminal A through the cell increases the internal energy of the cell. If the cell is now placed in a circuit in which it is the only source of energy, its internal energy will tend to decrease, which will mean that current will flow from A to B through the cell. In the circuit, the current will thus flow from B to A. Thus B is the positive terminal.

A circuit in which all of the circuit elements have an equal potential drop is a very special one. In practice, each of these elements — resistors, diodes, motors, and cell — can have widely varying characteristics in regard to potential drop and heat dissipated.

PROBLEM 19

(a) In Problem 17, what is the resistance in ohms of the coil?

(b) Show that the power dissipation from the coil is given by I^2R.

(a) The current in the coil was calculated to be 2.00 amperes, and the potential difference across it is 2.09 volts. The resistance is given by Ohm's law, $V = RI$:

$$R = \frac{V}{I} = \frac{2.09}{2.00} = \underline{1.05 \text{ ohms.}}$$

(b) The power dissipation is the rate at which energy is dissipated as heat. From Problem 17, this is 4.18 joules per second. But

$$I^2R = (2.00 \text{ coul/sec})^2(1.05 \text{ volts/amp}) = \underline{4.20 \text{ joules/sec.}}$$

Hence, this result is correct to within a half percent. This is no surprise, since we used the rate of heat transfer to calculate the current in the first place.

PROBLEM 20

If the conduction electrons in a metal move freely through it like the molecules of a gas, what is the mean thermal speed of the electrons at 27°C?

The temperature of a gas is related to the average kinetic energy of the center-of-mass motion of the gas molecules by

$$E_K = \tfrac{1}{2}mv^2 = \tfrac{3}{2}kT,$$

where k is the Boltzmann constant and T is the absolute temperature. Hence, for the electrons in a metal,

$$v^2 = 3kT/m$$

and

$$v = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ joule/°K})(300\text{°K})}{9.11 \times 10^{-31} \text{ kg}}} = \underline{1.17 \times 10^5 \text{ m/sec.}}$$

PROBLEM 21

(a) A copper wire has n conduction electrons per unit volume. The wire is carrying a current I produced by a systematic drift of the conduction electrons parallel to its length with an average velocity v. The area of cross section of the wire is A. How many electrons pass a marked point on the wire during the time Δt?

(b) If each electron carries a charge q, what is the current in the wire?

(c) Suppose that each copper atom in the wire contributes one conduction electron. Calculate the number of conduction electrons per cubic centimeter, taking the atomic mass of copper as 63 and its density as 8.9 gm/cm^3.

(d) A common size of wire has an area of cross section of 0.83 mm^2. A reasonable current for this wire is 5 amperes. What is the systematic drift velocity of the electrons along it?

(a) The electrons which are able to pass the marked point during Δt are just those which are within a distance $v\Delta t$ behind the mark at the start of the time interval. The volume they occupy is thus $Av\,\Delta t$ and the number of electrons within this volume is $\underline{nAv\,\Delta t}$.

(b) The current equals the total charge passing per unit time:

$$I = \frac{Q}{\Delta t} = q\frac{nAv\,\Delta t}{\Delta t} = \underline{qnAv}.$$

(c) Density = mass per volume
= (number of atoms per volume) (mass per atom).

Hence, n = density/mass per atom.
Since the atomic mass of copper is 63 amu, Avogadro's number of copper atoms has a total mass of 63 grams and the mass per atom is (63 grams)/N_0. Hence,

$$n = \frac{8.9\ \text{gm/cm}^3\ (6.02 \times 10^{23})}{63\ \text{gm}}$$
$$= \underline{8.5 \times 10^{22}\ \text{electrons/cm}^3}.$$

(d) From part (b) the drift velocity is

$$v = \frac{I}{qnA} = \frac{5\ \text{coul/sec}}{(1.6 \times 10^{-19}\ \text{coul})(8.5 \times 10^{22}/\text{cm})}$$
$$= \frac{5\ \text{coul/sec}}{(1.6 \times 10^{-19}\ \text{coul})(8.5 \times 10^{22}/\text{cm}^3)(8.3 \times 10^{-3}\ \text{cm}^2)}$$
$$= \underline{4.4 \times 10^{-2}\ \text{cm/sec}}\ \textit{or}\ 0.44\ \text{mm/sec}.$$

This result should be compared with the random velocity of thermal motion calculated in Problem 20. Because of the enormous number of conduction electrons in copper, even large currents produce only very slight disturbances in their ordinary thermal motion. The assumption of one conduction electron per atom is reasonable, but if it should be wrong by a factor of 2 or 5, the contrast in velocities will not really be affected.

PROBLEM 22* (See Short Answers.)

PROBLEM 23

We have two lengths of wire made of exactly the same metal and of exactly the same cross-sectional area. One wire is twice as long as the other.

(a) When the same difference of potential is imposed on the ends of both wires, what is the ratio of the currents that will pass through the wires?

(b) How does the resistance of a wire depend on its length?

Problems 23 and 24 together give the dependence of the resistance of a wire on its length (Problem 23) and its cross-sectional area (Problem 24).

(a) The drop in potential between the ends of wire B is the same as that between the ends of wire A. Since in B the ends are twice as far apart as in A, the electric field E in B is only half of the electric field in A. Because the electric current is the result of a small drift velocity of the electrons superposed on their rapid thermal motions, and the drift velocity is itself proportional to the electric field, then the drift velocity of the electrons in B is therefore only half of that in A, i.e., the current is halved. The *resistance* of the wire is defined by Ohm's law:

$$V = IR;\ R = V/I.$$

Since V is the same for both wires, the *resistance* of B is *twice* that of A.

(b) If we have two wires of identical material and cross section, but one is n times longer than the other (where n is a whole number), we can repeat the above argument to show that if the same potential is applied to each the current in the longer wire is $1/n$ times the current in the shorter wire. The *resistance* of the longer one is therefore n times that of the short one; i.e., *the resistance is directly proportional to the lengths.* It is then easy to see that this will hold for any lengths of the two wires; the resistance (for wires of identical cross-section and composition) is proportional to the length of the wire.

PROBLEM 24

There are two wires of identical material of exactly the same length and the same cross-sectional area.

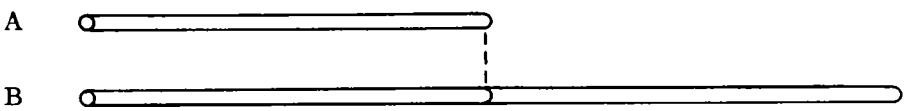
(a) If the same difference of potential is imposed across their ends, what is the ratio of the current that will pass through them?

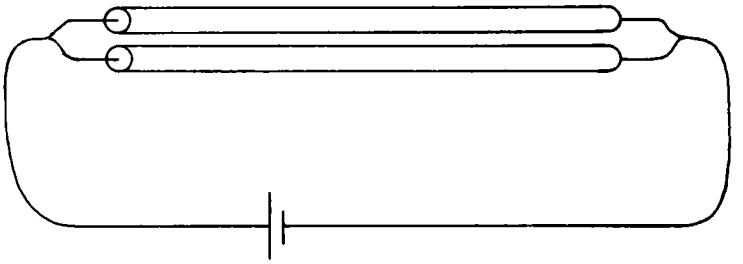
(b) If the wires side by side are viewed as a single conductor, what is the ratio of the current through this combined conductor to the current through one of the single wires with the same potential difference across its ends?

(c) If we have a wire of twice the cross-sectional area and the same length and material as each of the original ones, what current do you expect will flow through it compared to one of the original wires, with the same potential difference across its ends?

(d) What do you expect to be the dependence of the resistance of a wire on its cross-sectional area?

(a) If two identical wires have equal potential differences across their ends, the currents induced





must be identical. It does not affect this result if one source of potential is used for both wires simultaneously. As far as the electrons in the wires are concerned, the electric force they experience is just the same.

(b) The total current passed by the two identical wires is thus twice the current in either of them.

(c) The current flow is along the length of the wires, in the direction of the electric field. There is no flow of current across the cylindrical surfaces of the wires. Hence it does not matter if they touch along their length. Thus we could touch the two wires of part (b) together along their length and regard the pair as one wire. The current in this "new" wire would be twice as great as in either of the original parts, and the cross-sectional area would have been doubled.

We could imagine each wire to be made up of many smaller, parallel wires having the same total cross-sectional area. The total resistance would still be the same as that of the original wire. Imagining that each wire of the pair is made up in this way, we see that the resistance of the two depends only on the total cross-sectional area, and not on their shape. Thus one cylindrical wire having the same cross-sectional area as the sum of the two would have the same resistance.

Whether we have two identical wires or a single wire of equivalent cross-sectional area, a given potential difference will force twice as much current through them (or it) as it will through a single one of the original wires. By Ohm's law, $V = RI$. Since I is doubled and V is the same, the resistance must be halved.

(d) The argument of part (c) leads us to believe that the resistance is inversely proportional to the cross-sectional area.

PROBLEM 25

In Fig. 29–22(a), the current through a metal circuit is plotted as a function of the potential difference across its ends. In Fig. 29–22(b), we plot the current through a diode as a function of the potential difference across it.

(a) If a current of 5×10^{-3} ampere passes through both circuit elements in series, what is the potential difference from one end to the other of the resistor? What is the potential difference across the diode?

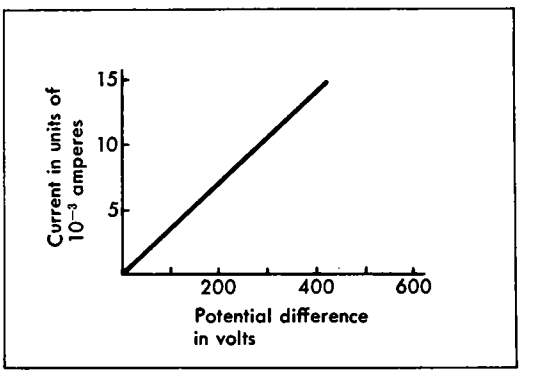
(b) If the diode and the resistor are connected in parallel as in Fig. 29–22(c), and the potential difference across the diode is 100 volts, what is the current through each of the circuit elements?

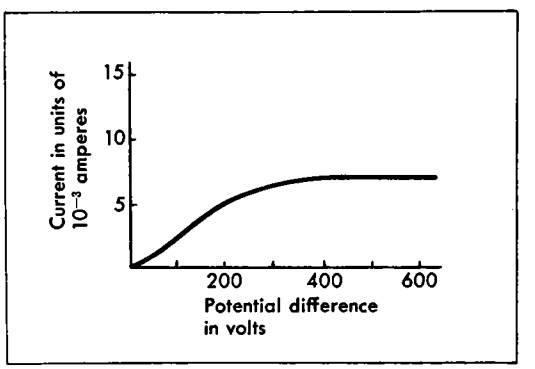
(c) What then is the total current passing point A in the figure?

(d) What is the resistance in ohms of the resistor?

(a) From Figure 29–22(a) we read that when $I = 5 \times 10^{-3}$ ampere, $V = 140$ volts. The same current in the diode (Figure 29–22(b)) is produced by a potential difference across the diode of 200 volts.

(b) In circuits of this kind (Figure 29–22(c)) one assumes that the wires themselves have negligible resistance. Thus the drop in potential between the positive terminal of the battery





and the adjacent end of the resistor is negligible. Similarly the corresponding ends of the resistor and diode effectively are at the same potential, so they have equal potential differences across them.

When the potential difference across the diode is 100 volts, the current it passes is about 2×10^{-3} ampere. We have argued above that the same potential difference will appear across the resistor, and from the graph we can find that 100 volts drives a current of 3×10^{-3} ampere through the resistor.

(c) The total current at A is the sum of the currents in the resistor and in the diode, i.e., $(2 + 3) \times 10^{-3}$ ampere = 5×10^{-3} ampere.

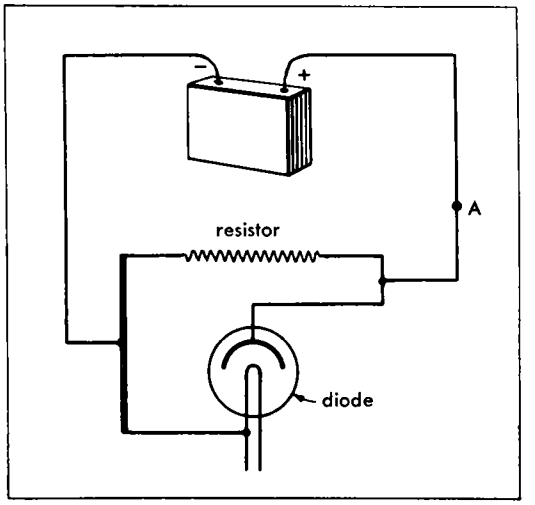
(d) The resistance of the resistor is, by Ohm's law, V/I. This is the reciprocal of the slope of the straight line in Figure 29–22(a). If the axes of the graph of Figure 29–22(a) were interchanged, then the resistance would be simply the slope of the straight line. We read on the graph that $V = 0$ when $I = 0$, and $V = 430$ volts when $I = 15 \times 10^{-3}$ ampere. Then R is given by:

$$R = \frac{430 \text{ volts} - 0 \text{ volts}}{15 \times 10^{-3} \text{ amp} - 0 \text{ amps}}$$
$$= 2.9 \times 10^{4} \text{ ohms}.$$

PROBLEM 26

The same diode and resistor as were used in Problem 25 are connected in series. The potential difference from one end of the circuit to the other is 1000 volts.
(a) What current flows through the circuit?
(b) What is the potential difference between one end and the other of the resistor?

(a) Since the behavior of the diode cannot be expressed simply in terms of Ohm's law, but must be read from the graph (Figure 29–22), the general problem would have to be solved graphically. For this particular case, however, the potential difference is so high that we can give the answer without actually drawing a graph.

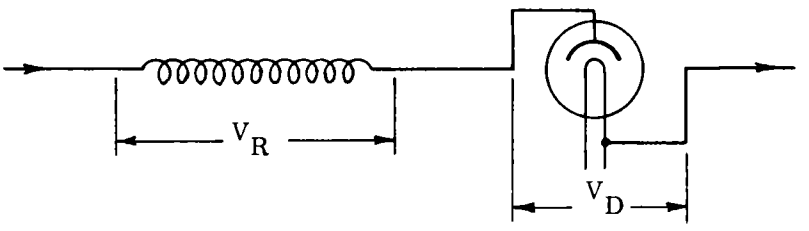


We are given the total potential drop across both diode and resistor, and it must be the sum of the potential drops across each:

$$V_R + V_D = 1000 \text{ volts}.$$

The same current flows through each. The maximum current we can get is limited by the diode, and from Figure 29–22(b) we read it to be 7×10^{-3} ampere. We shall get this provided V_D is greater than about 400 volts. We can now read from Figure 29–22(a) that a current of this magnitude requires that $V_R = 200$ volts. Then V_D must in fact be (1000 − 200) volts = 800 volts, which is greater than the potential required to give the saturation current in the diode. This completes the circle of our argument, and justifies our guess that the current is in fact the saturation current of the diode, 7×10^{-3} ampere.

(b) We have already calculated that the potential difference across the resistor is 200 volts. The general way to solve this problem for any applied potential is to consider the reverse situation, that the current is known, and calculate the total potential drop $V_R + V_D$. We can then draw a graph of $V_R + V_D$ vs. I,



CURRENT (in 10^{-3} ampere)	V_R (in volts)	V_D (in volts)	$V_R + V_D$ (in volts)
0	0	0	0
1	30	40	70
2	60	90	150
3	90	120	210
4	110	160	270
5	140	200	340
6	170	270	440
7	200	400 and greater	600 and greater

and interpolate graphically to find I for a given $V_R + V_D$. Reading from Figures 29–22(a) and 29–22(b), we can make up the table above. From this we then draw the graph below.

The solution to the original problem can now be read from this graph. We see that it lies in the high-voltage region, where the behavior is very simple. Suppose the total potential drop is maintained at 400 volts. We can then read from the graph that the current will be approximately 5.6×10^{-3} ampere, and that the potential drop across the resistor alone is 160 volts.

PROBLEM 27

Connect a 270-volt battery in series with a 1,000,000-ohm and a 2,000,000-ohm resistor.

(a) What potential difference do you expect across the megohm resistor?

(b) What potential difference do you expect across the 2,000,000-ohm resistor?

Now suppose you measure the potential differences with a voltmeter.

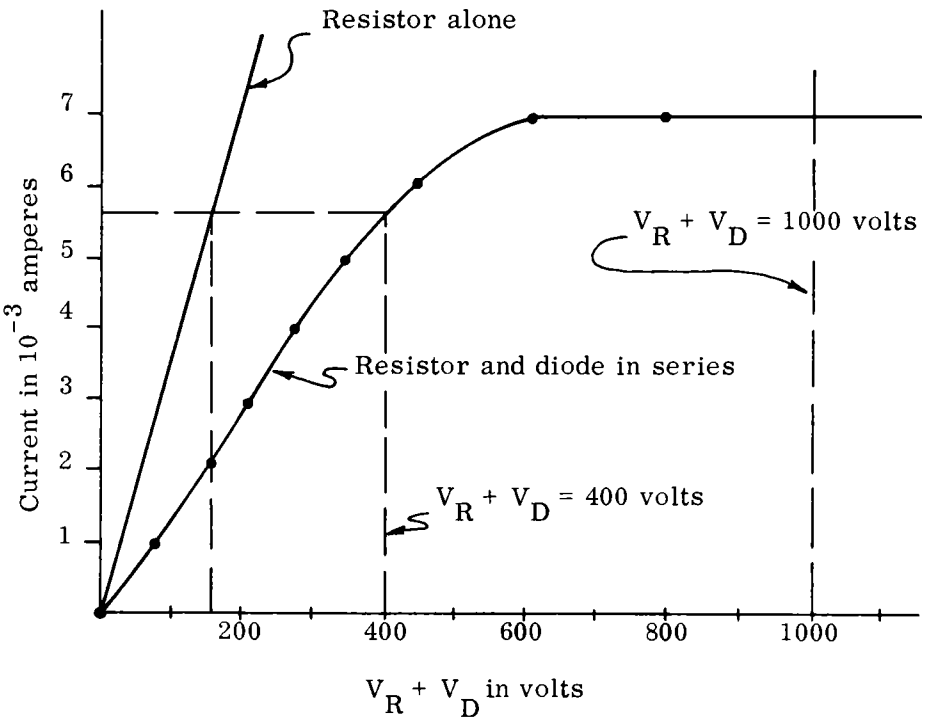
(c) If the voltmeter has a resistance of 10 megohms, what potential difference would you expect across the 2,000,000-ohm resistor?

(d) What potential difference would you expect to read across the megohm resistor?

This is a straightforward problem in the application of Ohm's law to a simple series-parallel circuit. If you discuss the problem in class, it is best to work through the solution following the method outlined in the text, without stressing "formulas" for circuits.

(a) We see that the sum of the potential differences across the two resistors is

$$V_1 + V_2 = 270 \text{ volts.}$$



Since the current is the same everywhere in the circuit,

$$V_1 = IR_1, \text{ and } V_2 = IR_2.$$

Thus,

$$V_1 + V_2 = I(R_1 + R_2),$$
$$I = V_1 + V_2/R_1 + R_2.$$

The potential difference across the megohm resistor is

$$V_1 = IR_1 = \frac{(V_1 + V_2)R_1}{R_1 + R_2}$$
$$= \frac{270v \times 1 \text{ meg}}{3 \text{ meg}} = \underline{90 \text{ volts}}.$$

(b) The potential difference across the 2-megohm resistor is

$$V_2 = IR_2 = \frac{(V_1 + V_2)R_2}{R_1 + R_2}$$
$$= \frac{270v \times 2 \text{ meg}}{3 \text{ meg}} = \underline{180 \text{ volts}}.$$

(c) With the meter placed in the circuit across the 2-megohm resistor, we now have the equivalent of a 10-megohm resistor in parallel with R_2. As in (a),

$$V_1 + V_2 = 270 \text{ volts},$$
$$V_1 = IR_1,$$
$$V_2 = I_aR_2 = I_bR_m.$$

Since the currents I_a and I_b must equal the total current,

$$I = I_a + I_b = V_2/R_2 + V_2/R_m = V_1/R_1.$$

But $V_1 = 270 \text{ volts} - V_2$, so we can write

$$\frac{270 \text{ volts} - V_2}{R_1} = V_2\left[\frac{1}{R_2} + \frac{1}{R_m}\right]$$
$$= \frac{V_2(R_m + R_2)}{R_mR_2},$$
$$270 \text{ volts} - V_2 = V_2R_1[(R_m + R_2)/R_mR_2],$$
$$270 \text{ volts} = V_2 + V_2\left[\frac{R_1(R_m + R_2)}{R_mR_2}\right],$$
$$= V_2\left[1 + \frac{R_1(R_m + R_2)}{R_mR_2}\right].$$

Hence,

$$V_2 = \frac{270 \text{ volts}}{1 \times \dfrac{R_1(R_m + R_2)}{R_mR_2}}$$
$$= \frac{270 \text{ volts}}{1 + \dfrac{1 \text{ meg (12 meg)}}{20 \text{ meg}^2}}$$
$$= \frac{270}{1 + \frac{3}{5}} = \frac{5}{8} \times 270 = \underline{170 \text{ volts}}.$$

(It may be interesting to solve for the other voltages and currents:

$V_1 = 270 - V_2 = 100$ volts;
$I = V_1/R_1 = 100 \text{ volts}/10^6 \text{ ohms} = 10^{-4}$ amp;
$I_a = V_2/R_2 = 170 \text{ volts}/(2 \times 10^6 \text{ ohms}) = 0.85 \times 10^{-4}$ amp;
$I_b = I - I_a = 0.15 \times 10^{-4}$ amp.

(d) Now $R_1 = 2$ meg, $R_2 = 1$ meg, so

$$V_2 = \frac{270 \text{ volts}}{1 + \dfrac{2 \text{ meg (11 meg)}}{10 \text{ meg}^2}} = \frac{270}{1 + \frac{11}{5}}$$
$$= \frac{5}{16} \times 270 = \underline{84 \text{ volts}}.$$

* * *

If they don't notice it, it is worthwhile to point out to students that the solutions to (a) and (b) do not add up to 270 volts even though the voltmeter would read 270 volts when placed across both resistances.

You might also ask whether the students can figure out how to use the two meter readings of parts (c) and (d) to find out what the voltages would have been without the meter. This is easy to do if they have solved for V_1 and V_2 with symbols. From part (c),

$$V_2 = \frac{270}{1 + \dfrac{R_1(R_m + R_2)}{R_mR_2}} = \frac{270R_mR_2}{R_mR_2 + R_mR_1 + R_1R_2},$$

and from (d),

$$V_1 = \frac{270}{1 + \dfrac{R_2(R_m + R_1)}{R_mR_1}} = \frac{270R_mR_1}{R_mR_2 + R_mR_1 + R_1R_2}.$$

Therefore, $V_2/V_1 = R_2/R_1$, so that even if the actual readings with the meter in place are not the same as without it, the ratios are the same.

Alternate solutions to part (c)

Noting that

$$I = V_2/R_2 + V_2/R_m = V_2\left(\frac{1}{R_2} + \frac{1}{R_m}\right),$$

we can define the equivalent resistance for the parallel combination R_2 and R_m by:

$$R_p = V_2/I, \text{ or } 1/R_p = 1/V_2 = 1/R_2 + 1/R_m.$$

Then

$$R_p = \frac{R_2 R_m}{R_2 + R_m} = \frac{2 \times 10}{12} = \tfrac{5}{3} \text{ meg.}$$

$$V_2 = IR_p = \frac{270 \text{ volts } R_p}{R_1 + R_p} = \frac{270 \times \frac{5}{3}}{\frac{8}{3}} = 170 \text{ volts.}$$

This more conventional method defines the equivalent resistance of a parallel combination of resistors, gives a convenient "formula" to memorize, and involves slightly simpler algebra. However, this approach tends to obscure what is actually happening to the currents and potential differences in the circuit.

chapter
30

The Magnetic Field

Chapter 30 introduces the magnetic field, its sources, the force it exerts on electric currents and moving charged bodies, the measurement of atomic masses with the magnetic mass spectrograph, and a description of the magnetic field in terms of its "circulation" around a current.

In the suggested schedules for the course it was pointed out that this chapter (together with Chapter 29 and most of Chapter 31) can be omitted without losing material essential to understanding the later chapters on atomic structure. For teachers who are pressed for time, it is worth emphasizing this point again. Chapter 30 presents a wealth of material on the magnetic field. Its approach is not the ordinary one. Rather than building the structure around concepts of magnetic poles and pole strengths, it introduces the idea of a current loop at the outset. This approach makes possible an effective organization of magnetic phenomena. While the details of the measurements that are described are not basically difficult, there are many facts. In our opinion, to do complete justice to this chapter will require about two weeks. Although the material can be treated in one week, any less time is almost sure to result in confusion. Similarly, a week will be required for Chapter 31. For this reason, if you do not have two weeks available for Chapters 30 and 31, we recommend that you skip ahead to Section 31–9.

CHAPTER SUMMARY

SECTIONS 1 THROUGH 3. The chapter begins with the use of the compass needle as a device to sense the existence of a magnetic field and its direction. Magnetic fields associated with several shapes of permanent magnets and current-carrying electric circuits are examined. A particular shape of current circuit, the loop, is used in conjunction with the earth's field to investigate the vector nature of the magnetic field. The addition of magnetic fields obeys the rules of vector addition provided that magnetic field strength is defined as proportional to the current producing it.

SECTIONS 4 THROUGH 6 introduce the force experienced by a current-carrying conductor in a magnetic field. The mutually perpendicular directions of the field, current, and force, as well as the magnitude of the force are described. A standard of magnetic field is established in terms of newtons, amperes, and meters. The application of magnetic forces in meters and motors is described as a reminder that the forces we are studying play an important role in technology. Finally, the force a magnetic field exerts on moving charged particles is derived and shown to be confirmed by experiment.

SECTIONS 7 AND 8 apply some of the ideas developed in previous sections by describing an experimental technique for measuring the masses of atomic particles.

SECTIONS 9 THROUGH 11 develop the idea of the "circulation" of the magnetic field around a steady current, starting with the field around a long straight wire, generalizing to other current configurations, and finally applying the circulation theorem to a solenoid in order to evaluate by experiment the proportionality constant between magnetic field strength and current.

SCHEDULING CHAPTER 30

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the volume summary section.

Chapter 30	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTIONS	CLASS PERIODS	LAB PERIODS	EXPERIMENTS	CLASS PERIODS	LAB PERIODS	EXPERIMENTS
1, 2, 3	1	1	IV–10	2	1	IV–10
4, 5, 6	1	1	IV–11	2	1	IV–11
7, 8	$\frac{1}{2}$	1	IV–12	1	1	IV–12
9, 10, 11	$1\frac{1}{2}$	0	—	3	1	IV–13

RELATED MATERIALS FOR CHAPTER 30

LABORATORY. Experiment IV–10, *The Magnetic Field of a Current*, uses a magnetic compass to explore the magnitude and direction of the magnetic field at the center of a loop of wire in which current is flowing. The experiment shows that magnetic fields add vectorially and are proportional to the current that produces them. This experiment is designed as an introduction to Section 3. The experiment should not take a whole period, so it can be combined with discussion.

Experiment IV–11, *The Measurement of a Magnetic Field in Fundamental Units*, uses a simple current balance to measure magnetic forces and establishes a unit of magnetic field strength. This high-priority experiment is intended to follow study and discussion of Section 4.

Experiment IV–12, *The Mass of the Electron*, uses a common "tuning-eye" radio tube and the coils used for Experiment IV–11 to determine the order of magnitude of the mass of an electron. This is a high-priority experiment. It should be preceded by Experiment IV–11, and can be done after Section 7 has been studied and discussed.

Experiment IV–13, *The Magnetic Field Near a Long, Straight Wire*, uses the methods of Experiment IV–10 to investigate the magnetic field near a current-carrying, straight wire. Magnitude as a function of distance from the wire is considered.

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILMS. "A Magnet Laboratory" by Dean Francis Bitter, M.I.T. A large magnet laboratory is used to demonstrate the magnetic effect of currents and the magnetism of iron. Running time: 20 minutes.

"Electrons in a Uniform Magnetic Field" by Professor Dorothy Montgomery, Hollins College. A spherical cathode-ray tube (containing gas to show the path of the electron beam) is used to give a fairly precise measurement for determining the mass of an electron. The electron beam is deflected by Helmholtz coils. Running time: 10 minutes.

"Mass of the Electron" by Professor Eric M. Rogers, Princeton University. A current-carrying loop of wire is used to deflect the electron beam in a cathode-ray tube. From circuit measurements and measurement of the deflection, the mass of the electron is calculated. Running time: 18 minutes.

The last two films cover similar material. One or the other, but in most cases probably not both, may be shown to advantage in connection with Sections 6 to 8.

30–1 The Magnetic Needle

PURPOSE. To introduce a familiar and yet fundamental device for sensing the existence and direction of a magnetic field.

CONTENT. A magnetic compass needle shows the existence of a magnetic field by lining up with it. The north tip of the needle points in the direction of the magnetic field.

EMPHASIS. The ideas are easy, but are basic to subsequent sections. Treat as a reading assignment, introductory to Experiment IV–10.

COMMENT. It is common to refer to the "North Pole" and the "South Pole" of a magnet as though there were equal and opposite magnetic charges at these points. As the text remarks, however, no one has been able yet to separate north and south magnetic charges although a number of experimental attempts have been made, some very recently. The text therefore does not introduce magnetic poles, but develops the picture of magnets and magnetic fields as created by electric currents. The current loop approach is quite different from the "pole" approach, even though both lead to the same answers to the questions dealt with in high school courses. Insofar as we know, no poles exist as such, only current loops.

See the Lab Notes on Experiment IV–10 for instructions on making a good compass from simple parts.

30–2 Magnetic Fields of Magnets and Currents — Magnetic Field Lines

PURPOSE. To describe the two main sources of magnetic fields: permanent magnets and electric currents; and to introduce the idea that the lines of the magnetic field, unlike the electric field, have neither beginning nor end.

EMPHASIS. Treat as a reading assignment, introductory to Experiment IV–10.

COMMENT. Oersted discovered that an electric current produces a magnetic field while giving demonstrations in a lecture on electricity in 1819. A prize had been offered fruitlessly in 1774 to anyone who could show any relationship between electricity and magnetism. Until Oersted's discovery, no one succeeded. As mentioned in the text, Oersted's experiment can be done easily; a single wire, practically any battery, and a compass needle are all that are needed. A current of 5 amperes will cause a large deflection of the compass needle.

DEVELOPMENT. This section is descriptive. It should give the student perspective for the quantitative material of the next section. Perhaps the best way of getting the ideas across would be to let the students play with bar magnets and compasses, perhaps at home. For instance, anyone with a long bar magnet and compass can see that the compass always lines up like the filings in Fig. 30–5a. The fields around bar and horseshoe magnets can easily be explored with iron filings. Rather large currents (an auto-

mobile battery is probably necessary) are needed, to produce such patterns as those shown in Fig. 30–5. Some students might like to make permanent their iron filing patterns on blue print or photographic printing paper.

30–3 The Vector Addition of Magnetic Fields

PURPOSE. To show the vector nature of the magnetic field provided that the magnetic field strength is defined to be proportional to the current producing it.

CONTENT. The vector nature of the magnetic field is investigated by testing the vector addition rule. Two fields of equal magnitude, the earth's field (horizontal component) and the field due to current in a circular loop (chosen to cancel the earth's field by experiment), are added at various angles and the vector addition rule is seen to be valid. Then, by defining field strength as proportional to the current producing it, we produce fields of different magnitudes and again find that the vector addition rules are obeyed.

EMPHASIS. The ideas of this section are essential to an understanding of the later sections of this and the next chapter. Experiment IV–10, and any extensions which the students wish to try as experimental verifications of the ideas of this section, can carry the major teaching burden for this section.

COMMENT. The burden of this section is to show that magnetic fields add vectorially, and it is here that the compass needle is a valuable measuring device. The choice of magnetic field strength as a quantity which is proportional to the first power of the current is shown clearly as a necessity in order to have B be a vector quantity.

This use of a compass needle to measure magnetic field in units of the earth's field (horizontal component) may be familiar to you as the tangent galvanometer. This method for measuring magnetic field is good only if there are no large amounts of magnetic material, like iron, nearby. If there are, the current in the circular loop will magnetize the iron and this will create a spurious field which will orient the compass needle incorrectly.

LABORATORY. Experiment IV–10 should be done before or along with this section.

30–4 Forces on Currents in Magnetic Fields — A Unit of Magnetic Field Strength

PURPOSE. For a current in a magnetic field, to find the direction of the force exerted on a current relative to the directions of the current and of the magnetic field, and to relate the magnitude of this force to the current, the field, and the geometry.

CONTENT. A right-hand rule is introduced to describe the mutual perpendicularity of I, B, and F. The force is proportional to B, I, and l where l is the length of wire in the field.

The practical unit of magnetic field is defined in terms of the force that a magnetic field exerts on a current. One unit of field is the amount that will exert a force of one newton on a wire one meter long carrying one ampere of current at right angles to the field. The unit is properly called one newton per ampere-meter and is the same as the weber/m^2 in the m.k.s. system, or 10^4 gauss in the c.g.s. system.

EMPHASIS. Understanding this section is basic to an understanding of the rest of the chapter. The student must know not only the direction of the force, but also the sense. In other words, he must know whether it is a push or a pull, up or down, right or left. This amounts to pure memorization, but it is necessary, due to arbitrariness of the relationship between the sense of rotation of a current and the direction we call positive for the field produced by the current.

COMMENT. Mnemonic devices like the right-hand rule are dangerous, and the student should realize that he still must remember what goes along the thumb and what along the fingers. He is not ready for vector product notation, but ordering the factors can be helpful. Memorize the order IB. The first goes along the thumb and the second along the fingers. The student who is familiar with right-handed coordinate systems may prefer to think of I along x, $B_\perp$ along y, and F along z. The teacher must decide whether or not to bring in the progression of a right-handed screw as I is twisted into B.

LABORATORY. Experiment IV–11 should accompany or follow discussion of this section in class. This is a high-priority experiment; if you have the time, it should be scheduled.

30–5 Meters and Motors

PURPOSE. To remind students that the forces on currents in magnetic fields are the forces that run meters and motors.

CONTENT. A D.C. ammeter and a D.C. motor are described qualitatively.

EMPHASIS. Treat as reading assignment.

COMMENT. Note that no attempt is made to derive an expression for the torque on the meter or on the motor armature.

Students should be clear that, from this introductory development of the subject, there is no way of proving that the force on a current is proportional to $B_\perp$, or that it is perpendicular to B and the current. These are experimental facts — although a detailed look at the logic of our definition of B, coupled with a theory of atomic magnetism in our compass needle would show that these facts are implicit in our

definition of B. The proportionality of the force to current and length of current-carrying wire are logical necessities.

Note for defense against a penetrating question.

An especially alert student may ask why we must use our right hand to find the direction of the force. Why is nature right-handed? This is a most difficult question to answer. The answer is that nature is not, in this regard at least, right-handed. We could have defined a C-field to be in the direction of the South Pole of a compass needle. Then the right-hand rule of Fig. 30–8 must be turned into a left-hand rule. Also, to get the force on a current in the C-field, you would put your left thumb along the current, the fingers on that hand along the C-field, and the palm would hit in the direction of the force. (Compare with Fig. 30–13.) The predictions thus made about the directions of the force would agree with experimental observations. Clearly a free choice of what we call field direction is possible; and this choice (not nature) determines the apparent handedness. Actually, several other choices can also be used to shift the apparent handedness. The observed forces are, of course, independent of these choices. There is little point in bringing this up unless the question is asked. These comments have been included simply as ammunition in case this rather penetrating question comes up.

30–6 Forces on Moving Charged Particles in a Magnetic Field

PURPOSE. To develop an expression for the force exerted by a magnetic field on a moving charged particle by assuming that the force on a current developed in Section 4 is simply the sum of the forces on the elementary particles of which the current is composed.

CONTENT. The relation between a stream of charged particles and a current is developed. The result is inserted into the expression for the force exerted on a current by a magnetic field, giving the force on a moving charge. This result is compared with experiment by using it to calculate the radius of curvature of the path of a proton of known momentum in a magnetic field.

EMPHASIS. It is important to recognize the equivalence of a current and a stream of charged particles as far as deflection in a magnetic field is concerned. Not only does this equivalence have practical applications, but its recognition gives an orderliness and a consistent aspect to the scheme and makes it less a matter of rote learning. This expression $F = qvB_\perp$, of course, has many applications.

DEVELOPMENT. The connection between the number of elementary charges per unit length of the wire and the number that pass a particular point per second is a slippery one that always gives some students trouble. Discussion of Fig. 30–19 should help clarify this point.

There is a lot of physics in the proton example at the end of this section and its further development in Section 7. Concepts of energy, centripetal force, and the conditions for circular motion all enter. If you have time it will be worthwhile to consider this in detail. If you don't have time to take this up in detail, it will be well to pass by quickly rather than give it an "intermediate" treatment. In this case, probably only a few students will understand anything of the detail, but the majority should be aware that such a measurement can be made.

LABORATORY. If you have the time, Experiment IV–12 is an excellent experiment. It must be preceded by Experiment IV–11. Since there is little laboratory work in subsequent chapters, this experiment may be delayed if your schedule suggests it.

30–7 Using Magnetic Fields to Measure the Masses of Charged Particles

PURPOSE. To indicate how the masses of charged particles can be measured by accelerating them to a known energy and observing their deflection in a known magnetic field perpendicular to their velocity.

CONTENT. The kinetic energy of the particle is computed according to the result of Sections 6 and 7, namely $\frac{1}{2}mv^2 = qV$, where q is the charge and V the accelerating potential. The momentum relation $mv = qBr$ is derived. These two relations are combined to solve for the velocity and then for the mass.

EMPHASIS. If you have the time, $1\frac{1}{2}$ class periods at least, you can treat this material thoroughly. It is elegant. However, if you cannot do it well, don't expect your average students to get it. If you are short of time, it is best to treat it as a reading assignment, and spend no class time on it.

COMMENT. Magnetic field strengths are given in newtons/(elem. chg./sec)(meters) in this section and the preceding. They seem to be just fine for a reasonable deflection, but the numbers are so small that they may not make much impression. It may be interesting to note that electromagnets with iron cores produce fields up to about 2 newtons/ampere-meter and to see that the fields cited indeed are reasonable. The procedure of plugging numbers into the equations immediately to solve explicitly for the velocity and then for the mass is clear and straightforward. Some students may prefer to solve algebraically for $m = qB^2r^2/2V$ before substituting numbers.

Note that the method given here for measuring the masses of atomic particles supplements the time-of-flight method given in Section 28–1. The time-of-flight method was introduced so the section on magnetism could be skipped. The magnetic method

of measuring masses is, however, much more elegant, practical, and commonly used.

FILMS. The films "Electrons in a Uniform Magnetic Field" and "Mass of the Electron" show two different experimental techniques for electron mass measurements.

30-8 What Alpha Particles Are

PURPOSE. To describe the use of crossed electric and magnetic fields to identify fast-moving, charged particles.

CONTENT. Two experiments on polonium alpha particles by Rutherford and Robinson are described. (1) The momentum of the alpha particles is measured in a magnetic field. (2) Crossed electric and magnetic fields adjusted for minimum deflection of the beam give the speed as $v = E/B$. Dividing the momentum by the speed gives the mass, which is the same as that of a helium atom.

EMPHASIS. The deflection of alpha particles will be used again in an important way in Chapter 32 as method of probing atomic structure. Like the mass spectrograph, the crossed field method of velocity determination is both an important technique and a good example of experimental design. Treat as little more than a reading assignement.

30-9 Magnetic Field near a Long Straight Wire
30-10 Circulation

PURPOSE. To establish the magnetic field at a distance r from a long, straight wire and to generalize this result using the concept of the "circulation" of magnetic field around the current producing it.

CONTENT. The experimental result of Biot and Savart for the field due to a long, straight current is described. Along a circular path concentric with the current, the circulation of B is defined as the strength of B multiplied by the length of the closed path to which it is tangent. Thus, for the concentric, circular path the circulation is just: $2\pi rB = KI$, where K is a proportionality constant. The circulation therefore depends only on the magnitude of the current passing along the axis of the circular path and not at all on the radius of the path!

Furthermore, this is shown to be only a special case of a more general circulation law which says: Along *any* closed path surrounging I, multiply each segment of path length by the component of the local B field lying along that segment and add together all of these contributions. This sum will be the "circulation," and it will still be exactly equal to KI, where I is now the total current threading through the closed path. The circulation depends *only* on the net current which threads the closed path.

DEVELOPMENT. Section 9 can best be handled through Laboratory Experiment IV-13 if you have the time. If not, you will simply have to present this as an historic fact which could be confirmed easily in the laboratory. Section 10 will offer considerably more trouble because of the abstract nature of the "circulation." The circulation around the long wire of Section 9 can easily be verified for the concentric, circular path, and most students should readily agree that this circulation is independent of distance from the wire. Those who wish to choose noncircular or nonconcentric paths can compute the circulation around a straight wire for such paths from a graphical analysis. Experimental measurements of circulation are quite tedious and time-consuming and are not really recommended at this point in the course. It is hoped that a film will be available in the near future to help in teaching these points. It should be quite easy for your students to convince themselves that the circulation is zero when the path does not surround a current for the two extreme cases of a uniform field (e.g., the earth's field) and for the field of the straight wire. Again, these must be done analytically rather than experimentally, and certain shaped paths are much easier than others.

30-11 Uniform Magnetic Fields

PURPOSE. To evaluate by experiment the constant in the circulation law and thereby obtain the complete equations for the magnetic field produced by various current configurations.

CONTENT. The circulation law is applied to the field of a long solenoid, taking a specially chosen rectangular path for the calculation. This enables us to predict the field inside the solenoid in terms of the current and the number of turns per unit length, except for the unknown constant in the circulation law. Using the equipment of Experiment IV-10, the current balance, it is then possible to measure both the current and the field in absolute units and establish the constant experimentally as

$$K = 2.00 \times 10^{-7}\ \text{nt/amp}^2.$$

The reason for such a simple number for the proportionality constant is shown to be due to a previous happy choice of units for the ampere in terms of elementary charges.

DEVELOPMENT. The only hard part of this section is the evaluation of the circulation over the special path around the solenoid. After this is convincingly established, it might be wise to set up the current balance again and allow some of the students to try the experiment mentioned here to evaluate the circulation constant.

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The following table classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1.* The filings would try to stand up at right angles to the paper.
2.* No; they are all closed loops of various shapes.
3.* Clockwise, in both cases.
4.* The magnitude of the resultant field is $\sqrt{2}$ times the magnitude of the field from one.
5.* The field is proportional to the magnitude of the current.
6. (a) 60° east of north; (b) If $\vec{\mathbf{B}}$ is east, $B = B_c$.
7. At 37° to the vertical.
8. Eight.
9.* Parallel to the field, in either the same or opposite direction as the field.
10.* Downward.
11.* $F = 2.1$ newtons.
12. (a) Zero;
(b) 2×10^{-4} newton downward;
(c) 2×10^{-5} kg;
(d) See discussion on page 30–9.
13. See discussion on page 30–9.
14. (a) 4.4 amperes;
(b) 0.15 newton/ampere-meter.
15.* The force is directly proportional to the angle of rotation.
16.* Switch the connections to the battery terminals, or reverse the direction of the magnetic field — but not both.
17.* The radius increases as the square root of the accelerating potential.
18.* The velocity of the charge, the sign and magnitude of the charge, and the component of magnetic field perpendicular to the velocity.
19. Their momenta are equal in magnitude.
20. See discussion on page 30–12
21. (a) $r = (2mE_K)^{1/2}/qB$; $T = 2\pi m/qB$.
(b) Radius would be twice as great, period unchanged.
(c) $T = 3.12$ microseconds; $r = 4.67$ meters.
22.* 360 electron volts $= 5.76 + 10^{-17}$ joule.
23. Accelerating potential difference, charge of particle, magnetic field strength, and radius of curvature of particle path in magnetic field.
24. (a) $r^{+} = 0.18$ meter; $r^{++} = 0.13$ meter;
(b)–(d) See discussion on pages 30–13 and 30–14.
25. (a) 8.8×10^{-4} newton;
(b) A parabola;
(c) 1.1×10^{-8} newton.
26. 5.0×10^{4} meters/sec.
27. 0.02 meter.
28.* $2\pi KI$.
29.* Zero.
30.* $B = 2\pi \times 10^{-3}$ newton/amp-m $=$ 6×10^{-3} newton/amp-m.
31. At P: $\vec{\mathbf{B}} = 7 \times 10^{-5}$ nt/amp-m, out.
At Q: $\vec{\mathbf{B}} = 4 \times 10^{-5}$ nt/amp-m, out.
At R: $\vec{\mathbf{B}} = 9 \times 10^{-5}$ nt/amp-m, in.
32. At P: $\vec{\mathbf{B}}_r = 2.2 \times 10^{-3}$ nt/amp-m, into page, 26.5° up from horizontal.
At Q: $\vec{\mathbf{B}}_r = 3.0 \times 10^{-3}$ nt/amp-m, up.
At R: $\vec{\mathbf{B}}_r = 2.2 \times 10^{-3}$ nt/amp-m, out from page, 26.5° up from horizontal.
At S: $\vec{\mathbf{B}}_r = 1.0 \times 10^{-3}$ nt/amp-m, down.
33. (a) 6.25×10^{12} electrons/sec.
(b) 2.08×10^{6} electrons/meter.

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1, 2, 3	1, 2, 3 4, 5	6	7#, 8		7#
4, 5	9, 10, 11, 15, 16		12, 14	13	13
6, 7, 8 9, 10, 11	17, 18, 22 28, 29, 30	19, 23, 26# 31	21, 25, 27# 32, 33#, 34	20, 24	20 33#

(c) 2.00×10^{-13} newton/amp-m.
(d) 1.0×10^{-7} newton.
(e) 4.8×10^{-14} newton.

34. (a) 1×10^{-4} nt/amp-m, left to right.
(b) 2.2×10^{-4} nt/amp-m, at 63° from direction of B_1.

COMMENTS AND SOLUTIONS

PROBLEM 1* (See Short Answers.)
PROBLEM 2* (See Short Answers.)
PROBLEM 3* (See Short Answers.)
PROBLEM 4* (See Short Answers.)
PROBLEM 5* (See Short Answers.)
PROBLEM 6

A horizontal uniform magnetic field $\vec{\mathbf{B}}$ is perpendicular to the horizontal component $\vec{\mathbf{B}}_e$ of the earth's magnetic field and points east.

(a) The ratio $B/B_e = \sqrt{3}$; in which direction will a compass needle point?

(*Note:* the compass needle rotates in a horizontal plane.)

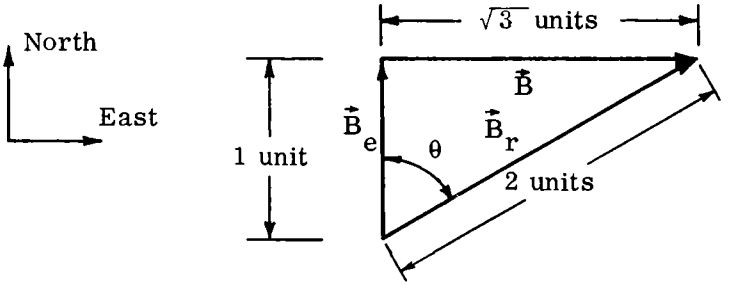
(b) If the compass needle points northeast, what is the magnitude of $\vec{\mathbf{B}}$ now?

(a) Since we know the directions and relative magnitude of both fields, we can draw a vector diagram to scale, and construct the resultant magnetic field vector $\vec{\mathbf{B}}_r$. From it we can find both the magnitude of $\vec{\mathbf{B}}_r$ relative to that of $\vec{\mathbf{B}}_e$ and the direction of $\vec{\mathbf{B}}_r$.

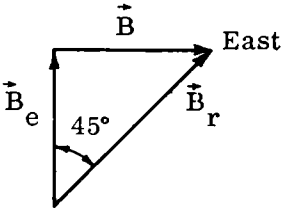
Some students will recognize a triangle whose sides are in the ratio of $1 : \sqrt{3} : 2$ as a 30°–60°–90° triangle, or we can find the angle θ from the diagram or by trigonometry:

$$\tan\theta = \frac{B}{B_e} = \sqrt{3}, \text{ therefore } \theta = 60\ .$$

The compass needle will point 60° east of north.

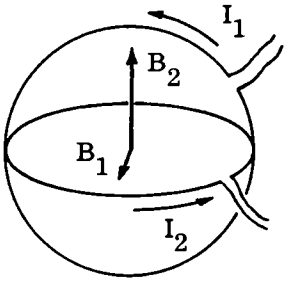


(b) If we continue to assume that $\vec{\mathbf{B}}$ points east we can find its magnitude by constructing the vector addition triangle of $\vec{\mathbf{B}}$, $\vec{\mathbf{B}}_e$, and $\vec{\mathbf{B}}_r$. These three vectors form a 45° right triangle, so $B = B_e$.



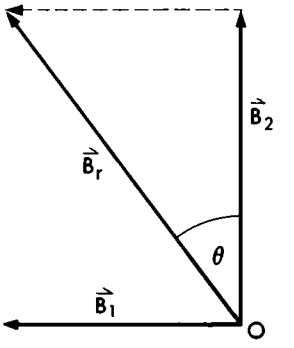
PROBLEM 7

Two circular wire loops of equal radius are set up at right angles to each other with a common center (Fig. 30–31). If $I_1 = 3.0$ amperes and $I_2 = 4.0$ amperes, what is the direction of the magnetic field at the center, 0?



The direction of the field produced by I_1 is directly out of the page, by the "right-hand rule"; the direction of the field produced by I_2 is vertically up (see drawing). Since the fields are proportional to the currents producing them, we can make a scale drawing to find the direction of the resultant. The angle θ between the resultant $\vec{\mathbf{B}}_r$ and the field $\vec{\mathbf{B}}_2$ produced by I_2 is 37°. This answer can also be obtained from trigonometry, since

$$\tan\theta = B_1/B_2 = 3.0/4.0 = 0.75.$$



PROBLEM 8

Suppose two loops are fixed at right angles and you can run a current in either direction through either loop. Neglecting the earth's field, how many different magnetic fields could you produce if a current of 2 amps flows whenever a loop is connected?

If we run a current through one loop at a time, there are four possible fields we can produce, corresponding to a current in each direction in each loop. If we run currents through both loops simultaneously, there are four other possible fields, corresponding to all possible combinations. Hence, the total number of different fields is eight.

PROBLEM 9* (See Short Answers.)

PROBLEM 10* (See Short Answers.)

PROBLEM 11* (See Short Answers.)

PROBLEM 12

The magnetic field inside a long solenoid is uniform and parallel to the axis of the solenoid.

(a) What is the force on a current-carrying wire inside the solenoid and parallel to the axis?

(b) What is the force on the wire shown in Fig. 30–32 which carries a current of 1.0 ampere, if the field B in the solenoid is 1.0×10^{-2} newton/amp-m and the length CD is 2 cm?

(c) How much mass m should be placed on the other end of the balance to keep it from tilting?

(d) Describe how you could use this setup as an ammeter.

(a) The current in a wire parallel to the axis of a solenoid is also parallel to the field of the solenoid; hence there is no component of $\vec{B}$ perpendicular to the current. The force on such a wire is zero. (The effects near the end of the solenoid, where $\vec{B}$ is not quite parallel to the current, are negligible.)

(b) As stated in (a), the wires carrying the current to and from the segment CD are parallel to the field; there is no force on them. The current in CD is perpendicular to the field. By the right-hand rule of Section 4 this force is down. The magnitude of the force is $BI\ell$:

$$F = 1.0 \times 10^{-2} \times 1.0 \times 0.02$$
$$= \underline{2 \times 10^{-4} \text{ newton, downward.}}$$

(c) Assuming that the balance is an equal-arm balance, the force found in (b) can be counterbalanced by a mass, m, whose weight ($w = mg$) must be equal to 2×10^{-4} newton. Solving for m ($g = 9.8$ m/sec^2),

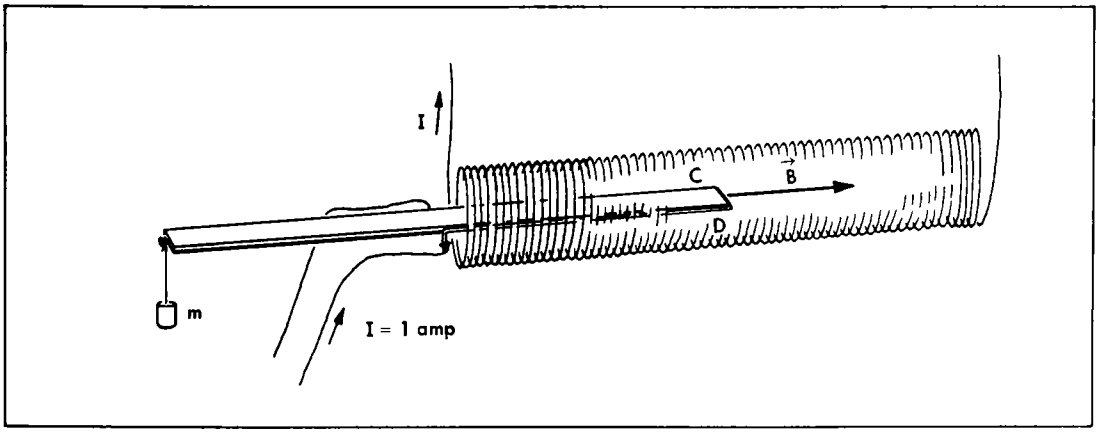
$$m = \frac{2 \times 10^{-4}}{9.8} = \underline{2 \times 10^{-5} \text{ kg.}}$$

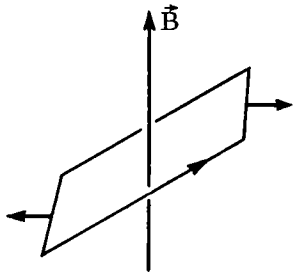
(d) The weight of the mass could be replaced by the force of an extended spring. The extension of a spring is approximately linear with the stretching force, and the magnetic force is linear with the current, so the scale of the spring balance could be marked in a linear way to read amperes directly.

PROBLEM 13

(a) Show that a square loop of wire carrying a current will always tend to align itself in a magnetic field so that the plane of the loop is perpendicular to the field.

(b) What do the magnetic forces tend to do to the loop in this position? (Hint: draw a side view.)



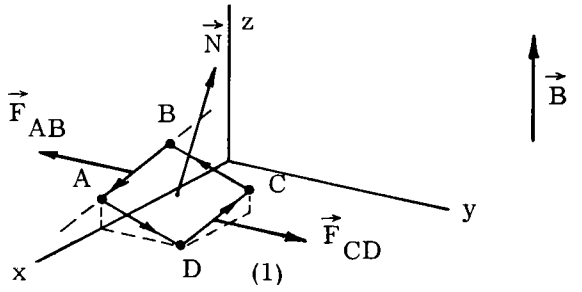


(a) Suppose the plane of the loop is not perpendicular to the direction of the magnetic field. In the position shown, the current in the loop will experience forces tending to pull the loop into the plane perpendicular to the magnetic field. More complicated orientations give more complicated forces, but they will still tend to rotate the loop into the plane perpendicular to the field.

* * *

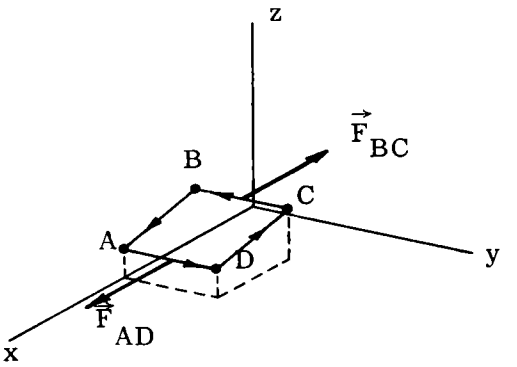
If you are forced into a more detailed discussion, the discussion below may be of help.

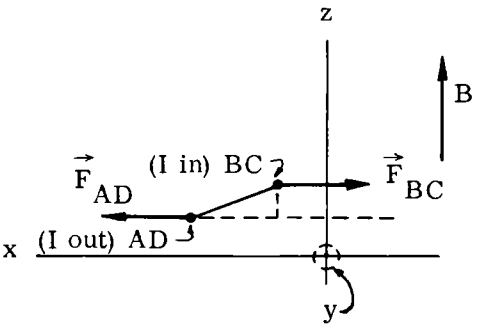
A completely arbitrary orientation of a square loop and the magnetic field direction is shown below. Here we have taken the direction of the field (we will assume it to be uniform) to be along the positive z-axis. The square loop $ABCD$ is initially oriented so one side AB is contained in the x-z plane. The arrowheads indicate the direction of current flow around the loop. The vector $\vec{N}$ is normal to the loop. Using the right-hand rule, we have drawn the forces on the sides AB and CD. (The field direction is parallel to the z-axis, the current AB lies in the x-z plane; the force is perpendicular to the plane defined by the current and the field; therefore the force $\vec{F}_{AB}$ is perpendicular to the x-z plane and it points in the negative y direction.)



The force on CD, $\vec{F}_{CD}$, is equal in magnitude and opposite in direction to $\vec{F}_{AB}$. This is true because CD is parallel to AB (the loop is square). Therefore CD will lie in a plane parallel to the x-z plane, and this plane also is parallel to the field. Furthermore, the component of the field perpendicular to the current is the same for CD as for AB, again because the sides are parallel.

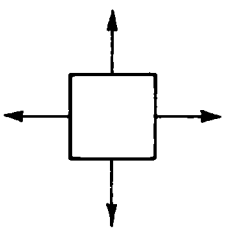
Let us assume that the loop is pivoted about the side AB. Then under the action of $\vec{F}_{CD}$ the loop will turn until the two sides AD and BC are parallel to the y-axis. This situation is shown immediately below.





In this diagram, we are looking along the y-axis toward the x-z plane. The arguments of the preceding paragraphs may now be applied to the pair of forces $\vec{F}_{AD}$ and $\vec{F}_{BC}$. They are equal and opposite (the component of the field perpendicular to the current is just the field itself). If the loop is pivoted about the side AD, the loop then tends to turn until it is perpendicular to the field (in this case, parallel to the x-axis).

(b) When the loop is perpendicular to the field, the forces are as shown in the diagram. Because the sides are all equal in length, and because the magnitude of the current is the same in all the sides, the forces are all equal in magnitude. Therefore, there is no net translational force on the loop. The forces tend to pull the loop apart, and they tend to keep the loop perpendicular to the field.



Notes on Problem 13.

(1) If the field were nonuniform the loop would still tend to rotate to a position where the plane of the loop would be perpendicular to the field in some average sense. (It would be possible to find fields which would not rotate the loop.) After this rotation there might be a net translational force on the loop. This unbalanced force can be resolved into two components: one parallel to the plane of the loop and the other perpendicular to the plane of the loop. It is easy to show that both of these components are in directions such as to translate the loop toward the region where the magnetic field is strongest.

(2) If we remember that a bar magnet is equivalent to some configuration of current loops with planes perpendicular to the magnet, we can use the converse of this fact to see the solution of part (a). Replace the square current loop with a proper configuration of bar magnets lying along the direction $\vec{N}$. They, we know, will tend to line up with the field. Thus the current loop will have its plane perpendicular to the field. Also, there is another position of equilibrium for the current loop. This is the inverted position. In that position the force on each side is inward instead of outward, and any small deflection from the exactly perpendicular position will lead to a flipping over of the coil by the forces acting on it. This kind of an equilibrium is therefore called unstable equilibrium. With the forces acting outward, the equilibrium is called stable, because the action of the forces will restore that position if any small deflection occurs.

PROBLEM 14

In the moving-coil ammeter of Fig. 30–16, the magnetic force on the coil is balanced by a torsion spring. An ammeter can also be constructed as shown in Fig. 30–33, using a small spring balance to measure the magnetic force on the coil. By adjustment of the position of the support, the conductors can be kept in the same position in the magnetic field for different currents.

One such coil, 4.0 cm wide by 25 cm high, was constructed of 100 turns and weighed 2.4 newtons. The spring balance had a scale of 0 to 5 newtons.

(a) When supported in a magnetic field, as shown, the spring balance changed from 2.4 to 2.7 newtons as the current was increased from 0 to 0.5 ampere. What is the maximum current that could be measured on this spring balance?

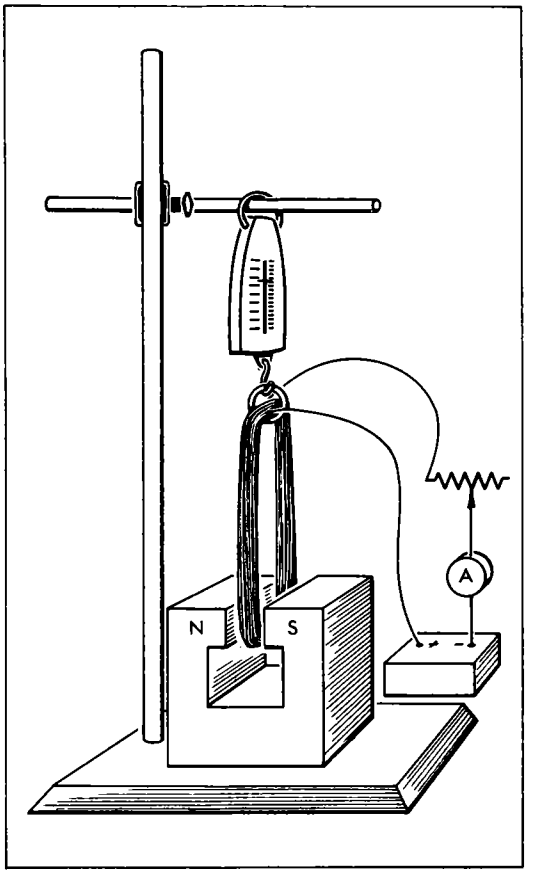
(b) What is the magnetic field in the air gap of the magnet?

(a) The useful range of our spring balance is from 2.4 to 5.0 newtons, an interval of 2.6 newtons. At 0.5 ampere per 0.3 newton, it would give us a maximum range of

$$(2.6 \text{ nt})(1.7 \text{ amp/nt}) = \underline{4.4 \text{ amperes.}}$$

(b) With 100 turns of wire, each contributing 4.0 cm of wire perpendicular to the field which will produce a downward force, the length is 4.0 meters.

$$B = \frac{F}{I\ell} = \frac{0.3 \text{ nt}}{(0.5 \text{ amp})(4.0 \text{ m})}$$
$$= \underline{0.15 \text{ newton/ampere-meter.}}$$



PROBLEM 15* (See Short Answers.)

PROBLEM 16* (See Short Answers.)

PROBLEM 17* (See Short Answers.)

PROBLEM 18* (See Short Answers.)

PROBLEM 19

How do the momenta of a proton and an electron compare if both are moving perpendicular to the same magnetic field in circles of the same radius?

The magnetic force F on a moving charged particle is $F = qB_{\perp}v$. Since it always acts perpendicular to the velocity of the particle, it causes the particle to move in a circle whose radius r is related to the magnetic force by $F = mv^2/r$. Hence, the momentum p of the particle is

$$p = mv = qB_{\perp}r.$$

Since a proton and an electron each have one elementary charge, then if they move in circles of the

same radius perpendicular to the same magnetic field, their momenta must be equal in magnitude.

Note that the momentum vectors of the proton and of the electron rotate in the opposite sense; if for one particle they rotate in the clockwise sense, for the other particle they rotate in the counterclockwise sense. In order to determine which particle is rotating in which sense, we must know the direction of the magnetic field.

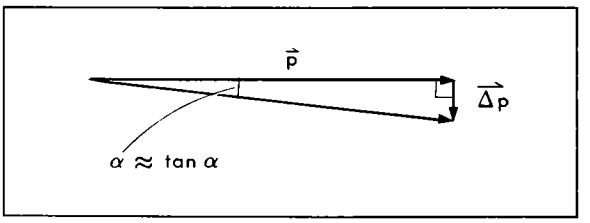
PROBLEM 20

Would it be easier for a mass spectrograph to separate the isotopes of light elements or heavy elements?

For isotopes which differ by a given mass difference Δm (say, 1 amn, the mass of an additional neutron), the relative increase $\Delta m/m$, where m is the atomic mass of the lighter isotope, is greatest for small m, that is, for the lightest elements. Hence, if the mass spectograph is sensitive to relative mass differences, it will be easiest to separate isotopes of the lightest elements.

We can examine the specific case of the spectrograph shown in Fig. 30–21. The ions are bent along a slightly curving path while they are between the poles of the magnet; afterwards they continue along a straight line undeflected until they hit the screen. Because the circular deflection in the magnetic field is small, we can approximate the circular part of the path by a section of a parabola (see Fig. 12–26). That is, we can say that the magnetic force is approximately constant in direction, perpendicular to the initial velocity of the ion. Then the change in momentum due to the magnetic force is perpendicular to the initial momentum $\vec{\mathbf{p}}$ and its magnitude is

$$|\Delta\vec{\mathbf{p}}| = F\,\Delta t = qvB\,\Delta t.$$



The time Δt it takes the ion to pass through the magnetic field equals the horizontal length L of the pole faces divided by the horizontal component v of the velocity. Hence,

$$|\Delta\vec{\mathbf{p}}| = qvB(L/v) = qBL.$$

The angular deflection of the ion is

$$\alpha = |\Delta\mathbf{p}|/p = qBL/p.$$

But in terms of the kinetic energy $E_K = qV$ which the ion attains by being accelerated through the potential difference V, the momentum with which it enters the magnetic field is

$$p = (2mE_K)^{1/2} = (2mqV)^{1/2}.$$

Hence, the angular deflection is

$$\alpha = \frac{qvB}{(2mqV)^{1/2}},$$

which varies inversely with the square root of the mass of the ion. For a given change, Δm, in the mass, the angular deflection will be changed the most for small m, as can be seen by making a sketch of α as a function of mass.

PROBLEM 21

A beam of protons of kinetic energy E_K is injected into a region of uniform magnetic field $\vec{\mathbf{B}}$ at right angles to the direction of the beam.

(a) Find expressions for the radius and the period of the resulting circular motion in terms of the magnetic field strength.

(b) How would your answers to (a) be affected if the incident energy were four times as great?

(c) Protons of energy 4.60×10^5 electron volts are injected into a proton synchrotron in which the magnetic field strength is 2.10×10^{-2} newton/amp-m. Calculate the period and the radius of the circular motion.

(a) Let m be the proton mass, q the charge on the proton, and v its speed. The radius of the circular motion at right angles to a magnetic field of strength B is

$$r = \frac{mv}{qB}.$$

In terms of the kinetic energy, the momentum is

$$p = mv = (2mE_K)^{1/2}.$$

Then the radius can be expressed in terms of the known quantities m, E_K, q, and B as

$$\underline{r = \frac{(2mE_K)^{1/2}}{qB}}.$$

The expression for the period is simpler. The period of a particle moving in a circle of radius r at constant speed v is $T = 2\pi r/v$. From our first expression for the radius, in terms of v, we see that $r/v = m/qB$. Hence,

$$\underline{T = \frac{2\pi m}{qB}}.$$

(b) If E_K were four times as great, the radius would be only twice as great, since the radius

varies as the square root of the kinetic energy. The period does not depend on the kinetic energy at all, and would be unchanged.

(c) Using the expressions found in (a), we have

$$T = \frac{2\pi m}{qB}$$
$$= \frac{2(3.14)(1.67 \times 10^{-27}\ \text{kg})}{(1.60 \times 10^{-19}\ \text{coul})(2.10 \times 10^{-2}\ \text{nt/amp-m})}$$
$$= 3.12 \times 10^{-6}\ \text{second}$$
$$= \underline{3.12\ \text{microseconds.}}$$

The radius is

$$r = (2mE_K)^{1/2}/qB$$
$$= \frac{\sqrt{2(1.67 \times 10^{-27}\ \text{kg})(4.60 \times 10^{5}\ \text{ev}) \times (1.60 \times 10^{-19}\ \text{joule/ev})}}{(1.60 \times 10^{-19}\ \text{coul})(2.10 \times 10^{-2}\ \text{nt/amp-m})}$$
$$= \underline{4.67\ \text{meters.}}$$

PROBLEM 22* (See Short Answers.)

PROBLEM 23

Charged particles are accelerated by a uniform electric field and then deflected by a magnetic field. What quantities must be measured to determine the mass of the particles from this experiment?

The quantities which must be measured are the potential difference through which the particle moves, the charge of the particle (these two quantities determine the kinetic energy of the particle as it enters the magnetic field), the magnetic field, and the radius of curvature of the path of the particle in the magnetic field (these last two quantities, together with the charge, tell us the momentum of the particle). From the kinetic energy and the momentum, we can then determine the mass of the particle.

PROBLEM 24

When helium gas is fed into the apparatus shown in Fig. 30–21, mounted in a chamber from which the gas can be pumped to keep the pressure low, He^+ and He^{++} ions are produced. These two ions form two spots on the screen below the position of the beam with no field. The face of the magnet is 3.0 cm by 1.5 cm and the center of the face is 10 cm from the screen. The strength of the magnetic field is 1.5×10^{-2} newton/amp-m.

(a) Compute the radius of the circular part of the path of each ion while it is in the magnetic field.

(b) Make a scale drawing and determine the position of each spot on the screen.

(c) What would be the path of a He^{++} ion which gains one electron while in the electric field? Describe it qualitatively.

(d) What would be the path of He^{++} ion which gains one electron while in the magnetic field? Describe it qualitatively.

(a) The radius of the path of a charged particle moving in a magnetic field perpendicular to the path is $r = mv/qB$. Since $mv = \sqrt{2mE_K}$ and $E_K = qV$, we have

$$r = \frac{mv}{qB} = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{2mV}{q}\left(\frac{1}{B}\right)}.$$

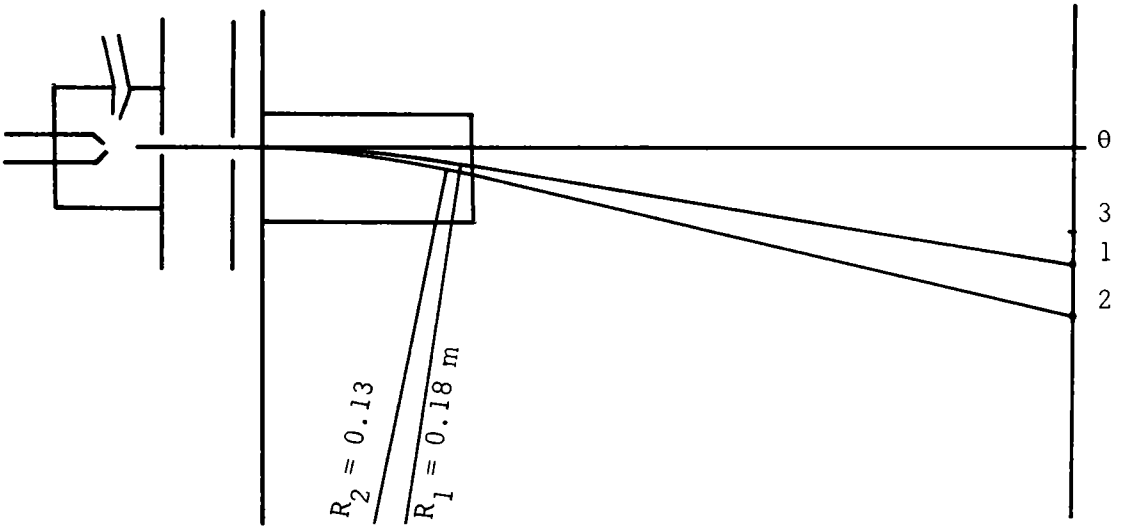
For the He^+ ion,

$$r^+ = \sqrt{\frac{2(6.6 \times 10^{-27}\ \text{kg})(90\ \text{volts})}{1.60 \times 10^{-19}\ \text{coul}}} \times \left(\frac{1}{1.5 \times 10^{-2}\ \text{nt/amp-m}}\right)$$
$$= \underline{0.18\ \text{meter.}}$$

For the He^{++} ion, all quantities are the same except the charge, which is twice as great, so

$$r^{++} = r_1/\sqrt{2} = \underline{0.13\ \text{meter.}}$$

(b)



(c) If an electron is gained in the electric field, the energy of the ion will be between ev and 2 ev. Its speed v as it enters the magnetic field will be between v_1 and v_2. Let's look at the solutions at the limiting values. If the electron is gained right at the beginning, $v = v_1$ and $R = R_1$. If the electron is gained just as the ion leaves the electric field, $v = v_2$ and $R = 2R_2$, which puts a spot above 1 in the figure about halfway from 0 to 2, or at 3. Therefore, gaining an electron in the electric field causes R to vary between R_1 and $2R_2$, and the spot to vary from position 1 to 3.

(d) If an electron is gained in the magnetic field, the speed of the ion will be v_2. We have already worked out the limiting cases. If the ion picks up an electron between the two fields, R will equal $2R_2$. If the electron is picked up at the end of the field, the radius of curvature will be R_2. The ion will travel on a path of radius R_2 until the electron is picked up and then it will travel on a path of radius $2R_2$ and will hit the screen somewhere between 2 and 3.

PROBLEM 25

A nearly uniform electric field is set up parallel to the surface of a glass-topped table by charging two large parallel, vertical metal plates. The plates are 0.50 m apart and the potential difference between them is 20 kv. A small Dry Ice puck with a total mass of 60 grams carries a copper-coated Ping-Pong ball which has a charge of 2.2×10^{-8} coulomb.

(a) What is the electric force on the ball when it is placed between the plates?

(b) What would be the shape of its path if it were given a brief push perpendicular to the field lines?

(c) Even in a restricted region, the largest magnetic field that can be obtained fairly easily is 0.50 newton/amp-m. If you projected the puck through such a field perpendicular to the field at a speed of one meter/second, what would be the magnetic force on the puck?

(a) The electric force on the ball is

$$F = qE = qV/d$$
$$= \frac{(2.2 \times 10^{-8}\ \text{coul})(2.0 \times 10^{4}\ \text{volts})}{0.50\ \text{m}}$$
$$= \underline{8.8 \times 10^{-4}\ \text{newton}}.$$

(b) The path would be a <u>parabola</u>.

(c) The magnetic force is

$$F_{\text{mag}} = qvB$$
$$= (2.2 \times 10^{-8}\ \text{coul})(1\ \text{m/sec})(0.50\ \text{nt/amp-m})$$
$$= \underline{1.1 \times 10^{-8}\ \text{newton}}.$$

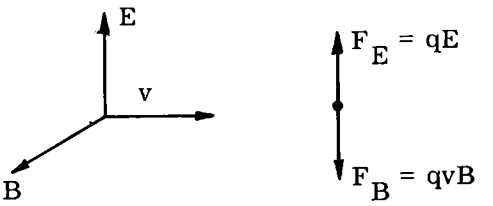
We see that in this case the electric force is 8×10^4 times as large as the magnetic force.

PROBLEM 26

A beam of singly charged ions moves into a region of space where there is a uniform electric field $E = 1.0 \times 10^3$ newton per coulomb, and a uniform magnetic field $B = 2.0 \times 10^{-2}$ newton/amp-m. The electric and magnetic fields are at right angles to each other and both are perpendicular to the beam so that the electric and magnetic forces on an ion oppose each other. What is the speed of those ions which move *undeflected* through these crossed electric and magnetic fields?

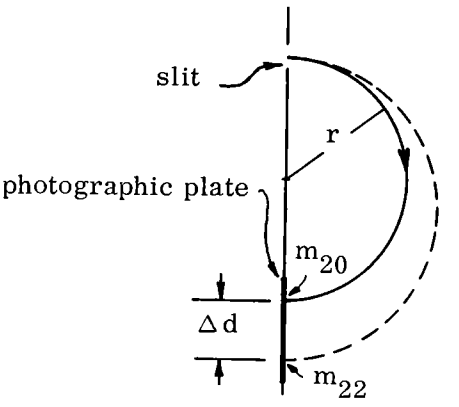
We must find the speed at which the magnetic force is equal and opposite to the electric force. The magnetic force and electric force are oppositely directed so that we may write $qE = qvB$. Cancelling q and solving for v, we have

$$v = \frac{E}{B} = \frac{1.0 \times 10^3}{2.0 \times 10^{-2}} \left[\frac{\cancel{\text{newtons}}}{\cancel{\text{coulomb}}} \times \frac{\cancel{\text{coulomb}}}{\cancel{\text{amp}}\text{-sec}} \right]$$
$$\frac{\cancel{\text{newtons}}}{\cancel{\text{amp}}\text{-meter}}$$
$$= \underline{5.0 \times 10^4\ \text{m/sec}}.$$



PROBLEM 27

The undeflected ions of Problem 26 are passed through a slit and move into a region where there is a uniform magnetic field $B = 0.09$ newton/amp-m at right angles to their motion. If the ions are a mixture of the neon ions of mass 20 and 22 atomic mass units, how far apart will these ions land on a photographic plate after they have moved through a semicircle?



(In this view, the direction of the field $\vec{B}$ is out from the page toward the viewer.)

The spot where the neon ions strike the plate will be a diameter away from the slit. We can calculate the radius of the circular path by the method described in Section 7. The magnetic force qvB produces a centripetal acceleration v^2/r. Thus, by Newton's law,

$$qvB = m\frac{v^2}{r},$$

$$mv = qBr, \text{ and}$$

$$r = mv/qB.$$

The ions have the same speed and the same charge, so the difference in the diameters of their paths is due only to the mass difference:

$$\Delta d = d_{22} - d_{20} = 2(m_{22} - m_{20})\frac{v}{qB}.$$

Since 1 amu $= 1.7 \times 10^{-27}$ kg,

$$\Delta d = \frac{2(2 \text{ amu})(1.7 \times 10^{-27} \text{ kg/amu}) \times (5.0 \times 10^4 \text{ m/sec})}{(1.6 \times 10^{-19} \text{ coul})(9 \times 10^{-2} \text{ nt/amp-m})}$$

$$= \underline{0.02 \text{ meter}}.$$

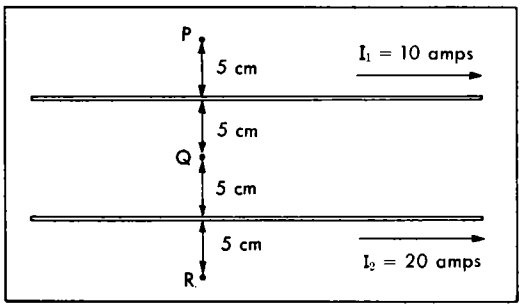
PROBLEM 28* (See Short Answers.)

PROBLEM 29* (See Short Answers.)

PROBLEM 30* (See Short Answers.)

PROBLEM 31

Two very long straight parallel wires, one carrying 10 amperes and the other 20 amperes, are separated by a distance of 10 cm (Fig. 30–35). Find the magnitude and direction of $\vec{\mathbf{B}}$ at the points P, Q, and R shown in the figure.



The magnetic field of each wire is perpendicular to the page at the points P, Q, and R. By the right-hand rule we can tell whether each field points into or out of the page at these points. The magnitude of each field at a distance r from the wire producing it is $B = KI/r$, where $K = 2 \times 10^{-7}$ nt/(amp)^2.
At point P:

$$\vec{\mathbf{B}}_1 = \frac{(2 \times 10^{-7})(10)}{5 \times 10^{-2}} = 4 \times 10^{-5} \text{ nt/amp-m, out.}$$

$$\vec{\mathbf{B}}_2 = \frac{(2 \times 10^{-7})(20)}{15 \times 10^{-2}} = 3 \times 10^{-5} \text{ nt/amp-m, out.}$$

Hence, the resultant field is

$$\vec{\mathbf{B}} = \underline{7 \times 10^{-5} \text{ nt/amp-m, out.}}$$

At point Q: $\vec{\mathbf{B}}_1$ has the same magnitude as at P but points *into the page;* $\vec{\mathbf{B}}_2$ points outward and has twice the magnitude of $\vec{\mathbf{B}}_1$, since the current producing it is twice as great. Hence, the resultant field is

$$\vec{\mathbf{B}} = \underline{4 \times 10^{-5} \text{ nt/amp-m, out.}}$$

At point R: $\vec{\mathbf{B}}_1$ points in and is only a third as strong as at point P:

$$\vec{\mathbf{B}}_1 = 1 \times 10^{-5} \text{ nt/amp-m, in.}$$

$\vec{\mathbf{B}}_2$ points in also and is as strong as at Q:

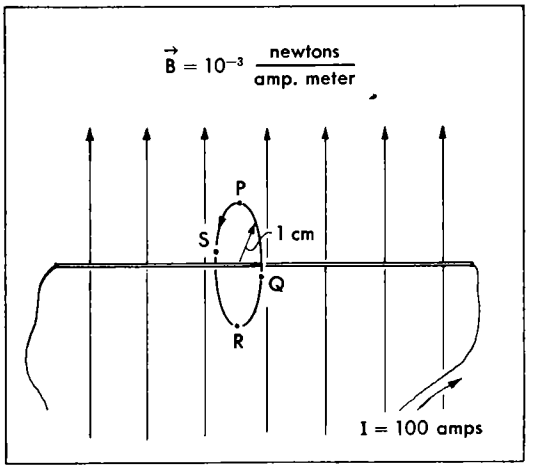
$$\vec{\mathbf{B}}_2 = 8 \times 10^{-5} \text{ nt/amp-m, in.}$$

The resultant is therefore

$$\vec{\mathbf{B}} = \underline{9 \times 10^{-5} \text{ nt/amp-m, in.}}$$

PROBLEM 32

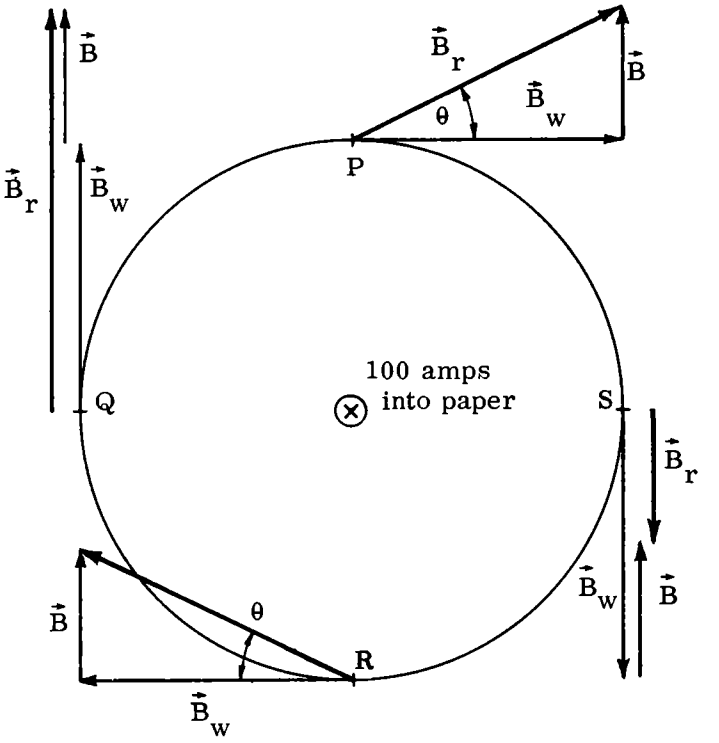
A long straight wire carrying a current $I = 100$ amperes is perpendicular to a uniform magnetic field of strength $B = 1.0 \times 10^{-3}$ newton/amp-m (Fig. 30–36). What is the strength and direction of the resultant $\vec{\mathbf{B}}$ field at the points P, Q, R, S on a circle of radius 1 cm around the wire?



We find the field $\vec{\mathbf{B}}_w$ produced at each point by the 100 ampere current in wire and add it vectorially to the field $\vec{\mathbf{B}}$ to obtain the resultant field $\vec{\mathbf{B}}_r$ at each point. The magnitude of $\vec{\mathbf{B}}_w$ is

$$B_w = 2 \times 10^{-7} \frac{\text{newton}}{\text{amperes}^2} \times \frac{10^2 \text{ amperes}}{10^{-2} \text{ meter}}$$

$$= 2 \times 10^{-3} \frac{\text{newton}}{\text{ampere-meter}}.$$



The direction of $\vec{B}_w$ at each point is shown in the diagram.

At P or R, $B_r = \sqrt{B^2 + B_w^2} = \sqrt{1^2 + 2^2} \times 10^{-3}$
$= \underline{2.2 \times 10^{-3} \text{ newton/amp-meter.}}$

At Q, $B_r = B_w + B = (2 + 1) \times 10^{-3}$
$= \underline{3 \times 10^{-3} \text{ newton/amp-meter.}}$

At S, $B_r = B_w - B = (2 - 1) \times 10^{-3}$
$= \underline{1 \times 10^{-3} \text{ newton/amp-meter.}}$

The directions of B_r are as shown in the diagram. For the angle θ,

$$\tan \theta = \frac{B}{B_w} = \frac{1 \times 10^{-3}}{2 \times 10^{-3}} = 0.5$$

$$\underline{\theta = 26.5°}.$$

PROBLEM 33

Suppose we have a beam of electrons moving at a speed $v = 3.00 \times 10^6$ m/sec and carrying a current of 1.00 microampere.

(a) How many electrons per second pass a given point?

(b) How many electrons are in 1.00 meter of the beam?

(c) What magnetic field does the beam produce at 1.00 m distance?

(d) What is the total force on all of the electrons in 1.00 m of the beam if it passes through a field of 0.10 newton/amp-m?

(e) What is the force on a single electron if you assume that each electron experiences the same force?

(a) 1 microampere = 10^{-6} ampere. This is 10^{-6} coulomb passing a given point every second. One coulomb is equivalent to 6.25×10^{18} elementary charges, and since a microampere is a microcoulomb per second,

$$1.00 \text{ microamp} = \frac{1.00 \text{ microcoulomb}}{\text{sec}}$$
$$= 1.00 \times 10^{-6} \text{ coul/sec}$$
$$\times 6.25 \times 10^{18} \text{ elem.ch./coul}$$
$$= \underline{6.25 \times 10^{12} \text{ electrons/sec.}}$$

(b) Since the speed of the electrons is 3.00×10^6 m/sec, 3.00×10^6 meters of beam will pass a given point every second. From (a) we know that this length of beam contains 6.25×10^{12} electrons. If the electrons are distributed uniformly along 3.00×10^6 meters of beam, then in 1.00 meter there are

$$\frac{6.25 \times 10^{12}}{3.00 \times 10^6} = \underline{2.08 \times 10^6 \text{ electrons.}}$$

(c) A current, I, in a straight wire produces a magnetic field $B = K(I/d)$ at a distance d from the wire. It is current that produces a field; a wire is simply a convenient container for charges in motion. Thus the electron beam constitutes a current and gives rise to a field:

$$B = (2 \times 10^{-7}\ \text{nt/amp}^2) \times \left(\frac{1.00 \times 10^{-6}\ \text{amp}}{1.00\ \text{m}}\right)$$
$$= \underline{2.00 \times 10^{-13}\ \text{nt/amp-m}}.$$

(d) A length ℓ of current I will experience a force $F = BI\ell$ in a field of strength B. For one meter of the beam in a field of 0.10 newton/ampere-meter, the total force is

$$F = 0.10 \times 1.00 \times 10^{-6} \times 1.00$$
$$= \underline{1.0 \times 10^{-7}\ \text{newton}}.$$

(e) From (b), one meter of beam contains 2.08×10^6 electrons. If the force is uniform for each electron, then the total force, 1.0×10^{-7} newton, divided by the number of electrons, 2.08×10^6, will give the force on each electron:

$$F = \frac{1.0 \times 10^{-7}}{2.08 \times 10^{6}} = \underline{4.8 \times 10^{-14}\ \text{newton}}.$$

This result follows from the same argument used in Section 6; we could have used the end formula for F:

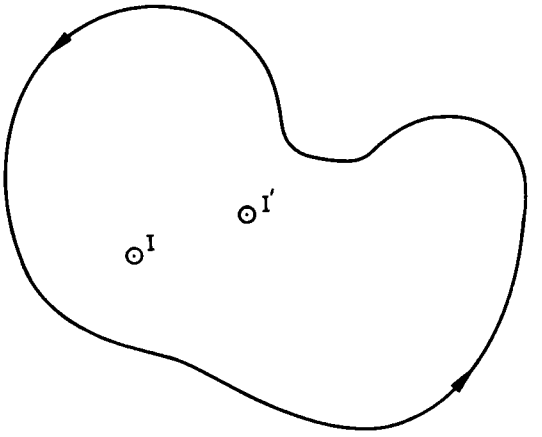
$$F = qvb = (1.6 \times 10^{-19}\ \text{coul})(3.00 \times 10^{6}\ \text{m/sec}) \times (0.10\ \text{nt/amp-m})$$
$$= \underline{4.8 \times 10^{-14}\ \text{newton}}.$$

PROBLEM 34

Point A is 2 cm from each of 3 currents I_1, I_2, and I_3, as shown in Fig. 30-37. All three currents are carried by long wires perpendicular to the paper.

(a) If $I_1 = I_2 = I_3 = 10$ amperes, and all three currents are directed out of the paper, what is the magnitude and direction of the magnetic field at A due to the currents in the wires?

(b) If $I_1 = I_2 = I_3 = 10$ amperes, but I_2 is into the paper while I_1 and I_3 are out, what is the magnitude and direction of the magnetic field at A due to the currents in the wires?



(a) Since A is equidistant from I_2 and I_3 on the line joining them and the two currents are parallel and both out from the paper, their contributions to the field at A cancel exactly. Thus, the net field at A results only from the current I_1:

$$B = B_1 = K(I/d) = (2 \times 10^{-7}\ \text{nt/amp}^2) \times \left(\frac{10\ \text{amp}}{2 \times 10^{-2}\ \text{m}}\right) = \underline{1 \times 10^{-4}\ \text{nt/amp-m}}.$$

The direction of the field is horizontal, *from left to right.*

(b) The fields $\vec{\mathbf{B}}_2$ and $\vec{\mathbf{B}}_3$ at A are now in the same direction, down. Since

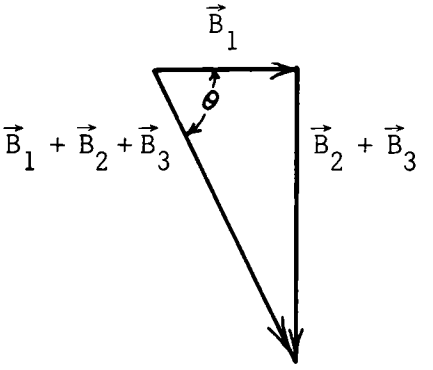
$$B_1 = B_2 = B_3 = 1 \times 10^{-4}\ \text{nt/amp-m},$$

the resultant field strength is

$$B = \sqrt{B_1^2 + (B_2 + B_3)^2}$$
$$= \underline{2.2 \times 10^{-4}\ \text{nt/amp-m}}.$$

The resultant field makes an angle θ with B_1 given by

$$\tan\theta = 2$$
$$\theta = \underline{63°}.$$



chapter
31

Electromagnetic Induction and Electromagnetic Waves

The ideas of the chapter are really the culmination of classical physics where electricity and light become one subject. While this material is abstract and difficult, it is reassuring that it may be treated profitably at all levels — in a completely qualitative way as a reading assignment amply fortified with classroom demonstration, or, with adequate time and able students, in a thoroughly quantitative manner.

The role of classroom demonstration should be emphasized here. Many of the ideas in this chapter can be adequately comprehended if the students get a qualitative understanding of what happens. Demonstrations can get these ideas across more effectively and faster than lecturing. Of course, if you have time, the quantitative aspects of this chapter can be handled as in previous work. There are, however, a lot of ideas presented in a condensed way in the text, and complete, quantitative comprehension can be expected only from good students and plenty of time.

CHAPTER SUMMARY

SECTIONS 1 THROUGH 3. The chapter begins by following in the footsteps of Michael Faraday to determine whether a magnet can produce a current. A series of experiments is described in which wire loops are pulled across a magnetic field, a magnetic field is pulled across a wire loop, etc. The concept of magnetic flux is developed as a unifying principle, and the prediction that induced current is the result of changes in magnetic flux is tested with the case in which both the magnetic field and the loop are fixed in space, but the magnetic field changes in magnitude with time.

SECTIONS 4 AND 5. Induced EMF is developed as the basic quantity which results from changes in magnetic flux, and quantitative results are obtained for predicting the magnitude and the direction of the induced EMF.

SECTION 6. A changing magnetic field is shown to cause an electric field in space.

SECTION 7. The circulation law of Chapter 30 is applied to a long, straight, current-carrying wire into which two parallel plates have been inserted. The current therefore varies with time, and the circulation law is seen to fail unless it is amended to include a second term, a sort of "equivalent current," which describes the change in electric flux which occurs between the parallel plates. The proportionality constant relating the circulation of magnetic field to the magnitude of the "equivalent current" is computed in the box on page 585.

SECTIONS 8 AND 9. In Section 8 it is shown that a boundary between a region of combined electric and magnetic fields and a region of no field can move through space by itself at one particular velocity, the velocity of light! Experimental confirmation of this prediction lends proof to the assumptions of Section 7 involved in the amendment of the circulation law. Section 9 describes various wavelengths of electromagnetic radiation in a qualitative way.

SCHEDULING CHAPTER 31

Important Note. Before starting this chapter it is essential that you know exactly which parts of the chapter you want your students to understand in detail and which parts you wish to treat qualitatively.

If you are skipping the study of magnetism as a possible alternative suggested in the introduction to this volume of the Guide, you will not want to cover the material of Sections 1–8. In this case, however, Section 9 should be assigned as a reading assignment. This is the point at which you should reenter the main stream of the text. A film is available to show experimental evidence associated with that section.

The first three sections form the scientific basis for understanding an electrical generator. They are straightforward extensions of Chapter 30 and should be handled readily by most students. A thorough treatment can be made in one class period.

Sections 4 and 5 extend the relatively straightforward material of the first three sections to consideration of the induced EMF in the circuit. If Section 3 is well understood, this material should go easily. All students will find the simplicity of Lenz's law appealing. Section 6 deals with electric fields created by changing magnetic fields. Some students

Chapter 31	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTIONS	CLASS PERIODS	LAB PERIODS	EXPERIMENT	CLASS PERIODS	LAB PERIODS	EXPERIMENT
1, 2, 3	1	0	—	2	0	—
4, 5, 6	2	0	—	3	0	—
7	1	0	—	1	0	—
8, 9	1	0	—	2	0	—

will find this material sufficiently more abstract than the previous two sections that they will find it difficult. Probably 2 class periods will be required as a minimum to get these basic ideas adequately planted.

In the treatment of Section 1 through 6, classroom demonstrations should play a prominent role. All of the phenomena described in these sections can be easily demonstrated to small groups with equipment usually readily available. For demonstration to a large class more elaborate setups would be necessary. Such demonstrations might well bear the major load of teaching this material. You may have your own favorites, but if not, suggestions are made under Demonstrations: "Induced Current and Voltages" in the section-by-section guide which follows.

Sections 7 and 8 will be rather hard going for the average class, but even a partial understanding of this material can be very rewarding to most students. Here is the real, surprise "pay-off" of the chapter when we find that electromagnetic radiation is predicted to travel at a unique velocity, the velocity of light. Section 9 can be handled as a reading assignment supplemented by a film if desired. Minimum time on these sections is about 2 days, but three or four days would be much more desirable.

RELATED MATERIALS FOR CHAPTER 31

LABORATORY. There are no experiments recommended for this chapter, but students who have not done so should be encouraged to shove magnets through coils and possibly to play with simple radio transmitters where these are available.

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILMS. "Electromagnetic Waves" by George Wolga of M.I.T. shows why we believe in the unity of the electromagnetic spectrum. Experiments are performed to show that radiation arises from accelerated charges, that this radiation consists of transverse waves that can be polarized, and that interference can be produced in four different regions of the spectrum: X ray, visible light, microwave, and radiowave. Running time: 33 minutes.

31-1 Induced Current

31-2 Relative Motion

PURPOSE. To show that a current is induced in a wire loop when there is relative motion in a direction perpendicular to the magnetic field.

CONTENT. (a) A current is induced in a wire loop moving perpendicular to a magnetic field.

(b) The magnitude and direction of the induced current on the magnitude and direction of the velocity relative to the field direction.

(c) The velocity of iimportance is the *relative* velocity between loop and magnetic field.

EMPHASIS. These two sections form an excellent experimental background for the subject of electromagnetic induction. These parts, with appropriate demonstrations, can be understood by all students in a relatively short time. If your schedule is tight, you may wish to do these sections, pull the experimental information together with Section 3 and then jump to Section 9.

COMMENT. We are following here the famous experiments of Michael Faraday.

DEVELOPMENT. These sections can best be taught with liberal use of demonstrations and class discussion. Many students will not have seen these magnetic induction experiments and they certainly should because these effects are so important to our modern technology.

DEMONSTRATIONS.

A. *General.*

It is worthwhile and easy to show qualitatively that a current is produced by a wire loop moving in a magnetic field. Of course, a quantitative demonstration would be more impressive, but unless you have special equipment, quantitative demonstrations are difficult. (It is not easy to produce or measure a uniform velocity. A large uniform magnetic field may not be available. The appropriate meter and circuit may not be available.)

Pedagogically it would be more effective to demonstrate with a single loop of wire. However, the size of the effect, and the meter you have available will probably make results with a single loop very quali-

tative (as described below). Except for the first, the suggestions below involve many turns of wire and can be made somewhat more quantitative.

B. *Qualitative Experiment with Single Loop of Wire.*

Equipment: Magnet and Microammeter (or sensitive D.C. oscilloscope).

A strong magnet is needed. If you do not have one, you might find strong permanent magnets at surplus stores; maximum fields of 0.1 nt/amp-m to 0.2 nt/amp-m (i.e., 1000 gauss to 2000 gauss) are usually available.

If you have a choice of microammeters, the smallest internal resistance will be most useful. An oscilloscope which responds to a fraction of a millivolt would be preferable.

Procedure. Make a closed circuit containing a long wire and the microammeter. If you use an oscilloscope, connect a large resistor (1000 ohms or greater should be suitable) in place of the meter and connect the oscilloscope across the resistor.

Move the wire quickly through the region between the magnet pole faces. The meter (or oscilloscope) should respond.

Typical Values and Possible Difficulties. Suppose your magnet has a field of 0.2 nt/amp-m and a pole area of 10 cm^2 (or $10^{-3}\ m^2$). The maximum flux through the circuit formed by the wire would be

$$\begin{aligned} BA &= 0.2 \times 10^{-3} = 2 \times 10^{-4}\ \text{nt-m/amp} \\ &= 2 \times 10^{-4}\ \text{nt-m sec/coulomb} \\ &= 2 \times 10^{-4}\ \text{joule sec/coulomb} \\ &= 2 \times 10^{-4}\ \text{volt sec.} \end{aligned}$$

If you move the wire so that this flux passes (or stops passing) through the area enclosed by the complete electrical circuit in, say, 0.5 second, an $\text{EMF} = \dfrac{2 \times 10^{-4}\ \text{volt sec}}{0.5\ \text{sec}} = 4 \times 10^{-4}$ volt will be produced. If your circuit uses a resistor with an oscilloscope across it, this voltage will be displayed on the oscilloscope. Changing the resistance by a factor of 10 will not produce a significant change in the voltage across the resistor. With a resistor-oscilloscope arrangement, you can show (qualitatively) the dependence on the speed, v.

If you use a microammeter, it may be much harder to show the effects of changing the resistance or the speed. Imagine an idealized microammeter with an internal resistance of 100 ohms. The 4×10^{-4} volt induced EMF would produce

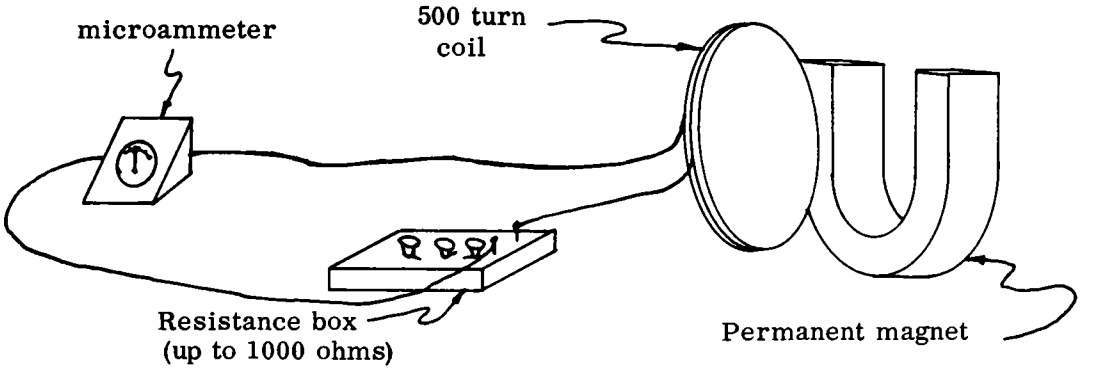
$$I = \frac{V}{R} = \frac{4 \times 10^{-4}}{100} \times 4 \times 10^{-6}\ \text{amp.}$$

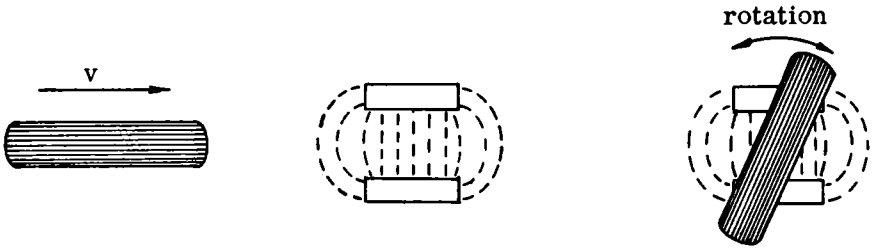
An actual meter might respond quite differently from the ideal meter. For one thing, the meter coil may be inductive. Thus, a back EMF may be generated in the meter itself. (The meter coil is in a magnetic field and the meter acts like a motor; when electrical energy is applied, there is a mechanical motion of the pointer.) Furthermore, the mechanical response time of the meter may be too long, and the meter may not have time to respond fully to the current. (You may recall from advanced courses in electricity that in the limit of very slow mechanical response, the meter receives an impulse from the burst of current, and its deflection depends only on the total charge; in this case, one has a ballistic galvanometer which you may have used to measure charge on a capacitance. When your meter acts like a ballister galvanometer you measure the total flux change and cannot detect the effects of changing v. If you want to do this demonstration, try it first.)

It will not be wise to take the time to explain these extraneous effects to the class. Before using such a demonstration in class, it will be wise to determine whether your microammeter responds when you change v.

C. *Induced Current Using Many Turns of Wire.*

With the microammeter you have available, you may be able to get much better results by using a coil consisting of many turns of wire. (For example, 50 turns of wire might be wrapped around





a 6-inch diameter, improvised wooden spool.) With 50 turns, the EMF will be 50 times larger or $50 \times 4 \times 10^{-4} = 2 \times 10^{-2}$ volt. This larger induced voltage makes it possible to use a relatively large resistance in series with the ammeter. (For example, with a 100-ohm meter, you might use 1000 ohms in series with the ammeter and still get a current of $I = \dfrac{2 \times 10^{-2}}{1.1 \times 10^{3}} = 18 \times 10^{-6}$ ampere. You could then show qualitatively that a 5000-ohm or 10,000-ohm resistor reduces the current as expected.) With this larger moving coil and a large series resistor, it is less likely that the electrical or mechanical properties of the meter will influence the measurement. Thus, the dependence of the induced current on velocity may be demonstrated. Even if the velocity is constant, the current will not be constant unless you have a large region over which the magnetic field is constant. In many experimental arrangements, the maximum current (which is most easily observable) clearly increases with increasing velocity.

The demonstrations can be extended by rotating the coil about a diameter as shown in the figure above. This part of the demonstration goes particularly well with Section 3, however. If you use a resistor and oscilloscope (rather than a microammeter) the output voltage of the coil can be seen to reverse sign. Even with a microammeter, the reversal of the induced current is visible because the needle deflection will reverse. A microammeter with a center zero-point will show the symmetry between positive and negative voltages. For this type of demonstration, it is necessary to stress that $B_\perp$ rather than B is the important parameter.

D. *Relative Motion.*

Connect the terminals of one of the solenoids used in Experiments IV–11 and IV–12 to a microammeter or a sensitive galvanometer. Move a bar magnet in and out of the coil and, while holding the magnet stationary, move the coil to show that the presence of an induced voltage in the coil depends only on the existence of relative motion between the magnetic field and the coil. Several magnets may be tied in a bundle to give a stronger field.

31–3 Magnetic Flux Change

PURPOSE. To introduce a unifying concept to explain the previously observed induction effects and to predict the nature of several other effects which can be confirmed by simple experiments.

CONTENT. (a) The magnetic flux, ϕ_B, through a circuit is defined as the product of the perpendicular component of the magnetic field, $B_\perp$, and the area, A, enclosed by the circuit.

(b) To get induced current in a circuit, the magnetic flux through the circuit must be changing.

(c) This statement is shown to agree with the previously observed effects and to predict several other possible ways of producing induced currents: by changing the magnetic field strength in an electromagnet whether the wire of the loop is in the field or simply surrounding the field, by allowing the area of a loop (made of a spring) to shrink, by rotating a small fixed loop around an axis perpendicular to the direction of the field.

EMPHASIS. This section pulls together a large number of observed induction effects under one simple concept, the change of magnetic flux through a circuit. This is very important material which the students should definitely learn.

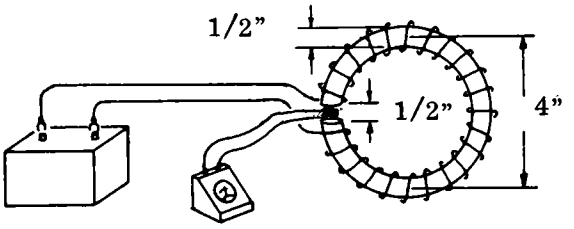
DEVELOPMENT. Again the main burden of teaching can rest on the demonstrations you are able to present, using them as a basis for discussion. The concept of magnetic flux should not be too hard for most students, although there may be some conceptual difficulty when the field is nonuniform (see the boxed material on page 576).

DEMONSTRATIONS.

A. *Induced Current Produced by a Changing Magnetic Field.*

(1) With a stationary coil and a magnet in motion, show that the induced current increases with the rate of change of the magnetic field through the coil. Use a weaker magnet which requires a higher speed of motion to give the same effect, thereby showing it is the rate of change of the flux, not the speed of the magnet, that is important. Demonstrate the difference in strength of the magnets by the quantity of nails each can lift from a pile.

(2) A simple electromagnet experiment can be done using the solenoid of Experiment IV–11. Wind several turns of heavy insulated wire around the solenoid and connect this wire in series with a knife switch and a dry cell and show each of the following: (a) When the switch is closed, an induced current flows while the magnetic field in the coils is building up (the flux in the coil is increasing); (b) An induced current flows when the switch is opened and the flux is decreasing; (c) There is no induced current while the switch stays closed and the flux produced by the loop of wire is constant.



(3) Effects more like that shown in Fig. 31–10 can be demonstrated if you have an electromagnet. If you do not, a simple version can be constructed quite easily. The illustration of Fig. 31–10 can be duplicated effectively with a soft iron ring from which a piece has been cut. Lacking anything better, you can make a ring by bending a piece of concrete-reinforcing rod. With a ring of diameter 10 cm and cross-sectional area about 2 cm^2, you can obtain a sufficiently large magnetic field by winding the ring closely with one layer of 18 gauge *insulated* copper wire. About 100 to 300 turns should be sufficient. Using a 6-volt car battery as a current-source, connect the electromagnet as shown above.

For connections to the battery, use clip-on cables so that they may be attached and detached easily. With a current of 6 amperes, there would be a field of about 0.03 nt/amp-m in the gap of the iron ring. With a few hundred turns of wire the current drawn from the battery should be reasonable — less than 10 amperes. If your coil draws too large a current, it can be cut down with a resistance of a few ohms. About eight 150-watt light bulbs (or two 500-watt heating elements) in parallel will be sufficient. Ten amperes may burn up No. 18 wire if left on continuously.

A small coil of about ten turns of wire can now be connected to the microammeter, and inserted in the gap, as shown in the figure. Upon *reversing* the current in the electromagnet, with the ten-turn coil absolutely still, a deflection will be seen on the microammeter. There has been a change of flux through the coil because, with fixed geometry, the magnetic field has changed (from B to $-B$).

Alter the orientation of the ten-turn coil so that its plane is *along* the field. Repeat the experiment now, and no deflection will be observed. The reason is that although B changes, ϕ does not.

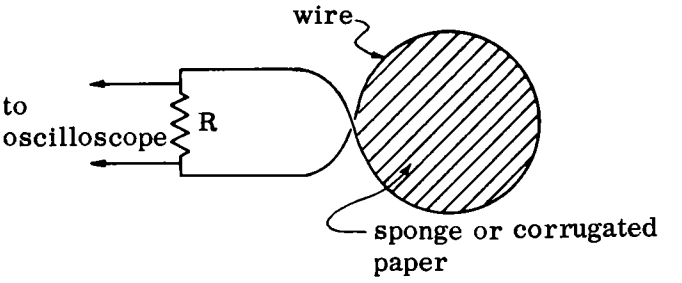
(4) Transformer.

If you have a transformer, you can show your students how it works. Use an alternating current supply and an oscilloscope (or an A.C. meter). Point out that the ratio of secondary to primary voltage is essentially the same as the ratio of secondary turns to primary turns.

If you have an extra transformer, unwrap it so that the students can see the windings. Point out the similarity to the iron ring electromagnet (the primary) and the coil hooked to the microammeter (the secondary). If there is room on the core, wind some turns on and show the students that an EMF is induced on this third winding if either of the others has an energizing alternating current.

B. *Currents Induced by Area Changes.*

(a) Wire on Flexible Form (a variation of Fig. 31–10).



The original loop area should be a bit smaller than the area of strong magnetic field. If you pull the ends of the wire, the loop area will decrease, and the flux will decrease proportionately. This decrease in flux would produce a clockwise induced current if B is into the page. If your meter is too insensitive or your magnet not strong enough to show a visible deflection with a single turn, wrap several turns on a sponge and squash it with your hand. A sensitive oscilloscope will be better than a meter as a detector.

Rotate the plane of the loop by 90° and show that there is no induced voltage if B is in the plane of the loop, i.e., $B_\perp = 0$ even though B and A are large.

(b) Simple Flexible Wire.

Using a simple flexible wire, you can easily change the area by twisting the wire. The main problem will be the small size of the signal you can get from the single turn of wire.

C. *Currents Induced by Changing the Orientation of the Circuit Relative to the Magnetic Field.*

A small coil can be wound and taped to the end of a dowel or stick. The wires from the coil can be twisted together and brought out along the stick to an ammeter or to a resistor with a sensitive oscillo-

scope across it. Slow rotation of the coil will reproduce the effects shown in Fig. 31–11.

31-4 Induced EMF

31-5 Direction of the Induced EMF

PURPOSE. Quantitative experiments show that the change in magnetic flux produces an EMF as the fundamental result, rather than an induced current. The size of this EMF can be related to the flux change through a conservation of energy argument. The ultimate energy source is the mechanical energy required to maintain the motion.

CONTENT. (a) If the induced EMF in a moving wire produces a current I, mechanical energy is required to maintain the velocity of the wire.

(b) The required mechanical force for constant velocity motion of the wire is equal to the magnetic force on the wire, $F = I\ell B$; the mechanical energy that must be supplied then is just $W = -I\ell Bv\,\Delta t$.

(c) The amount of electrical energy needed to push the charge $I\,\Delta t$ around the circuit is just the EMF times the charge, $\mathcal{E}I\,\Delta t$.

(d) Thus the induced EMF is given by equating these energy terms, using the previous definition of change of magnetic flux, $\Delta\phi_B = B\ell v\,\Delta t$, to give

$$\mathcal{E} = \frac{\Delta\phi B}{\Delta t}.$$

(e) The minus sign tells us that the direction of the induced current is such that its own magnetic flux is opposite in direction to the *change* in flux which produced it.

EMPHASIS. This is the all-important section which gives a quantitative result for all of the induced effects we have been observing. It is a particularly important example of the use of conservation of energy, but students can appreciate it without much detailed work. Most students will realize that electrical generators require mechanical forces. It is not necessary to solve quantitative problems using this principle. On the other hand, if you have enough time to treat the chapter thoroughly, you can use this interesting material to give the students worthwhile practice.

DEVELOPMENT. An ounce of demonstration is worth a pound of argument! The following demonstrations, which use the same apparatus as that used for the preceding sections, with inexpensive additions, illustrate in a convincing way the two points which are made in the text; (1) there must be a force *on the coil* when it is moved in the magnetic field; and (2) the force depends only on the relative motion. The demonstration is so simple and persuasive that it should be given *before* a detailed discussion of the text. The text material can then be regarded almost as a series of generalizations of these simple experiments.

It is difficult to verify the results of these two sections *quantitatively*. Once the demonstrations have established them qualitatively, the quantitative equality of mechanical and electrical power should be discussed in class. This quantitative discussion should follow the arguments in the text, and can be used to review electrical power and the magnetic force on a current.

DEMONSTRATION. We wish to show that when a closed circuit is moved in a region containing a magnetic field, the induced currents give rise, in turn, to a force which retards the motion of the coil. Since the students will have seen already (Chapter 30) that a wire carrying a current does experience a force when placed in a magnetic field, this particular result is not completely new. It will, however, emphasize the fact that the electrical energy dissipated in the circuit must come from some source of mechanical energy.

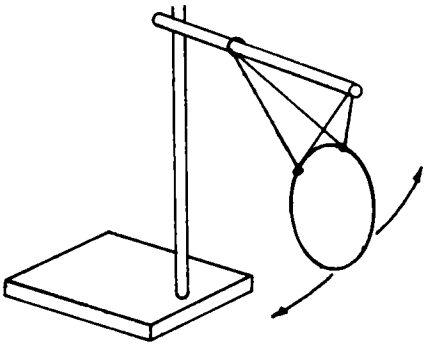
(1) *Lenz's Law and Conservation of Energy.*

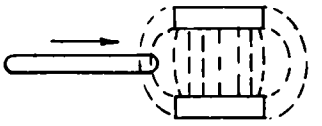
From the direction of the induced current as shown by the meter and the direction in which the coil is wound, find the direction of the magnetic field produced by induced current. Show that this field reacts with the field of the magnet to give a force on the magnet opposite in direction to the magnet's motion and that therefore the direction of the induced current is consistent with the conservation of energy.

A very convincing demonstration of the transfer of mechanical energy can be done as follows: Suspend a strong bar magnet by one end from the spring used in Experiment III–11 so that it can oscillate up and down inside the solenoid used in Experiments IV–11 and IV–12. Connect the solenoid in series with a knife switch and an ammeter (about 0–3 amp), using heavy wire. With the switch open, set the magnet to vibrating, noting that it loses little energy and rises to very nearly the same height at the end of each oscillation. With the magnet still vibrating, close the switch and note that the vibration is quickly damped and the magnet loses mechanical energy, transferring it to the coil circuit where it appears as heat loss due to the flow of current in the wire. A low-resistance meter and heavy wire must be used to get a large enough rate of transfer of energy to make the damping of the vibrations apparent.

(2) *Loop Swinging Freely as Pendulum.*

Make a circular loop of copper tubing (about $\frac{1}{4}$ inch is a convenient size) or of very thick copper wire. (If you have a multiturn coil of the proper loop area, you may use it.) Make a sturdy, electrically low-resistance clamp which can be used to connect the ends of the tubing which forms the loop. If you cannot arrange to close and to open the electrical circuit (loop) easily, make two identical loops, but solder the joint of only one while leaving the other electrically disconnected.





Suspend the loop so that it will swing freely as a pendulum and so that the area of the loop can swing between the magnetic poles, cutting the maximum flux.

When the electrical circuit of the coil is closed, the pendulum motion will be slowed noticeably as it passes through the magnetic field. The stronger the field, the more noticeable the effect (it goes as the *square* of the field strength) so a large surplus-store magnet is much better than a small one. If the electrical circuit is open (or if the magnet is removed) the pendulum will swing freely. Note that if the area of the loop is smaller or bigger than the magnetic pole area, the coil can make small oscillations with no change in the flux. Consequently no currents are induced and the oscillations become small; in this case, small oscillations may persist.

(3) *A Solid Metal Slab as a Circuit.*

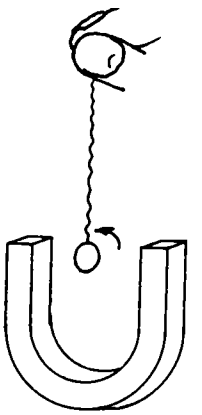
The loop may be replaced by a flat piece of metal without changing the basic effect. If this metal swings between the poles of the magnet, electrical currents (called "eddy currents") are set up and the forces on these currents oppose the motion. A flat metal plate, provided it is thick, experiences a big retarding force if the motion is slow. Although the effect is large, it may not be so instructive for the students because they cannot visualize the current so easily. Another similar metal slab with many fine air gaps (produced by a saw blade) will swing much more freely.

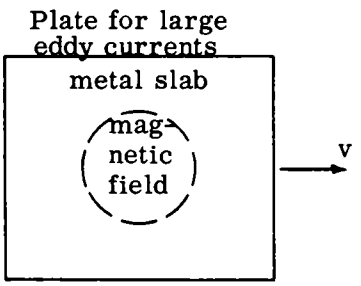
Note. This effect is often used to damp the motion of sensitive chemical balances. Magnetic damping has the important advantage that the retarding force opposing the motion is large when the velocity is great but reduces to zero when the motion is stopped or almost stopped.

(4) *Retarding Force Opposing a Twisting Motion.*

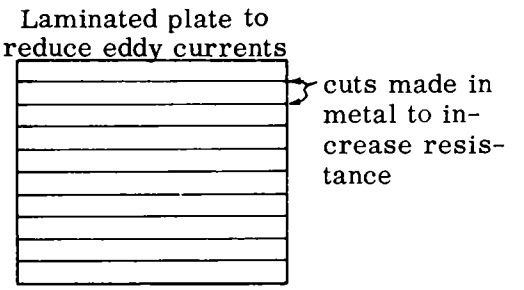
Make a loop of wire using thick wire (large cross-sectional area) but make the area of the loop small enough so that it can be rotated between the jaws of the magnet. Suspend this loop on a fine string which twists easily.

When the loop is not near the magnet, twisting the top of the string will easily rotate the suspended loop. However, when the loop is between the jaws of the magnet, the loop moves more slowly.





If B is into the page in the circled region current circulates



Furthermore, the retarding force varies during the course of rotation. The maximum retarding force exists when the loop is moving and the plane of the loop is along the magnetic field. (This same effect comes up in Problem 1.) Students must realize the importance of $B_\perp$ to appreciate this.

DEVELOPMENT. *Quantitative Conversion of Mechanical to Electrical Power.*

The quantitative material of this section is the basis of the electrical generator. Most students can follow the logic of the text from their experience with the force on a current (Section 30–5). But they need the type of practice given in Problems 13 through 22, if they are expected to be able to use these ideas solving problems.

QUIZ PROBLEMS.

(1) A loop of copper wire is pulled through a magnetic field at a certain velocity. If the strength of the field is doubled, what happens to the force required to maintain the original velocity? (It is four times larger. The current goes up by a factor of 2 because the field is doubled. The force is proportional to I and B. Since both double, the force quadruples.) What happens to the power needed to maintain the motion? (Since the force is 4 times bigger, and v is the same, the power needed is multiplied by 4.) What happens to the power going into electrical form in the loop? How does it compare with the old value? (The induced EMF is doubled and the current is doubled. Therefore $I\mathcal{E}$ is 4 times its old value, in agreement with the four-fold power fed in.)

(2) A 5-turn coil encircles a rectangular area, 2 cm × 3 cm, through which there is a uniform (perpendicular) magnetic field of 4×10^{-2} nt/amp-m. If the magnetic field is reduced to zero in 10^{-3} second, what will be the EMF induced in the coil?

Answer:

$$\text{Flux} = B_\perp A = 4 \times 10^{-2} \times 2 \times 10^{-2} \times 3 \times 10^{-2}$$
$$= 24 \times 10^{-6} \frac{\text{nt-m}}{\text{amp}}$$
$$= 2.4 \times 10^{-5} \text{ volt sec.}$$

$$\text{EMF in one turn} = \frac{\Delta\phi}{\Delta t} = \frac{2.4 \times 10^{-5}}{10^{-3}}$$
$$= 2.4 \times 10^{-2} \text{ volt.}$$
$$\text{EMF in 5 turns} = 1.2 \times 10^{-1} \text{ volt.}$$

This quiz can be made easier by leading students through the steps: ask for (a) the flux, (b) the EMF per turn, and (c) the EMF in the coil.

The quiz problem can be made more difficult by (a) stating a flux change which differs from simply the initial flux going to zero, or (b) having the coil area make an angle with the magnetic field.

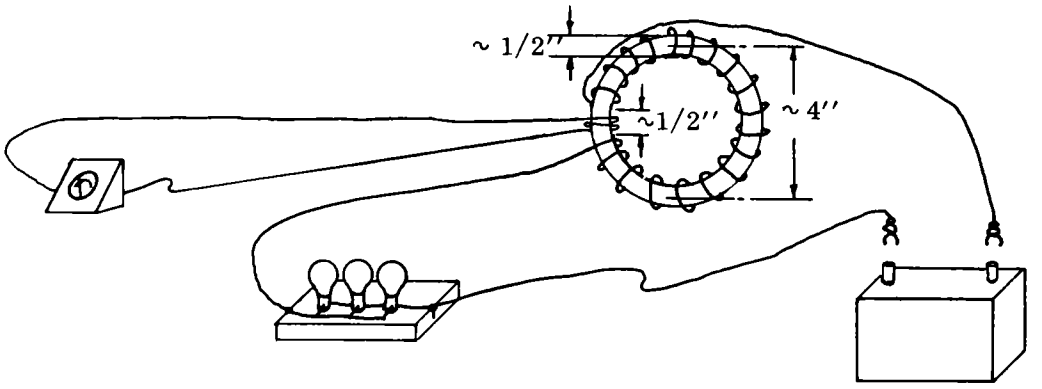
31-6 Electric Fields Around Changing Magnetic Fluxes

PURPOSE. To see that the electric field is induced by a changing magnetic flux; to set the stage for the mechanism of electromagnetic waves.

CONTENT. The EMF induced due to $\frac{\Delta\phi}{\Delta t}$ implies the existence of a field, $\vec{\mathbf{E}}$.

EMPHASIS. This section is important if you plan to discuss the mechanism of electromagnetic radiation in detail. However, you need not take much class time because the existence of the electric field can be inferred from the operation of a transformer.

COMMENT. Many people find it hard to believe that a changing magnetic field produces an electric field at points in space where there is no magnetic field. There is no way of proving this except experimentally. The betatron is probably the best example, but it is an unfamiliar one to students. Be sure to remind your class that the operation of a transformer is very much like the circular electromagnet described in the last section. The battery causes a current in the electromagnet which results in a magnetic field in the magnet. If the gap is plugged with iron as shown here almost no magnetic field will exist outside the ring. Yet a voltage is induced in a coil wound around the ring when the current is cut off. There must have been an electric field induced even where there was no magnetic field.



DEVELOPMENT. Beyond your discussion of the points made in the text, be sure to point out how far we have come from the original ideas of Section 1. The force on a moving charge led directly to the idea of an induced EMF in a wire which moves in a magnetic field. Step by step, further experiments led to abandoning the necessity of motion of a wire in favor of any change of flux through the loop of wire. Finally in this section, the wire is abandoned, and we are led to the very abstract idea of a changing magnetic field producing an electric field.

COMMENT. The last paragraph of this section contains more than may meet the eye. That the changing magnetic field forms a kind of "magnetic current" is a direct preparation for Section 7 where it is shown that a changing electric field is equivalent to an electric current and thus produces a magnetic field just like a real electric current.

31-7 Magnetic Fields Around Changing Electric Fluxes

PURPOSE. To set the stage for the mechanism of electromagnetic waves.

CONTENT. A changing electric field produces a magnetic field.

EMPHASIS. A qualitative understanding of this material is necessary to an understanding of the nature of electromagnetic radiation discussed in the next section.

COMMENT. Even the best students have trouble with a quantitative treatment of induced magnetic fields unless a sizable amount of time is allotted to these ideas. The main purpose of the quantitative material shown boxed on page 585 is to indicate to students that these ideas can be treated quantitatively with the material thus far covered.

You will probably not have time to explain this quantitative material to average students.

* * *

31-8 The Mechanism of Electromagnetic Radiation

PURPOSE. To show that the principles of electricity and magnetism presented thus far make possible the prediction that electric and magnetic fields can generate each other in such a way that they can propel themselves through space, thus giving rise to electromagnetic radiation.

CONTENT. A very particular combination of changing electric and magnetic fields can propagate each other through space as an electromagnetic wave.

The wave always moves with the "velocity of light," $c = 3 \times 10^8$ m/sec.

The electric and magnetic fields are perpendicular to each other, and both are perpendicular to the direction of propagation.

EMPHASIS. The mechanism of electromagnetic waves is the climax of the sections on induced electric and induced magnetic fields. However, a qualitative treatment may have to be sufficient. Class time probably can be spent most profitably in going over Fig. 31-19.

31-9 Evidence for Electromagnetic Radiation; The Electromagnetic Spectrum

PURPOSE. To introduce the wide variety of electromagnetic waves.

CONTENT. Electromagnetic radiation is produced by accelerating electrical charges. Different physical situations produce different frequencies, f (or wavelengths, λ). For all electromagnetic waves, $f\lambda = 3 \times 10^8$ m/sec.

EMPHASIS. This is an important section and even if you have been rushed and have omitted considerable material on electricity and magnetism, some class time should be given to discussing electromagnetic waves and the electromagnetic spectrum. Above all, for material which follows, students should know that accelerating charges produce electromagnetic waves even if they have only a vague idea of what an electromagnetic wave is.

Furthermore, at some point, you should stress that the wave model of light discussed in Part II involves electromagnetic waves. The speed of light, the effects caused by light, and the origin of light can all be understood in terms of electromagnetic effects.

COMMENT. *The Mechanism for Producing Electromagnetic Waves is Quite Simple (Compared with Mechanism of Propagation).*

Be sure to emphasize the simple classical criterion which indicates whether an electromagnetic wave is propagated: whenever a charge accelerates, electromagnetic radiation is produced. (The rate of emission of radiant energy is proportional to the square of the acceleration.)

Many students, struggling to understand the mechanism of propagation, miss this very simple feature of the production of electromagnetic waves. Once it is pointed out (and perhaps supplemented with the understandable examples given below), students will be able to use this production mechanism as a clue to atomic structure (and the need for quantum mechanics).

DEVELOPMENT. *Production Mechanism for Apparently Different Electromagnetic Waves.*

To show that accelerating charges are responsible for electromagnetic radiation, it may be wise to discuss examples which cover a very broad wavelength (or frequency) interval.

At the long-wavelength, low-frequency end of the spectrum, the frequency is low enough to make the mechanism apparent. In radio antennas, an alternating electric field accelerates charges, first in one

direction, and then in the other. The emitted radio waves depend on this acceleration; their frequency is the same as the number of cycles of acceleration. This mechanism commonly gives frequencies from about 10^4 to 3×10^{11} per second, corresponding to wavelengths from 3×10^4 meters to 10^{-3} meter.

At the higher frequencies associated with infrared, visible, and ultraviolet light, accelerated charges in atoms are the usual source. These electromagnetic radiations are also produced by high-energy electrons undergoing centripetal acceleration in a modern particle accelerator. The energy loss (due to radiated electromagnetic energy) is a crucial factor in designing electron accelerators with energies above about 10^8 electron volts.

Because the radiation varies with acceleration, for a given force, the radiation varies inversely with the mass (actually as m^{-2}), and electrons radiate much more than do more massive charged particles, such as protons in the electric fields.

Still higher frequencies, up to the highest that can be produced, are generated when charged particles are decelerated. This deceleration mechanism produces not only the standard X rays (with frequencies above 3×10^{16}/sec) but also the highest frequency or highest energy gamma rays. Several accelerators now produce electrons of 10^9 ev and these, in stopping, radiate gamma rays with frequencies of about 2×10^{23}/sec. Cosmic rays generate even higher frequency γ rays.

COMMENT. *Electromagnetic Character of Light Waves.*

Be sure to emphasize in class that light is an electromagnetic wave. Explain that if time were available you could return to the characteristics of light and analyze them. For example, electric fields cause motion of charges in metal. When a light wave hits a metal the charges near the surface acclerate and send back a reflected wave. When light interacts with matter, the charges within matter are basically what respond.

Point out too that knowing the electromagnetic character of light allows us to use light, (and ultraviolet, and X rays, and gamma rays) to explore the position of, and the forces on, charged particles in matter. In this way, electromagnetic waves become probes of the structure of matter.

DEVELOPMENT. *Electromagnetic Spectrum.*

Students should appreciate the tremendous range of phenomena associated with electromagnetic waves.

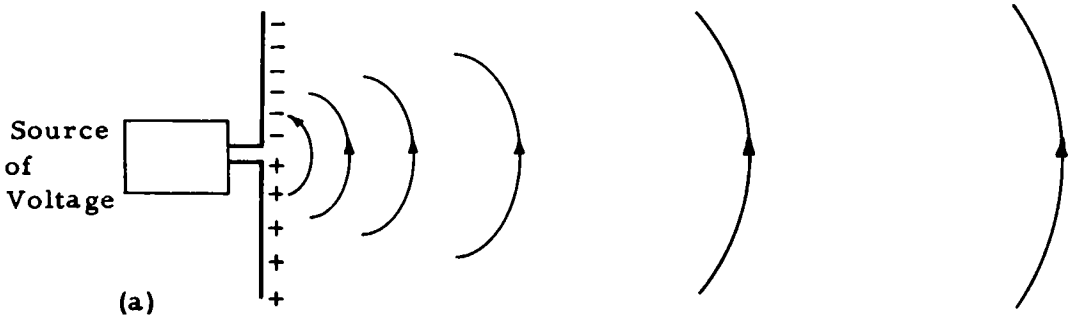
Using the formula $F\lambda = c = 3 \times 10^8$ m/sec you can find the wavelength corresponding to a known frequency, or the frequency corresponding to a known wavelength. Supplement these numbers with specific physical examples showing both differences and similarities between different electromagnetic waves. The similarities are clear: the speed of 3×10^8 m/sec, the origin in accelerating charge, and the interaction which is always due to the motion of charge. The differences include the wavelength and frequency, the size of the system emitting the radiation, and the size and characteristics of the systems which interact with the wave.

Note that many phenomena do not vary smoothly with wavelength or frequency. For example, consider the penetration of electromagnetic waves through a piece of paper. At low frequencies, transmission is high; radio and radar waves go through paper unimpeded. At higher frequencies, such as infrared, there is a strong absorption; paper will burn when sunlight is focused on it. At still higher frequencies, in the visible range, there is selective reflection and absorption depending on the exact frequency (which with visible light is associated with color). In the ultraviolet, absorption again takes over. At still higher frequencies, the far ultraviolet and X rays, there is some slight absorption but most of the radiation is transmitted.

Note that at this stage of the course (i.e., before you discuss photons and photon energies $E = h\nu$, Chapter 33, page 622, the principal clue to what a wave will do is its wavelength. As you discuss different phenomena, be sure to write the relevant wavelengths on the blackboard.

Of course, the wavelength varies smoothly from one end of the spectrum to the other. The overall pattern of electromagnetic interaction also varies smoothly although there are many detailed features in some parts of the spectrum. With long wavelengths, at frequencies below infrared, electromagnetic interactions depend on free charges. Any metal behaves like other metals do; the key to electromagnetic effects in this range is "how free are the charges."

When the wavelengths become smaller, the detailed structure of matter plays a more important role. Infrared radiation affects molecules easily. Visible and ultraviolet radiation move charges relative to one another within an atom, especially at certain natural frequencies. These wavelengths are still large compared with the atom (by about a factor of 10^2 or 10^3), so that the crude picture of wavelength determining the size of the object with which interaction occurs is not exact. (It may miss by several orders of magnitude. But even several orders of magnitude are not much in the entire electromagnetic spectrum.) As the wavelength becomes still smaller, the interaction is with the innermost atomic electrons (X rays). A further decrease in wavelength involves either interaction with individual electrons (the Compton effect) or interaction including the atomic nucleus and the closest electron (the atomic photoelectric effect). Finally, at still shorter wavelengths the main interaction is nuclear. For X rays and gamma rays the interaction is governed by the charge of the nucleus and not by the arrangement of atoms (i.e., in molecules, in crystals, in metals, etc.)



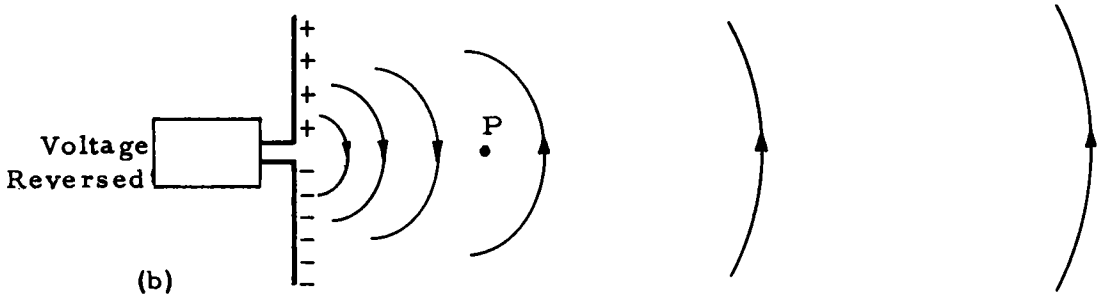
DEVELOPMENT. *Antennas.* These days your students see television antennas atop most houses and it may not be amiss to describe the action of a simple antenna. You cannot at this level *prove* anything, but the class may get a feel for some aspects of electromagnetic radiation.

At (a) we have shown a simple, very familiar antenna which consists of two rods which are connected near the center to a voltage source — for the moment just a battery. The voltage charges the upper rod negatively, the lower one positively. An electric field extending through all space is set up by these charges. We have shown it only to the right, but of course it extends in all directions. Now let us suddenly switch the battery to the other polarity so that the top rod becomes positively charged and the lower one negatively. We then have the situation shown at (b).

There was no mistake made in the drawing. The field beyond *P* is shown just as it was before, while only the field near the antenna has changed. The point is that the regions beyond *P* have not yet had time to "learn about" the change of the polarity of the antenna. Why not? When the electric field was changed in direction, a *magnetic* field was *generated* perpendicular to the paper. But if a magnetic field appeared where none was before, it *changed.* This *changing magnetic* field generated an *electric field* which was just such as to compensate for the change in the original electric field for points beyond *P.* The result of all this is that points distant from the antenna can "learn about" the change in polarity of the antenna only through a changing electric and perpendicular magnetic field which can travel only at the speed of light as discussed in the last section.

It only remains to add that the antenna itself must produce not only an electric field but also a magnetic field; for if the bottom was originally positive and later on the top was, positive charge must have run up the antenna or negative charge down. In either case a current ran up the antenna. A current going up the antenna would produce the necessary magnetic field. Thus we see that in every case a changing electric or a changing magnetic field implies the other; the two are irrevocably coupled together.

To get back to our antenna and battery, we see that if now the battery is switched back and forth, changing electric and magnetic fields sweep out across space alternating in direction. They try, in a sense, to carry the message as rapidly as they can, i.e., with the speed of an electromagnetic wave, as to which way the charge is on the antenna at a given instant. The result is an oscillating electromagnetic wave. Notice that if the transmitting antenna is horizontal the electric field is always horizontal, and an antenna used to pick up the signal must also be horizontal. To receive a signal, a wire or antenna is placed in this field and the electrons in it are forced back and forth in the wire or antenna, producing a current in the receiver. If a wire is vertical in a horizontal electric field no electrons will flow. In this country convention is that the electric fields for TV transmission will be broadcast horizontally. Hence all TV antennas are oriented horizontally. This need not be so. Vertical orientation would work just as well and is indeed used in England.



FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The table below classifies problems according to their estimated level of difficulty and the section with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1.* The positive charges will be at Q, the negative charges at P.
2.* (a) The force on positive charges is directed outward from the page; the force on negative charges is directed into the page.
3. See discussion on page 31-14.
4.* There would be no current as long as the entire loop was in the region of uniform field.
5.* (a) When the plane of the loop is parallel to the magnetic field.
(b) When the plane of the loop is perpendicular to the magnetic field.
6.* The flux through the loop will not change.
7.* $\phi_B = 1 \times 10^{-2}$ newton-m/amp.
8. $\phi_B = 1.6 \times 10^{-6}$ newton-m/amp.
9.* Yes. The force you must exert on the loop is proportional to the current, which would be reduced if the electrical resistance of the loop were increased.
10.* At high speed, since the pulling force would have to be greater.
11.* $\mathcal{E} = 1 \times 10^{-4}$ volt.
12.* $\mathcal{E} = 0.5$ volt.
13. (a) $\mathcal{E} = 0.1$ volt;
(b) $I = 2$ amps;
(c) $F_{mag} = 0.1$ newton.
14. (a) Replace battery by external circuit and mechanically rotate the coil.
(b) The plane of the wire loop should be perpendicular to $\vec{\mathbf{B}}$.
15. (a) Clockwise in both cases.
(b) The induced currents would be the same.
(c) Induced EMF is directly proportional to loop area.
(d) $\mathcal{E} = 0.72$ volt.
16. (a) $\mathcal{E} = 0.16$ volt;
(b) $\mathcal{E} = 0.16$ volt;
(c) $\mathcal{E}I = 0.32$ watt;
(d) $F_{wire} = 1.6$ newtons.
17. See discussion on page 31-17.
18.* No; the magnetic force on the charges in the moving loop, which is proportional to the speed of the loop, would increase until it exactly cancelled the applied force, and the loop would then move with constant velocity.
19. (a) The pointer swings toward P; the pointer swings toward Q.
(b) The pointer swings toward P.
(c) The pointer still swings toward P.
20. (a) No induced currents for equatorial orbits; induced currents for polar orbits.
(b) They will oppose the motion and might deflect the satellite.
21. (a) See discussion on page 31-19.
(b) No; the force opposing acceleration depends on IB.
22. See discussion on page 31-21.
23.* No; the lines are closed loops, since there are no free charges on which they can begin or end.
24. 9.6×10^6 electron volts.
25.* The rate at which charge is flowing onto either plate, which in turn determines the rate of change of electric flux through the region between the plates.
26.* (a) Yes; with a constant current.
(b) No; it is impossible to change the magnetic

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1, 2	1, 2		3#		3#
3	4, 5, 6, 7		8		
4	9, 10, 11, 12	13	14, 16#, 17	15#	15#, 16#
5	18		19, 20, 22	21	21
6	23			24	24
7	25, 26, 27				
8			28		28
9	29	31#	30#		30#, 31#

flux within a region at a constant rate indefinitely.

27.* (a) Negative charges on the top plate, positive charges on the bottom plate.
(b) The electric fields point upward.

28. See discussion on page 31-33.

29.* (a) 3×10^8 m/sec;
(b) 3×10^8 m/sec;
(c) 3×10^8 m/sec.

30. (a) $t_{stop} = 6.7 \times 10^{-18}$ sec.
(b) $f = 1.5 \times 10^{17}$ sec^{-1}.
(c) $\lambda = 2.0 \times 10^{-9}$ meter.

31. Sound does not travel through empty space; it does not go fast enough; it does not have electrical origin nor electrical interactions, etc.

COMMENTS AND SOLUTIONS

PROBLEM 1* (See Short Answers.)

PROBLEM 2* (See Short Answers.)

PROBLEM 3

(a) Analyze the situation of Fig. 31-5 entirely from the point of view of the forces on charges in the wire as it moves through the magnetic field, and show why there is no induced current.
(b) Continue this line of argument to show why there is induced current as one side of the loop gets into the weaker field at the edge of the magnet.

(a) Since $\vec{\mathbf{v}}$ is to the left and $\vec{\mathbf{B}}$ points down, the force on all positive charges in the wire is toward the front. Since the entire wire is in a region of uniform magnetic field, the magnitude of the force is the same everywhere. Positive charges between P and Q are pushed toward P, but positive charges between R and S are pushed toward S with the same force, and there is no net current. The charges between Q and R and between S and P are pushed perpendicular to the wire and cause no current.

(b) When RS reaches a region of weaker field, the force on positive charges within it is still toward S but is not as great as the force pushing positive charges from Q toward P. Hence, positive charges can move from Q to P to S to R, creating a current which is clockwise from our point of view.

PROBLEM 4* (See Short Answers.)

PROBLEM 5* (See Short Answers.)

PROBLEM 6* (See Short Answers.)

PROBLEM 7* (See Short Answers.)

PROBLEM 8

What would be the magnetic flux inside a long solenoid 4.0 cm in diameter with 10 turns per cm of length when the current is 1.0 amp?

From Section 30-11, the magnetic field B inside a solenoid carrying a current I, where N is the number of windings per unit length, is $B = 2\pi KNI$. The cross-sectional area of a solenoid of diameter d is $\pi d^2/4$. Hence the flux is

$$\phi_B = BA = (2\pi KNI)(\pi d^2/4) = \pi^2 KNI d^2/2.$$

For the given solenoid,

$$\begin{aligned}\phi_B &= (3.14)^2(2 \times 10^{-7}\ \text{nt/amp}^2) \\ &\quad \times (1.0 \times 10^3\ \text{turns/meter})(1.0\ \text{amp}) \\ &\quad \times (4.0 \times 10^{-2}\ \text{m})^2/2 \\ &= \underline{1.6 \times 10^{-6}\ \text{nt-m/amp}}.\end{aligned}$$

PROBLEM 9* (See Short Answers.)

PROBLEM 10* (See Short Answers.)

PROBLEM 11* (See Short Answers.)

PROBLEM 12* (See Short Answers.)

PROBLEM 13

Suppose that in Fig. 31-2 the magnetic field strength is 0.5 newton/amp-m, l is 10 cm, the resistance of the loop is 0.5 ohm, and the loop is moved at a constant speed of 2 m/sec.
(a) What is the EMF generated in the wire?
(b) What is the current in the circuit?
(c) What is the magnetic force acting on the wire?

(a) The EMF is

$$\begin{aligned}\mathcal{E} &= vB_\perp \ell \\ &= (2\text{m/sec})(0.5\ \text{nt/amp-m})(0.1\ \text{m}) \\ &= 0.1\ \text{joule/amp-sec} = \underline{0.1\ \text{volt}}.\end{aligned}$$

(b) $I = \mathcal{E}/R = 0.1\ \text{volt}/0.05\ \text{ohm} = \underline{2\ \text{amps}}.$

(c) $F_{mag} = B_\perp I \ell$
$= (0.5\ \text{newton/amp-m})(2\ \text{amp})(0.1\ \text{m})$
$= \underline{0.1\ \text{newton}}.$

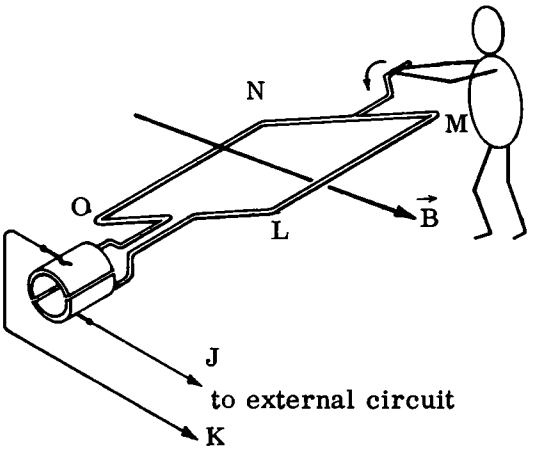
PROBLEM 14

Figure 30-17 is a diagram of an electric motor.
(a) Explain how you could use the same arrangement of magnetic field and wire loop to make a generator that would produce current in only one direction in an external circuit.
(b) How should the wire loop be oriented in the magnetic field when the commutator interchanges the connections between the ends of the loop and the wires of the outside circuit?

(a) Some students may give a short answer which is correct but which they have not checked properly. There should be three steps in a complete answer:

(1) Arrange to rotate the coil with an external mechanical force. (The fact that an external mechanical force will be needed follows from Section 4, but probably can be "guessed" before.)

(2) Put the external circuit in place of the battery.
(3) Check the polarity to be sure that the current goes in only one direction. This check essentially incorporates the answer to part (b).



When the plane of the coil is parallel to the field, and the right-hand side is moving upward while the left-hand side is moving down, the flux through the coil is momentarily zero, but the flux is changing with time and there is an induced EMF. The direction of the induced current can be found by considering the force exerted by the magnetic field on a moving positive charge. A positive charge along LM is pushed from L to M, according to the right-hand rule, and a positive charge along NO is pushed from N to O. The current therefore flows from K, through the external circuit, and back to J.

As the coil continues turning, the flux through the coil keeps changing until the coil is perpendicular to the field. At this point, the flux reaches its extreme value, and further turning causes a reverse change in the flux. If the flux was increasing, it is now decreasing. (See Fig. 31–11.) Hence, the current through the coil reverses, and is momentarily zero as it changes direction. From the point of view of the force on a positive charge, the velocity of the charge is parallel to the field, so there is no force when the coil is perpendicular to the field. Note that the commutator changes the connections between the external circuit and the coil just at this point. Hence the current through the external circuit remains in its previous direction. This line of argument can be extended for further orientations of the coil.

(b) The plane of the wire loop should be perpendicular to $\vec{B}$ when the commutator interchanges connections because the induced EMF and current are then zero. See part (a) above.

PROBLEM 15

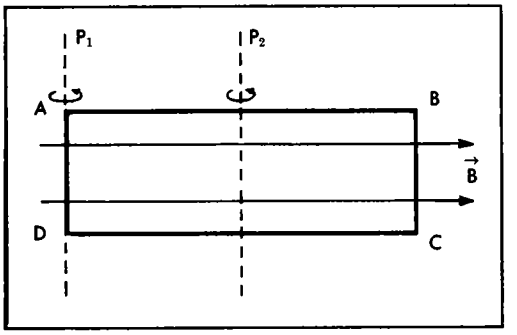
The rectangular wire loop shown in Fig. 31–22 is in a uniform magnetic field.

(a) In what direction (clockwise or counterclockwise) is the induced current when the loop is in the plane of the paper and BC is moving into the page about P_1 as an axis? About P_2?

(b) For a given speed of rotation, how would the induced currents compare when the loop is rotated about P_1 and then about P_2?

(c) For a given speed of rotation, how does the induced EMF depend on the area of the loop?

(d) If the magnetic field has a magnitude of 1.5 newtons/amp-m, AB = 10 cm, and BC = 4 cm, what is the maximum induced EMF (in volts) when the loop is rotating about P_1 at 19.1 revolutions per second $\left(\frac{120}{2\pi}\text{ rps}\right)$?



This is a straightforward problem about induced EMF, but it is listed as difficult because it involves circular motion. It emphasizes the importance of flux. This problem is probably better suited to class discussion than a home assignment.

(a) The direction of current can be found by determining the direction of the force on a positive charge. Use the right-hand rule. Consider the length BC, and consider the rotation about P_1.

Point the thumb in the direction of $\vec{v}$, into the page. Point the forefingers along the magnetic field. The open palm shows the direction of the force on a positive charge, from B to C. There is no force along AB or DC. AD is not moving. The induced current is clockwise in the loop.

For the rotation about P_2, the force in BC is in the same direction. There is also a force in AD. In this case, AD is moving out of the page. The thumb points out of the page. The fingers point toward the magnetic field. The palm is up; the force on a positive charge goes from D to A. This contributes to the clockwise induced current.

(b) The speed of rotation determines the rate at which the loop rotates, and hence the rate at which the component of the magnetic field perpendicular to the plane of the loop changes with time. Hence, when the loop is in the plane of the paper, the rate of change of flux must be the same for rotation about either axis, since the area of the loop itself does not change. We conclude that the current is the same in both cases.

(c) The induced EMF is proportional to the rate of change of flux, which is proportional to the area of the loop. Hence, the induced EMF is itself proportional to the area of the loop.

(d) The maximum EMF is induced when the plane of the loop is parallel to the field, as in the figure, since at this orientation the flux is changing at the greatest rate. To calculate the maximum EMF, we recall from part (a) that the only side along which an EMF is induced is BC. The EMF along a wire of length ℓ moving perpendicular to a field B at speed v is $\mathcal{E} = vB\ell$. In one complete revolution at frequency f, side BC covers a distance $2\pi \times$ (length AB) in time $T = 1/f$, so its speed is

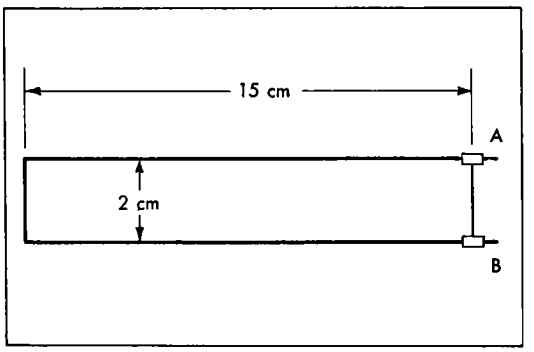
$$v = \frac{2\pi \times (\text{length } AB)}{T} = 2\pi f \times (\text{length } AB).$$

Hence, the EMF induced along BC when the loop is parallel to the field is

$$\begin{aligned}\mathcal{E} &= 2\pi fB \times (\text{length } AB) \times (\text{length } BC)\\ &= 2\pi\left(\frac{120}{2\pi}\text{ cycles/sec}\right)(1.5\text{ nt/amp-m})\\ &\qquad \times (0.10\text{ m})(0.04\text{ m})\\ &= 0.72\text{ nt-m/amp-sec} = 0.72\text{ joule/coul}\\ &= \underline{0.72\text{ volt}}.\end{aligned}$$

PROBLEM 16

A U-shaped wire (Fig. 31–23) has a movable wire AB connected to it. This arrangement is in a uniform field perpendicular to, and into, the page.



(a) If the magnetic field strength is 40 newtons/amp-m, what is the induced EMF (in volts) when AB is in the position shown and moving at 20 cm/sec? Calculate this first from the rate of flux change and then from the magnetic force on the elementary charges in the wire.

(b) What is the induced EMF when AB is 5.0 cm from the left end and moving at 20 cm/sec? At 10 cm/sec?

(c) At what rate is energy fed into the loop when AB moves at 20 cm/sec and the induced current is 2.0 amperes?

(d) If the induced current is 2.0 amperes, what force is needed to move AB at 20 cm/sec?

(a) To calculate the induced EMF from the flux change we use

$$\mathcal{E} = \frac{\Delta\phi_B}{\Delta t}.$$

(We ignore the minus sign in the equation because we are not told whether AB is moving to the right or left, so we shall be concerned only with magnitudes.) Now, the change in flux is due to the change in area $\Delta A = \ell v\,\Delta t$, where v is the speed of the movable wire AB and ℓ is the length of AB. Hence,

$$\begin{aligned}\mathcal{E} &= \frac{\Delta\phi_B}{\Delta t} = \frac{B\,\Delta A}{\Delta t} = \frac{B\ell v\,\Delta t}{\Delta t} = Bv\ell\\ &= (40\text{ nt/amp-m})(0.20\text{ m/sec})(0.02\text{ m})\\ &= \underline{0.16\text{ volt}}.\end{aligned}$$

Alternate Solution

Although Section 31–4 stresses the relation between flux and induced EMF, some students may prefer to work the problem from the point of view of forces. The force on an elementary charge q moving at a velocity $\vec{\mathbf{v}}$ perpendicular to a magnetic field $\vec{\mathbf{B}}$ is $F = qvB$. The EMF is the energy per charge needed to move a charge the entire distance between A and B. Since work is force times distance, the work done per charge when a charge is moved the distance ℓ is

$$\mathcal{E} = (F/q)\ell = vB\ell.$$

This is identical to the expression we derived by considering the change in flux, so again $\mathcal{E} = \underline{0.16\text{ volt}}$.

(b) The induced EMF depends on the time rate of flux change. In this example, the flux change is due to a change in area. Because the rate of change of area depends on the speed of AB but not on the area itself, the induced voltage is $\underline{0.16\text{ volt}}$ if $v = 20$ cm/sec as in part (a), even though 15 cm has been changed to 5 cm and the area is different.

(c) By conservation of energy, the energy fed into the loop is the energy given to the charges which move through a potential difference given by the EMF. Since the EMF is the energy per charge, the total energy fed in is equal to the EMF times the total charge, and the *rate* at which energy is fed in is equal to the EMF times the rate of flow of charge, or the EMF times the current. In this case, using the value of $\mathcal{E}$ found in (a),

$$\begin{aligned}\mathcal{E}I &= (0.16 \text{ volt})(2.0 \text{ amps}) \\ &= 0.32 \text{ joule/sec} = \underline{0.32 \text{ watt}}.\end{aligned}$$

(d) The force needed is equal in magnitude and opposite in direction to the magnetic force on the wire. Hence, the magnitude of the force is

$$\begin{aligned}F_{\text{wire}} = \ell IB &= (2 \times 10^{-2} \text{ m})(2.0 \text{ amps}) \\ &\quad \times (40 \text{ nt/amp-m}) \\ &= \underline{1.6 \text{ newtons}}.\end{aligned}$$

Alternatively, the rate at which energy is supplied, or the power, is equal to the applied force times the speed of the wire:

$$\begin{aligned}F_{\text{wire}}v &= \text{power, or} \\ F_{\text{wire}} &= \text{power}/v = 0.32 \text{ watt}/0.20 \text{ m/sec} \\ &= 1.6 \text{ joules/m} \\ &= \underline{1.6 \text{ newtons}}.\end{aligned}$$

PROBLEM 17

Suppose that the moving circuit of Problem 13 has a mass of 200 grams, and that the external force is suddenly removed when it is moving at 2 m/sec. What will its motion be like afterwards?

This problem can be discussed at several different levels, depending on the time available and the abilities, interest, and preparation of the class.

Most simply, as long as the wire is moving there is an EMF induced in it, and this EMF produces a current in the circuit. While there is a current in the wire there is a magnetic force on it, and the right-hand rule shows that this force is opposite in direction to the motion of the wire. When the external force is removed, the wire begins to slow down.

When the wire moves more slowly, the induced EMF decreases, and the magnetic force decreases in proportion with it. When the force decreases, so does the acceleration of the wire, so the wire loses speed at a decreasing rate.

To calculate the change in speed of the wire, we begin with the induced EMF, $\mathcal{E} = B\ell v$. This produces a current $I = \mathcal{E}/R = B\ell v/R$. The magnetic force on the wire is

$$F = BI\ell = B\left(\frac{B\ell v}{R}\right)\ell = \frac{B^2\ell^2}{R}(v) \text{ in magnitude.}$$

Since this force opposes the motion, vectorially we have $\vec{\mathbf{F}} = -\frac{B^2\ell^2}{R}\vec{\mathbf{v}}$. Finally,

$$m\,\Delta\vec{\mathbf{v}} = \vec{\mathbf{F}}\,\Delta t = -\frac{B^2\ell^2}{R}\vec{\mathbf{v}}\,\Delta t.$$

So

$$\begin{aligned}\Delta v &= -\frac{B^2\ell^2}{mR}v\,\Delta t = -\frac{(0.5)^2(0.1)^2}{(0.2)(0.5)}v\,\Delta t \\ &= (-0.25/\text{sec})v\,\Delta t.\end{aligned}$$

The change in speed is proportional to the time interval over which we watch the wire, and also to the speed itself.

An exact solution of this equation requires calculus, but it is not difficult to make a numerical approximation. If we take $\Delta t = 1$ sec, $\Delta v = -0.25\,v$, we can find the speed at the end of any second by subtracting a quarter of the previous speed. For the first 5 seconds of motion this gives:

Time (sec)	0.00	1.00	2.00	3.00	4.00	5.00
v(m/sec)	2.00	1.50	1.13	0.85	0.64	0.48
Δv(m/sec)		0.50	0.37	0.28	0.21	0.16

These values are represented by the points on the graph. To find the distance the wire went, we use the area under the velocity curve, which we approximate by connecting the points with straight-line segments. The area of the five trapezoids gives a distance of 5.36 meters.

Students who are taking calculus may want to integrate the equation for v to obtain

$$v = v_i e^{-0.25t}.$$

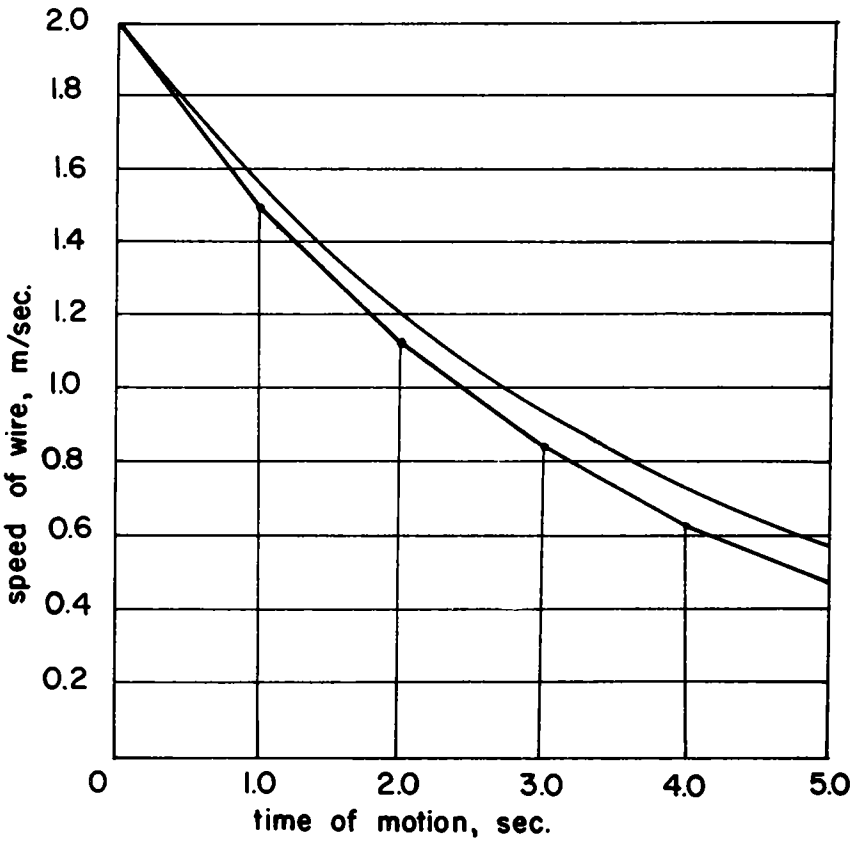
The curved line on the graph represents this equation. The distance of travel is

$$\begin{aligned}s = \int_0^t v\,dt &= \int_0^t v_i e^{-0.25t}\,dt \\ &= v_i/0.25(1 - e^{-0.25t}).\end{aligned}$$

This gives, for the distance of travel in 5 seconds, 5.72 meters.

The numerical approximation can be improved in accuracy by calculating the Δv's for shorter intervals of time.

According to the exact solution, the wire will continue to move for an infinite time, but since the area under the velocity curve is finite, it will travel only the finite distance of 8.00 meters.



PROBLEM 18* (See Short Answers.)

PROBLEM 19

A loop of wire *JKL* lies between the pole pieces of a magnet with the plane of the loop parallel to the pole pieces, as shown in Fig. 31–24(a). It is connected to a sensitive ammeter whose pointer stands at the center of the scale when there is no current, and which is so constructed that the pointer swings toward the terminal (*P* or *Q*) at which the current enters the ammeter.

(a) Which way does the pointer swing when the loop is pulled out of the field of the magnet? When it is put back as it is in Fig. 31–24(a)?

(b) Which way does the pointer swing when the loop is put back as it is in Fig. 31–24(b)?

(c) How does the pointer move when the loop is turned over from the first position to the second without being taken out of the field of the magnet?

(a) If we apply the right-hand rule to the portion of the loop at *K*, we find that positive charges are moved toward us, and go around the loop in the sense of *LKJ*. The current enters the ammeter at *P*, and the pointer swings toward *P*.

When the loop is put back, the original action is reversed, the current flows away from us at *K*, and the pointer swings toward *Q*.

(b) If the loop is put back in the position of the second drawing, the current flows away from us at *K*; so it goes around the loop in the sense of *LKJ*. The pointer swings toward *P*.

(c) Turning the loop from its first position through 90°, so that its plane lies parallel to the magnetic field, brings it into such a position that there is no flux through it. This is equivalent to drawing it out of the field. The pointer swings toward *P*. Turning it through another 90° in the same direction to bring it to the second position is equivalent to putting it back as we did in part (b). The pointer still swings toward *P*.

PROBLEM 20

(a) Will the earth's magnetic field induce currents in an artificial satellite with a metal surface that is in orbit around the equator? Around the poles?

(b) If so, how would these currents affect the motion of the satellite?

This is a qualitative problem but is difficult because it requires students to consider the flux through a satellite in the magnetic field of the earth. Students find it hard to visualize currents in a spherical shell.

(a) An orbit around a circle of constant magnetic latitude provides a case in which the flux through the satellite does not change. The orbit about the equator is the only possible such orbit. (The center of the earth must lie in the plane of the orbit.) Thus there will be no induced currents in the metal of a satellite

which is orbiting around an equatorial circle. (Strictly these statements apply only to the magnetic equator which does not coincide with the geographical equator.) In other orbits, since $\vec{B}$ will be changing in magnitude and direction, the flux will be changing, and there will be induced currents.

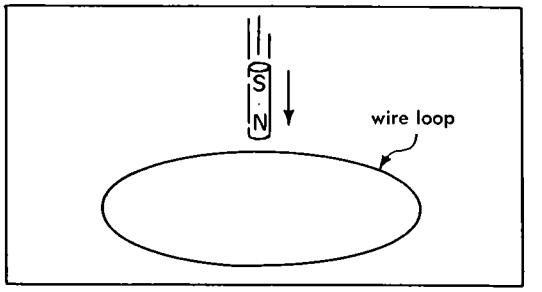
(b) The currents produce a magnetic field, and therefore the satellite will itself act like a small magnet. It will experience a small magnetic interaction with the earth so that the force is not purely gravitational. There will therefore be deflection from the gravitational path the satellite would take if it moved in the gravitational field alone. Part of the effect of the interaction must be to cause the satellite to lose energy dissipated in heat by the induced currents. Paradoxically, if it loses energy it will speed up. See Section 24–5. It is hard to estimate the magnitude of these forces.

PROBLEM 21

A bar magnet is dropped through a wire loop as shown in Fig. 31–25.

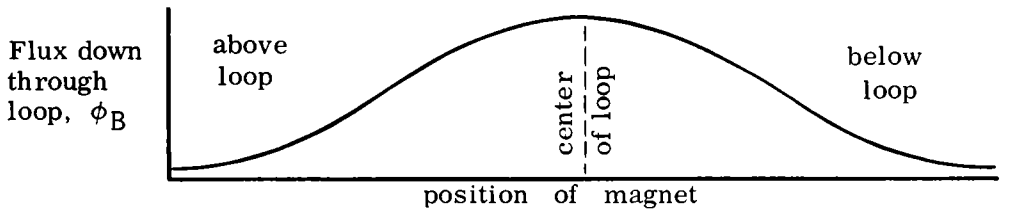
(a) Describe the changes in the direction and magnitude of the induced current in the loop as the magnet falls through the loop.

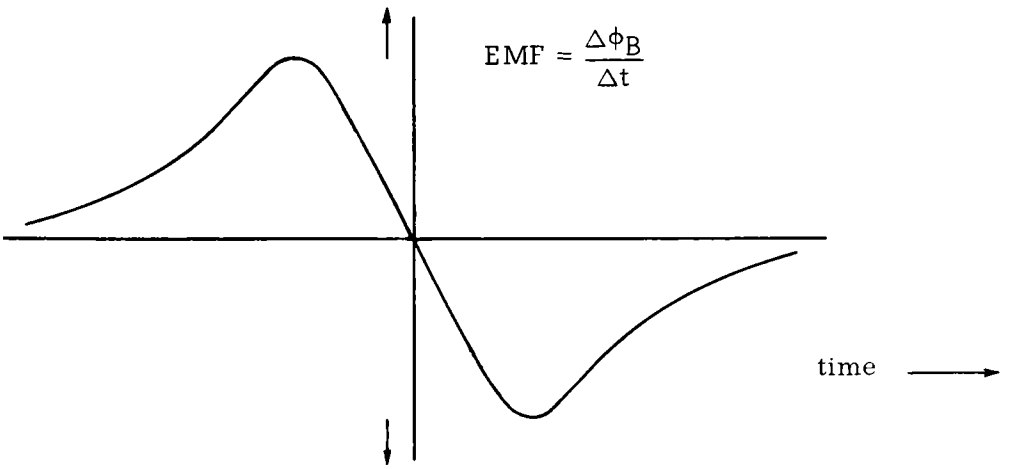
(b) Neglecting air resistance, will the acceleration of the falling magnet be constant?



(a) As the magnet nears the loop, the magnetic field inside the loop due to the magnet increases in strength. Hence, there is a change in flux within the loop and we know a current is induced. By Lenz's law, the induced current must be in the direction which will oppose the flux change inducing it. Hence, it must decrease the strength of the magnetic field within the loop; that is, it must itself produce a magnetic field which points *upward* inside the loop. Curling our fingers upward inside the loop, we see that our thumb points counterclockwise around the loop. This is the direction of a current which would oppose the flux change.

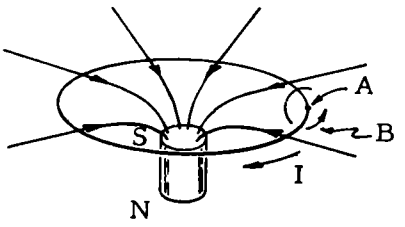
To find the magnitude of the EMF generated in the loop by the change of flux from





the moving magnet, we need to find the rate of change of flux $\Delta\phi_B/\Delta t$. Let us therefore sketch the flux ϕ_B from the magnet through the loop as a function of the position of the magnet (preceding page). The flux is zero when the magnet is far above the loop, rises to a maximum with the magnet in the center of the loop, and then decreases to zero again when the magnet is far beyond the loop.

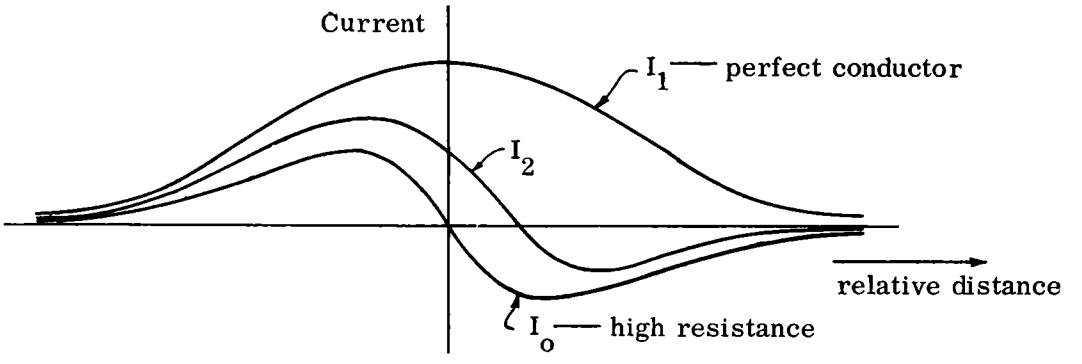
If the magnet falls at more or less steady speed the rate of change of this flux then looks as in the drawing. This is the EMF induced in the loop because of the change of flux from the magnet. Note that it is counterclockwise as seen from above as the magnet approaches the loop. It is zero when the magnet is falling through the center of the loop because there is then no rate of flux change. It is clockwise as the magnet falls beneath the loop because the downward flux is then decreasing instead of increasing. This reversal of direction can also be understood by looking at the figure below. In the frame of reference where the magnet stands still the loop is rising through fields which are opposite in direction from before.



The foregoing discussion has concentrated on the induced EMF. If the loop has a rather large resistance, the current will follow the variation of the EMF, as does the current I_0 below. This is likely to be the answer given by the students. On the other hand, if the wire loop is a nearly perfect conductor, instead of following the EMF induced by the falling magnet, the current will rise to a maximum and will decrease to zero only when the magnet falls through. The current fails to follow the EMF generated by the falling magnet because the induced current itself makes a magnetic field through the loop and its changes tend to maintain the current in the loop. For a high-resistance loop the current is small and its magnetic field can be neglected compared to that of the falling magnet. Therefore the total induced current follows the flux ϕ_B rather than the EMF $\Delta\phi_B/\Delta t$ from the falling magnet. This behavior is shown approximately by the curve I_1. For an imperfect conductor the current will be intermediate between the two cases, as shown for I_2.

(b) No. The induced EMF causes a current which in turn causes a magnetic field which in its turn causes a force on the magnet. The magnet therefore falls with a non-uniform acceleration.

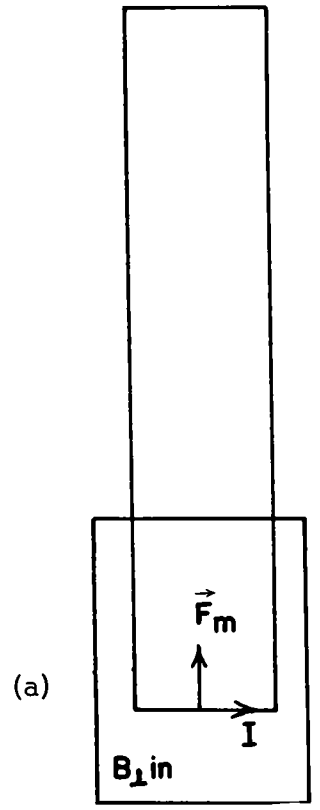
If the loop is highly resistive (the I_0 case), energy is continually being turned into heat in the loop. This energy comes from the kinetic energy of the magnet. The force is always a retarding force. If the loop has no resistance, no energy can be dissipated. The initial force is retarding, but the loop repels the magnet after it has passed through and speeds it up. These statements can be qualitatively verified by a direct application of the laws on direction of forces for the two current situations I_0 and I_1.

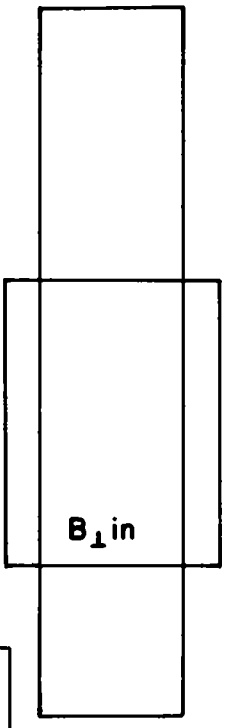


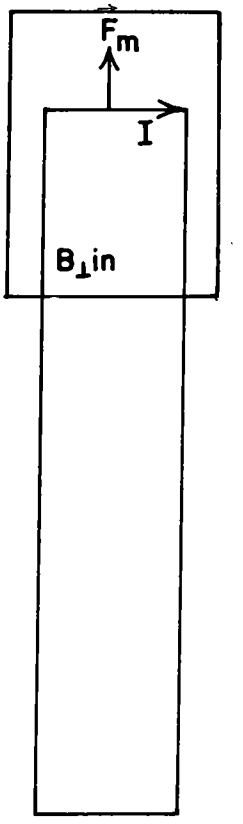
PROBLEM 22

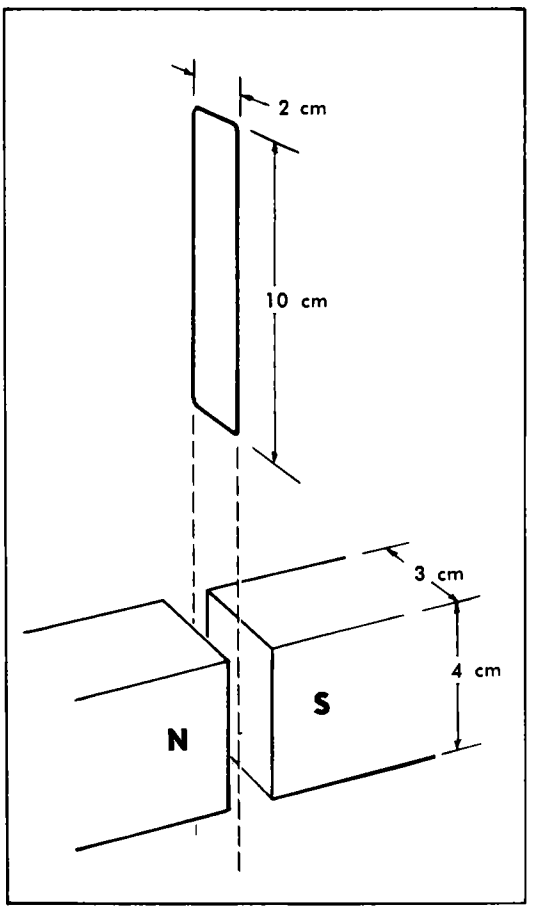
A rectangular loop of copper wire is dropped between the poles of a magnet as shown in Fig. 31–26. Describe qualitatively the motion of the loop. Neglect air friction.

While the loop is above the magnetic field, it will fall freely with constant acceleration.









When the lower side of the loop enters the magnetic field, and as long as it stays in the field, the magnetic flux through the loop will be increasing. This changing flux in the loop induces an EMF which causes a current as shown in diagram (a). The magnetic force $\vec{F}_m$ on the lower side of the loop decreases the acceleration of the loop.

After the bottom side of the loop leaves the field the loop will fall freely with the acceleration of gravity until the top side of the loop enters the field. During this time the flux in the loop is constant as can be seen in diagram (b).

As the top side of the loop enters the magnetic field the magnetic flux in the loop begins to decrease. This induces an EMF in the loop which causes a current as shown in diagram (c). The magnetic force, $\vec{F}_m$, on the current again opposes the motion and decreases the acceleration of the loop.

As soon as the loop is entirely out of the magnetic field, it falls freely once more with constant acceleration.

Further Discussion

Since the upward force $\vec{F}_m$ is proportional to the instantaneous speed of the loop, it may be more or less than the gravitational force on the loop. If more, the loop will slow down until the two forces are equal, then the speed will remain constant. If this force is less it will decrease the acceleration, but the velocity will continue to increase towards that constant speed for which the two forces are equal. This speed will probably not be reached in the short distances of this experiment.

PROBLEM 23* (See Short Answers.)

PROBLEM 24

In a betatron electrons are accelerated to very high energies by a changing magnetic flux. The electrons move in a circle around the inside of an evacuated glass "doughnut" and speed up when the flux through the hole in the doughnut increases. A separate magnetic field passes through the doughnut perpendicular to its plane to keep the electrons moving in a circle.

An electron is traveling in a circle of 50 cm radius at 98 percent of the speed of light in the doughnut of a betatron. If the flux inside the doughnut increases at a rate of 24 newton-meters/amp-sec for $\frac{1}{240}$ second, how much energy (in electron volts) will be transferred to the electron? (*Note:* Assume that the speed stays constant at approximately the speed of light.)

This problem involving induced EMF can be used to emphasize the electric field produced by a changing magnetic flux. It also contains some interesting facts about modern nuclear physics accelerators.

The energy gained by the electron will be equal to the work done on it. This work can be calculated from Fd, where F is the electric force and d is the total distance traveled. The electric force can be calculated from the induced EMF:

$$\mathcal{E} = \frac{\Delta\phi_B}{\Delta t} = \frac{24 \text{ nt-m}}{\text{amp-sec}} = 24 \text{ joule/coul} = 24 \text{ volts.}$$

The electric field, E, is found from the fact that this EMF corresponds to the effect of an electric field E acting through the distance of $2\pi r$. (See page 580.)

$$\mathcal{E} = 2\pi r E$$
$$E = \mathcal{E}/2\pi r.$$

The distance the electron travels while it experiences this field is $d = vt$. The equivalent voltage is

$$V = Ed = \mathcal{E}/2\pi r(vt)$$
$$= \frac{(24 \text{ volts})(3.0 \times 10^8 \text{ m/sec})(1/240 \text{ sec})}{2(3.14)(0.50 \text{ meter})}$$
$$= 9.6 \times 10^6 \text{ volts.}$$

The electron has one elementary charge, so it gains an energy of 9.6×10^6 electron volts. In nuclear physics, one would use the abbreviation Mev for 10^6 electron volts; thus, the problem is about a 10-Mev betatron.

Alternate Solution

One can arrive at the same expression for the energy by realizing that in each revolution the electron is accelerated through a potential difference $\mathcal{E} = 24$ volts, so the total energy it gains must equal 24 electron volts multiplied by the total number of revolutions. In each revolution the electron goes a distance $2\pi r$ and altogether it goes a distance vt, so it must make $vt/2\pi r$ revolutions. Hence the energy gained is

$$\mathcal{E}(1 \text{ elem.ch.})vt/2\pi r.$$

Note on Relativity. Relativity will not be discussed in any detail athough it will be mentioned briefly (on pages 623–4). But some of your students may wonder why the speed of the electron is taken as 3×10^8 m/sec despite its increase in energy. For example, some students may try to find the "actual" final speed of a 10-Mev electron by using the formula $E_k = \frac{1}{2}mv^2$. If they did this they would get:

$$v = \left(\frac{2E_k}{m}\right)^{1/2} = \sqrt{\frac{2(1.6 \times 10^{-12} \text{ joule})}{9 \times 10^{-31} \text{ kg}}}$$
$$= 2 \times 10^9 \text{ m/sec,}$$

which is more than six times the speed of light. The difficulty is that the kinetic energy is not $\frac{1}{2}mv^2$ when an object approaches the speed of light, c. The correct formula is

$$E_k = mc^2\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right];$$

this reduces to $\frac{1}{2}mv^2$ only if v is much less than c.

PROBLEM 25* (See Short Answers.)

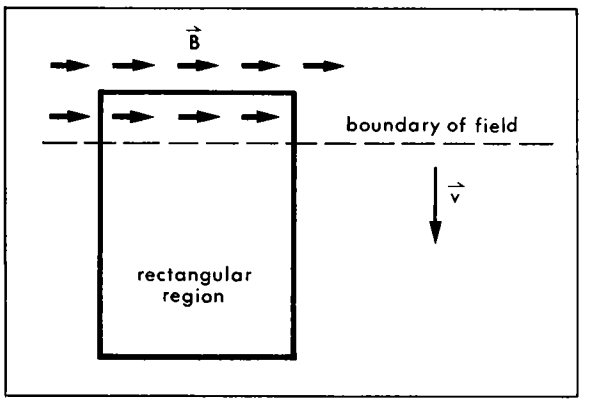
PROBLEM 26* (See Short Answers.)

PROBLEM 27* (See Short Answers.)

PROBLEM 28

Suppose that at the moment there is no electric field where you are, but a short time later there will be an electric field pointing vertically down. The boundary is moving straight toward you. When it has reached you, what will be the direction of the magnetic field at your position?

The following argument parallels the argument given in the text for the moving boundary of magnetic field. Consider a rectangular region of space which is perpendicular to the direction of the electric field and which is almost entirely in front of the boundary; one edge is just behind the boundary and parallel to it. Since the electric flux through this region is changing, there must be a magnetic circulation around the edge of the region. There is no field at all in front of the boundary. The only place there could be a magnetic field is along the edge just behind the boundary. Since the electric field points down and the area through which it passes is increasing, there is an equivalent current directed downward. Pointing our thumb down, we see that behind the boundary our fingers point to the right, so the magnetic field just behind the boundary is to our right as the boundary moves toward us.



We can now extend our region back a little further behind the boundary. The diagram shows what we would see if we looked down on the region. The electric field is down into the page. The magnetic field along the side edges is zero in front of the boundary and is perpendicular to the edges just inside the boundary. Hence, there is no contribution to the magnetic circulation from the side edges. The only place a magnetic field could contribute to the circulation is along the back edge. Again, it must point to the right. Extending this argument further, we see that everywhere behind the moving boundary there is a magnetic field to the right.

PROBLEM 29* (See Short Answers.)

PROBLEM 30

When electrons moving at 3×10^7 m/sec hit the target in an X-ray tube, they are brought to rest in about an atomic diameter — in about 10^{-10} m.

(a) What is the stopping time?

(b) If you use one divided by this time to estimate the frequencies you might expect to see in the electromagnetic radiation arising from this deceleration, what frequency do you get?

(c) What wavelength does this frequency give?

This problem emphasizes the relation between acceleration and electromagnetic radiation. It also gives typical values of λ and frequency for low-energy X rays.

(a) To estimate the time, assume a constant deceleration. First, find v_{average}:

$$v_{\text{average}} = \frac{v_{\text{initial}} + 0}{2} = \frac{3 \times 10^7 \text{ m/sec}}{2}$$

$$= 1.5 \times 10^7 \text{ m/sec.}$$

$$t_{\text{stop}} = \frac{d}{v_{\text{av}}} = \frac{10^{-10} \text{ m}}{1.5 \times 10^7 \text{ m/sec}}$$

$$= \underline{6.7 \times 10^{-18} \text{ sec.}}$$

(b) An estimate of the frequency, f, as $1/t_{\text{stop}}$ is:

$$f = \frac{1}{6.7 \times 10^{-18} \text{ sec}} = \underline{1.5 \times 10^{17} \text{ sec}^{-1}}.$$

(c) This frequency corresponds to a wavelength:

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/sec}}{1.5 \times 10^{17} \text{ sec}}$$

$$= \underline{2.0 \times 10^{-9} \text{ meter}}.$$

Note: In the calculation above, the frequency that we estimate depends upon 1/time of stopping. Other effects may enter and determine the highest frequency seen, as we shall find out in Chapter 33. The highest frequency depends, according to quantum theory, on the energy of individual electrons. If that energy is low, no light of high frequency will be seen even if the stopping time is very short.

PROBLEM 31

What evidence is there that sound is not electromagnetic radiation?

This easy problem can be used to review the properties of electromagnetic waves. A discussion of all the reasons for believing that sound is not electromagnetic radiation could go on interminably. A few are listed below.

(1) Sound does not travel through empty space.
(2) Sound does not travel with the speed of electromagnetic radiation.
(3) Sound is not produced by accelerating charges. It can be produced by means which are clearly nonelectric.
(4) Electromagnetic radiation of the same wavelengths as sound is well identified (ham band to radar wavelengths) and is different from sound.
(5) Sound is often primarily a longitudinal wave which cannot be polarized (while polarization is not discussed in the text, it will still have meaning to most students).

chapter
32

Exploring the Atom

Starting with the classical laws of physics (Newton's law and Coulomb's law) Rutherford attempted to explore the structure of individual atoms. The attempt was both a success and a failure. Rutherford's experiments led to a "planetary model" of the atom, in which a massive positive nucleus is encircled by one or more electrons which are held to the nucleus by the Coulomb force, much as planets are held by the gravitational attraction of the sun. This model gave an understanding not only of the results of the Rutherford scattering experiments, but also, at least qualitatively, of some of the other properties of atoms. The attempt was a "failure" only in that it led to a model which demonstrated rather conclusively the inability of classical physics to account for phenomena on the atomic scale; according to classical physics such an atom as Rutherford visualized would be unstable and should collapse in an extremely short time by emission of electromagnetic radiation.

CHAPTER SUMMARY

INTRODUCTION. The introduction briefly reviews what students should already know about the constituents of atoms. Then, as background for understanding the general design of the Rutherford experiment, the introduction discusses a simple, analogous procedure: probing a bale of hay with rifle bullets, determining the location and shape of "hidden" objects from a study of the entering and leaving paths of the bullets.

SECTION 1. If alpha particles are shot into a thin gold foil about 400 atoms thick, most of them pass through without being deflected. This indicates that most of an atom is empty space (or filled with a tenuous material which does not impede the alpha particles). Occasionally an alpha particle is deflected through a very large angle indicating that there is some concentration of mass in the atom which can deflect the alpha particle. This evidence led Rutherford to devise a planetary model of the atom.

SECTIONS 2, 3, AND 4. Rutherford's planetary model not only predicted qualitatively the results of the alpha particle scattering experiments, but predicted exactly the trajectory to be followed by a particular alpha particle which would miss the heavy core or nucleus by a distance, b. Averaging over all possible b's, since atoms are too small to be aimed at, gives the expected scattering of the alpha particles. Experiment and theory agree and, in fact, give us both the number of charges on each kind of atom and our first ideas of the size of the atomic nucleus.

SECTION 5. The Rutherford model raised more questions than it answered. Most of these questions centered on the fact that, according to classical electromagnetic theory, an atom should radiate away its energy as light and quickly collapse. Atoms do not collapse. The light radiated does not have the expected frequencies. At this point, classical theory is powerless. Enter the quantum theory in Chapters 33 and 34.

DEVELOPMENT. Most of the material in this chapter can be understood in a qualitative way by any interested person, with very little previous knowledge of physics. It is important that the obvious qualitative features not be obscured by an excessively detailed treatment. On the other hand many students will want to try very hard to apply all that they have learned about mechanics and electricity. For each class a balance must be found between the extreme of a purely qualitative treatment and the other extreme of getting lost in quantitative detail. How you handle this problem depends on the ability of your class and the time you have to spend on this chapter.

SCHEDULING CHAPTER 32

The following table suggests possible schedules for this chapter consistent with the schedules outlined in the volume summary section. There are no experiments specifically designed to go with this chapter.

Chapter 32	10-*week schedule for Part IV*	15-*week schedule for Part IV*
SECTION	CLASS PERIODS	CLASS PERIODS
Intro., 1	1	2
2	1	2
3, 4	2	2
5	1	2

RELATED MATERIALS FOR CHAPTER 32

LABORATORY. There are no experiments directly related to this chapter.

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILMS. "Rutherford Atom," by Robert I. Hulsizer of the University of Illinois, is a performance of the classical Rutherford Scattering Experiment of alpha particles by a thin gold foil. The experiment and its results are thoroughly discussed so that this film can well bear the major burden of developing this chapter. Running time: 40 minutes.

DEMONSTRATIONS — CAUTION. With radioactive alpha sources now readily available from several places, it may be tempting to think of demonstrating alpha-particle scattering. *Do not try it* unless you are highly expert and are very well equipped. Any source strong enough to do this experiment would be extremely dangerous. Alpha sources are dangerous because the alpha particles cannot penetrate the wall of a Geiger-counter and are therefore not readily detected if spilled. Often these sources "creep" and get spread around their containers — at least if packaged by an amateur. Confine yourself to observing the scintillations from very weak sources: the PSSC spinthariscope or radioactive watch dials.

INTRODUCTION TO THE CHAPTER

The introduction to this chapter contains more than the ordinary amount of material. It should be discussed in class.

DEVELOPMENT. Before starting these last three chapters you may wish to have the class discuss what they already have learned (from this course) about the structure of matter. If you have had time to emphasize the atomic nature of electricity they should already know quite a bit:

From Parts I and III

(a) All matter is composed of a relatively few elements. 7–7.

(b) Light emitted by each element has a characteristic spectrum. 7–8.

(c) Matter is composed of atoms. 8–1.

(d) We know the approximate size of atoms. 8–2, 3, 4; Experiment I–6.

(e) We know the number of atoms in a mole and hence the mass of an atom. 8–7, 10.

(f) We can guess at the structure of crystals. 8–14.

(g) We have a pretty model of a gas. Chapters 9 and 25.

From Part IV

(a) Matter contains positive and negative electric charge. 26–2.

(b) The negative and positive charges can be separated, e.g., 26–9.

(c) Electric charge comes in multiples of a natural unit. 27–5.

(d) Electrons, found in all materials (26–12), have one negative elementary charge. 27–9.

(e) We know the mass of individual electrons and protons. 28–1.

(f) Newtonian mechanics applies to some motions of atomic particles. 28–1. (In Chapters 33 and 34 we shall discuss some of the limitations.)

(g) Accelerated charges emit electromagnetic radiation and so do atoms — of about the "right" frequencies. 31–9.

Although "lists" are often to be discouraged, such a list as described above might be a good preliminary assignment as a base for a class discussion which is introductory to this and the final two chapters.

* * *

The analogy between the search for contraband in bales of hay and the study of the structure of atoms is simple and should require little discussion. You may want to consider the suggestion above. You may want to ask your class to read the introductory section and to do Problem 1 before you take up the chapter in class. Problem 1 illustrates two important points. We can get quantitative information about the structure of the atom from scattering experiments. They

may not, by themselves, lead to a unique conclusion about the structure, but can certainly be used to test the validity of an assumed model.

In a very real sense a scattering experiment is just a modified version of what we do when we look at an ordinary object in ordinary light. When we look at an object, we see it and thereby learn its size and shape by observing, with our eye, the light which the object reflects, scatters, or intercepts. In very much the same way, we "look" at atoms by observing their effect on a beam of alpha particles, electrons, protons, neutrons, or of electromagnetic radiation of suitable frequencies.

32-1 The Deflection of Alpha Particles and the Rutherford Model of Atoms

CONTENT. In 1911 Rutherford proposed an atomic model. According to this model an atom is made up of a small, positively-charged nucleus, containing almost all of the mass of the atom, and of a number of electrons circling about the nucleus similar to the planets circling the sun. Rutherford saw that the the validity of his ideas could be tested by observing the deflections of fast-moving charged particles as they traversed thin foils of various substances.

EMPHASIS. This section needs thorough discussion because it presents in a qualitative manner most of the content of the chapter.

DEVELOPMENT. The planetary model proposed by Rutherford should not be discussed in detail at this point. We shall come back to it in Chapter 34. For the present we need only the qualitative picture of a massive, positively-charged nucleus surrounded by light, negatively-charged electrons. This is sufficient to discuss the scattering experiments. Be sure students realize that in Rutherford's time it was already known (as they know) that the negative particles of atoms were all of the same kind and were the light particles in atoms — that the positive parts were heavy.

You should also be sure they understand that a collision of an alpha particle with an electron cannot deflect the alpha particle appreciably since the mass of an alpha particle is about 7400 times greater than the mass of an electron. This is something like a cannon ball hitting a ball bearing. To be deflected appreciably the alpha particle must collide with something having at least a comparable mass.

Students may wonder what happens if an alpha particle passes outside a neutral atom. Unless the alpha particle passes inside the electron orbits, no appreciable deflection is expected. Since the atom as a whole is neutral, there is no field outside. It is only when the alpha particle gets inside the electron orbits that it is acted on by an electric force.

In trying to learn about the structure of atoms it is useful to study the dynamics of a collision between an alpha particle and a single atom. It would be difficult to draw simple conclusions if the observed deflections were due to the cumulative effect of many collisions. For this reason very thin foils are used in scattering experiments. Whether the foils are thin enough to make multiple scattering unimportant can be checked by observing how the fraction of alpha particles scattered through a given angle depends on the foil thickness. It will increase in direct proportion to the thickness only if the deflections are due to collisions with single atoms.

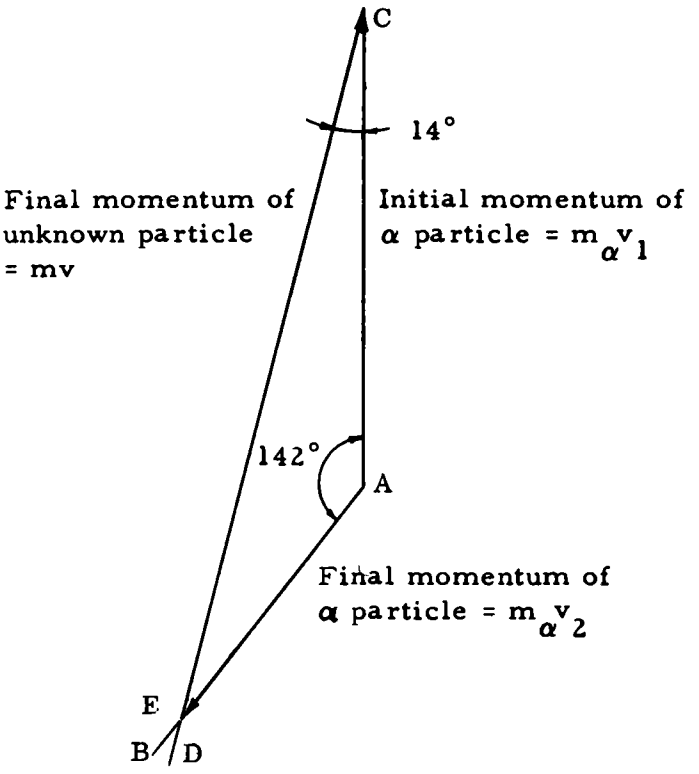
The fact that the majority of alpha particles can traverse a foil many atoms thick without deflection disposes of the idea that atoms are solid chunks of matter, but does not, by itself, tell us that an atom is mostly empty space. The mass of the atom could be distributed uniformly over its entire volume like a thin jelly. However, with such a model we could not explain the observation that in a few cases, very large deflections occur. It is only from a thorough study of the angular distribution of deflected alpha particles, as discussed in the following sections, that we can draw sound conclusions about the structure of the atoms.

A careful analysis of the collision in Fig. 32–3 can lead to an excellent review of mechanics. The analysis can serve to remind students that the mechanics they have studied is still applicable in the fields of atomic and nuclear physics.

How can we tell that the event shown is, in fact, an elastic collision between an alpha particle and a nitrogen atom? Presumably we know the identity of the incident alpha particle but wish to identify the particle that was struck. We can do this by making use of the conservation laws as follows: If we measure the angles between the three tracks* we can apply conservation of momentum to calculate the final velocities of the alpha particle and of the unknown particle in terms of the initial alpha particle velocity. We can then write down the equation which expressed the conservation of energy and solve it for the unknown mass.

In the figure following we have drawn the triangle representing the configuration of the three tracks of the photograph. The lengths of the three sides of the triangle must be proportional to the magnitudes of the corresponding momenta. We know the original direction of the momenta of the alpha particle ($m_\alpha v_1$) by its incoming track. We draw such an arbitrary vector as shown. All other momenta and velocities will be obtained in terms of this arbitrary one. We know the final direction of the alpha particle so we lay off a line AB at 142° to the original momentum; we do not yet know the length. We lay

* This would really require a steroscopic picture, i.e., two views from different points, but we can assume that the collision is in the plane of the photograph. It nearly is.



off the final momentum of the unknown particle along a 14° line CD as shown. AB and CD intersect at E and the momentum triangle is now defined since the initial momentum must equal the sum of the final two. Measuring the scale drawing we find

$$\frac{m_\alpha v_2}{m_\alpha v_1} = 0.60;\ v_2 = 0.60\, v_1$$
$$\frac{mv}{m_\alpha v_1} = 1.51;\ v = 1.51\, \frac{m_\alpha v_1}{m}.$$

By the law of conservation of energy, we know:

$$\tfrac{1}{2} m_\alpha {v_1}^2 = \tfrac{1}{2} m_\alpha {v_2}^2 + \tfrac{1}{2} m v^2,$$

or, substituting the expressions just obtained for the velocities,

$$\tfrac{1}{2} m_\alpha {v_1}^2 = \tfrac{1}{2} m_\alpha (0.60^2 {v_1}^2) + \tfrac{1}{2} m\, 1.51^2\, \frac{{m_\alpha}^2 {v_1}^2}{m^2}.$$

Cancelling $\frac{1}{2} m_\alpha {v_1}^2$ we have,

$$1 = 0.36 + 2.28\, \frac{m_\alpha}{m};\ m = \frac{2.28}{0.64}\, m_\alpha = 3.56\, m_\alpha.$$

Since the mass of the alpha particle is 4 atomic mass units, $m = 14.2$, i.e., nearly 14. The unknown atom is nitrogen.

Since the measurement of angles is difficult, no great accuracy should be expected from the students — 12–16 is fine.

32–2 The Trajectories of Alpha Particles in the Electric Field of a Nucleus

PURPOSE. To discuss the *orbits* of alpha particles scattered by the nucleus according to the Rutherford atomic model.

CONTENT. From Newton's law of motion and Coulomb's law, it is possible to calculate the possible paths of a charged particle in the electric field of another charged particle. The deflection angle θ depends on the "aiming error" b.

DEVELOPMENT. This section is qualitative and easy to grasp, but quite important. It should be clear to your class that if one knows the force law, the orbits can be calculated by a straightforward application of Newton's laws of motion. It turns out that if the force is given by Coulomb's law one can calculate the orbits, but the mathematics that is required is somewhat more advanced than the usual secondary school level. It should be clear, however, that in principle one could always calculate the orbits by applying Newton's law repeatedly, on successive small portions of the orbit, using the correct force in each interval to calculate the change in velocity produced by the force. It may be helpful and interesting, if you have the time, to go through an approximate calculation of the small deflections which go with large aiming errors. Such a calculation is given in Appendix 4 at the back of this volume of the Guide.

Before you introduce the potential energy arguments given in the text it might be helpful to question the class about the nature of the path of an alpha particle past a heavy nucleus. Start with a head-on case. Students should see that the alpha particle will simply slow up, stop, and retrace its path. Next take a path with such a large aiming error that the force is always negligible. Such a path will be a straight line. Now take an intermediate case. The direction of the force alone will show the students that the particle is deflected to the side.

Students may wonder why the atomic nucleus does not move when struck. It does, of course. In fact its change in momentum, $m\,\Delta\vec{v}$, is as great as for the alpha particle. However, since the gold nucleus has a mass much greater than that of an alpha particle, its change in velocity is correspondingly less. We may usually neglect its motion. However, if the struck nucleus is light, we must consider its recoil. See Fig. 32–3.

The mechanical model shown in Figs. 32–6 and 32–7 should be very helpful in providing an understanding of the dependence of the scattering angle θ on the "aiming error" b. To avoid possible disappointments, however, it must be pointed out that it is a difficult task to build such a model with sufficiently high precision and with sufficiently low friction to yield anything but crude results. It is possible to play the game on the figures themselves.

If a crude model which is easy to make will suffice, stretch a rubber sheet over a frame and poke a piece of dowel rod or a pencil up in it to raise a hump. There will be a good deal of friction when a marble is rolled on such a model, and the force law it duplicates is $1/r$ rather than $1/r^2$, but it is easy to make and illustrates many of the features of alpha-particle scattering.

At this stage students should know:

(1) For an alpha particle of given energy — or marble on a hill — a particular scattering angle corresponds to each aiming error.

(2) Large b's, little scattering; small b's, large scattering.

32-3 Angular Distribution of Scattering

32-4 More Information from Scattering

PURPOSE. To describe the actual alpha-particle scattering experiments and show that they are quantitatively explained by Rutherford's planetary atomic model.

CONTENT. From Newton's law and Coulomb's law, Rutherford was able to calculate the probability that an alpha particle would be deflected through various angles in traversing a thin foil. The experiments of Geiger and Marsden were in full agreement with this calculation and provided strong evidence in favor of the atomic model proposed by Rutherford. These experiments also gave an upper limit to the size of the atomic nucleus. Similar experiments by Chadwick gave a measurement of the electric charge of several atomic nuclei.

DEVELOPMENT. We can calculate the orbits corresponding to different aiming errors. This provides a relationship between aiming error and scattering angle. The relationship depends on the assumed force law. To make the matter concrete, it should be helpful to carry out the computation for one or two simple cases. Problem 15 and Fig. 32–16 refer to the case of an abrupt force such as would arise in the elastic collision of two billiard balls or Dry Ice pucks. It will be worthwhile to discuss this simple case in detail.

The calculation for the more pertinent case of the Coulomb force is rather more complicated and cannot be taken up here (a brief derivation is given in Appendix 4 at the back of this volume of the Guide). The result itself is quite simple and you may wish to use it to make the subsequent discussion more concrete. The Rutherford result (see Fig. 32–11), relating the "aiming error" b to the scattering angle θ is $2\tan\frac{\theta}{2} = \frac{r_0}{b}$, with $r_0 = \frac{kQ_\alpha Q_N}{E_\alpha}$, where r_0 is the distance of closest approach for a head-on collision; r_0 is the distance at which the electric potential energy $\frac{kQ_\alpha Q_N}{r_0}$ equals the original kinetic energy E_α. For small angles of scattering θ, the relation is $\theta = \frac{kQ_\alpha Q_N}{bE_\alpha}$.

* * *

The relationship between "target area" and scattering probability is simple enough, but will probably require a thorough discussion. A simple example such as the following may help.

We have an array of identical targets distributed uniformly (or randomly) over a certain area, e.g., 15 targets per square meter. If we spray bullets uniformly (or at random) over the target area, what is the probability of hitting within a circle of radius 5 cm on any target? What is the probability of hitting within a circle of radius 2 cm on any target? In the first case, each target has an effective area of $\pi \times (0.05)^2 = 0.008\ \mathrm{m}^2$. There are 15 targets in a square meter so that the effective target area in one square meter is $15 \times 0.008 = 0.12\ \mathrm{m}^2$. The fraction of shots that will hit the targets (within the 5-cm circles) is just the ratio of the effective target area to the total area, or in our example:

$$\frac{0.12\ \mathrm{m}^2}{1\ \mathrm{m}^2} = 0.12.$$

If we fire 10,000 shots, we expect $0.12 \times 10{,}000 = 1200$ hits. If we wish to be more choosy and only count hits within the smaller circle of 2 cm radius, the number of counted hits will be smaller. In this case the effective area of each target is $\pi b^2 = \pi(0.02)^2 = 0.001\ \mathrm{m}^2$. We still have $N = 15$ targets in $1\ \mathrm{m}^2$, so that the total effective target area in $1\ \mathrm{m}^2$ is:

$$N_t \times (\pi b^2) = 0.015\ \mathrm{m}^2 \text{ per square meter.}$$

The probability that any one shot will hit a 2-cm radius target is in this case 0.015. Out of 10,000 shots we shall get (on the average) only 150 hits.

The situation is much the same in the scattering experiments. If we have N_a atoms per square meter in a foil, each with an effective target area of πb^2 meters, the probability of a hit within a distance b from the center will be $N_a \pi b^2$. We know that if we hit within a distance of b we shall get a deflection through an angle in excess of θ_b, related to the aiming error b. For instance, with a Coulomb force we have:

$$b = r_0 \Big/ \left(2\tan\frac{\theta}{2}\right),\quad r_0 = \frac{kQ_\alpha Q_N}{E_\alpha}.$$

The probability of a deflection of more than θ_b is then given by

$$N_a(\pi b^2) = \frac{N_a r_0^{\,2}}{\left(2\tan\frac{\theta}{2}\right)^2}.$$

Figure 32–15 in the text is a graph of this probability vs. θ.

* * *

The dependence of the probability of scattering of an alpha particle on its energy is contained in the above result. The probability is proportional to r_0^2 and therefore inversely proportional to the square of the kinetic energy of the alpha particle.

The dependence of the scattering on the energy is another test of the validity of the hypothesis that the scattering is due to a force which varies inversely as the square of the distance. The fact that this test was carried out reveals the thoroughness of the investigations of Rutherford and his co-workers.

* * *

It should be clear, qualitatively, that since the force exerted by a nucleus on an alpha particle depends on the nuclear charge, a scattering experiment can be used to determine this charge.

The scattering probability varies as r_0^2, and r_0, varies directly with the nuclear charge Q_N; the scattering probability therefore varies as the square of the nuclear charge.

Since an atom as a whole is electrically neutral, the number of elementary charges on a nucleus is equal to the number of electrons which surround the nucleus. Chadwick's measurements, therefore, give the number of electrons in each atom. This number turned out to be identical to the "atomic number," which had been previously used to classify elements on the basis of their chemical properties. This is closely related to the fact that the chemical properties of atoms are due to the arrangement of the atomic electrons.

* * *

Rutherford's alpha-particle scattering experiments gave only an upper limit to the size of nuclei, since no departures were observed from what is calculated under the assumption that the alpha particles do not penetrate the nucleus. It may be interesting to point out that later, scattering experiments (many of them going on today) particularly with high-energy electrons, lead to quite a detailed knowledge of the size of nuclei and about the distribution of charge within the nuclei. These experiments are even giving information about the size and charge distributions of the nucleons (neutrons and protons) themselves.

32-5 Troubles

PURPOSE. To prepare the way for the introduction of quantum mechanics in the final two chapters.

CONTENT. The Rutherford model explains many observed facts. There are as many or more facts which it cannot account for. Experimentally, atoms are stable, but according to classical electromagnetic theory, the electrons in Rutherford's atom should radiate energy as they circle around and, as they lose energy, they should spiral towards the center. As this process occurs, light with a continuous spectrum should be emitted. This also is not observed.

DEVELOPMENT. The main point of this section is that many facts about atoms remain unexplained, and in violent disagreement with the predictions of classical physics — that is, the physics of Newton, Coulomb, and Maxwell. An attempt to resolve these difficulties must, however, be postponed until we have gone through the systematic development of the next two chapters.

The previous chapters led to a kind of climax of classical electromagnetic theory — that accelerated charges give rise to electromagnetic radiation. Experiments with accelerating electrons and protons in gas discharge tubes seem to indicate that these atomic particles obey the laws of classical physics (28–1). If Rutherford's atomic model is accepted, the atomic electrons should, according to this same classical theory, radiate light at a truly enormous rate. An ordinary-sized light bulb (about 100 cm^3) filled with hydrogen should emit several hundred watts of light. According to the same calculation it would not take very long (about 10^{-11} second), before our "light bulb" would go out, each atom having collapsed to the size of its nucleus (about 10^{-15} meter). All of this obviously does not occur. Hydrogen atoms emit no light at ordinary temperatures, and atoms apparently have stable sizes of about 10^{-10} meter. A quite drastic revision of the whole structure of physics is required to resolve these difficulties; classical physics is adequate to explain what happens on a macroscopic scale, but apparently fails at the atomic scale.

QUIZ QUESTIONS AND PROBLEMS. Most of the following questions are too long for quizzes. You may find a portion of a question useful for class discussion, and use the remainder as a quiz.

(1) This is really a series of review questions relating to Section 32–2.

Bullets are fired into a "foil" 10^{-1} meter thick and 1 meter square. The "foil" is known to contain 10^5 "atoms."

(a) What is the approximate distance between the centers of the "atoms" in our "foil"?

(b) How many layers of atoms are there in the foil?

Out of 10^6 bullets fired, all but 10^4 suffered no deflection, while the remaining 10^2 bullets were scattered more or less uniformly in all directions.

(c) Were the observed deflections due to single collisions or multiple collisions? Why?

Answers:

(a) $v_{foil} = 10^{-1}\ m^3$. $v/atom = 10^{-6}\ m^3$. This is a cube of 10^{-2} meter on a side. The "atoms" must be about 10^{-2} meter apart.

(b) Since the foil is 10^{-1} m thick and each layer is

10^{-2} m, there are about 10^{-1} meter/$10^{-2}\left(\frac{\text{meter}}{\text{layer}}\right)$ or about 10 layers of atoms.

(c) The probability of a bullet making a single collision is no greater than $10^2/10^6 = 10^{-4}$. The probability of making 2 collisions cannot be greater than about $(10^{-4}) \times (10^{-4}) = 10^{-8}$ [$\frac{1}{2}$ of this really, but it doesn't matter]. Surely the observed deflections are mostly due to encounters with a single target.

(2) Information from scattering.

Bullets are sprayed uniformly over a target area of 2 m^2 in which we know there are 10^2 identical targets. Out of 10^5 bullets, only 500 are deflected. Some of these were scattered through about 180° and were to found to come out with a speed of 90 m/sec, the initial velocity being 100 m/sec.

(a) What is the effective area presented by each target?

(b) If the mass of a bullet is 10^{-3} kg, what can be said of the mass of the targets?

Answers:

(a) The effective area of all targets is the fraction $\frac{500}{100{,}000} = \frac{1}{2} \times 10^{-2}$ of the total area, or $\frac{1}{2} \times 10^{-2} \times 2\ m^2 = 10^{-2}\ m^2$. Each target has an effective area of $\frac{10^{-2}\ m^2}{\text{No. of targets}} = 10^{-4}\ m^2$.

(b) Since the bullets bounce back with nearly all of their kinetic energy, the targets must be rather massive compared to the bullets.

If a quantitative answer is required, the momentum change of the bullet is

$$mv_2 - mv_1 = 10^{-3}\ \text{kg}\left(-90\ \frac{\text{m}}{\text{sec}} - 100\ \frac{\text{m}}{\text{sec}}\right) = -0.19\ \text{kg-m/sec}.$$

The target must recoil forward with a momentum of 0.19 kg-m/sec.

The kinetic energy lost by the bullets was

$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_2^2 = \tfrac{1}{2} \times 10^{-3}\ \text{kg}\left(100^2\ \frac{\text{m}^2}{\text{sec}^2} - 90^2\ \frac{m^2}{\text{sec}^2}\right) = 0.95\ \text{joule}.$$

If the collision was elastic, an equal amount was gained by the target; if the collision was inelastic, the amount gained is less than 0.95 joule. Since $m_t v_t = 0.19$,

$$\frac{1}{2}\,m_t v_t^2 = \frac{1}{2}\,\frac{m_t^2 v_t^2}{m_t} = \frac{1}{2}\,\frac{(0.19)^2}{m_t} \leq 0.95.$$

Therefore $m_t \geq \frac{1}{2}\,\frac{(0.19)(0.19)}{0.95} = 0.019$ kg.

Unless we know that the collisions are elastic we can only conclude that the mass of the targets is at least 19 times the mass of the bullets.

(3) Nuclear billiard balls.

In an experiment, alpha particles traverse a cloud chamber full of helium gas (the nuclei of He atoms are alpha particles). Suppose that the alpha particles have speeds of 10^7 m/sec. Remember that the potential between two alpha particles is

$$U = k\,\frac{2 \times 2}{r} = \frac{9 \times 10^{-28}}{r}\ \text{joule}\ (r \text{ in meters}),$$

and that the mass of an alpha particle is 6×10^{-27} kg (in round numbers).

Suppose an alpha particle makes a head-on collision.

(a) What are the speeds of the alpha particle and the struck nucleus at the distance of closest approach?

(b) What is the total kinetic energy at this point?

(c) What is the distance of closest approach?

(d) What would be seen in the cloud chamber?

Answers:

(a) At the distance of closest approach the two speeds are the same. By conservation of momentum:

$$m_\alpha \times 10^7\ \text{m/sec} = m_\alpha v + m_\alpha v,$$
$$v = \tfrac{1}{2} \times 10^7\ \text{m/sec}.$$

(b) $E_k = 2(\frac{1}{2}\,m_\alpha v^2) = 6 \times 10^{-27}(\frac{1}{2} \times 10^7)^2 = 1.5 \times 10^{-13}$ joule.

(c) The total energy is

$$\tfrac{1}{2}\,m_\alpha v_0^2 = \tfrac{1}{2} \times 6 \times 10^{-27}(10^7)^2 = 3 \times 10^{-13}\ \text{joule}.$$

Therefore 1.5×10^{-13} joule $= U = \frac{9 \times 10^{-28}\ \text{joule}}{r}$, $r = 6 \times 10^{-15}$ meter.

(d) An undeflected alpha particle track.

(4) Scattering in a gas.

A gas at normal temperature and pressure contains about 3×10^{25} molecules per cubic meter. We may think of these molecules as little hard balls with a cross section (πR^2) of about $10^{-19}\ m^2$.

(a) How far apart are the molecules in the gas?

(b) How thick a layer of gas will have 10^{19} molecules per square meter?

(c) Roughly, how far does a molecule travel between collisions?

Answers:

(a) The volume per molecule is about $\frac{1\ m^3}{3 \times 10^{25}} \approx 30 \times 10^{-27}\ m^3$. This is a cube about 3×10^{-9} meter

on a side, a cube with faces of area $(3 \times 10^{-9}\ \text{m})^2 = 10^{-17}\ \text{m}^2$.

(b) A layer of gas 3×10^{-9} meter thick (!) would have an average of 1 molecule behind each $10^{-17}\ \text{m}^2$ of surface area. In order to have an average of 1 molecule behind each $10^{-19}\ \text{m}^2$ of surface we must increase the thickness of the layer a hundredfold to 3×10^{-7} meter.

(c) When we have a molecule behind each $10^{-19}\ \text{m}^2$ of surface, the target area ($10^{-19}\ \text{m}^2$/molecule) is equal to the surface area. If we shoot a bullet through such a layer, the average number of collisions will be one, since area of targets = total area. We conclude that molecules travel about 3×10^{-7} meter between collisions. In a more refined estimate we would include other effects (both bullets and targets have a finite size, both move), which would not change the order of magnitude of our answer.

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The table below classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1. See discussion on page 32–9.
2. About 800 sheets.
3. (a) 5×10^{-7} meter;
   (b) 3×10^{-10} meter;
   (c) 2×10^{3}.
4.* The large number of particles that come through the foil undeflected, compared with the small number that are deflected, tells us that the chance for one collision is very small; hence, the chance that one alpha particle will have two collisions is much much smaller.
5.* An atom must be mostly empty space, with something in it which can exert a strong force on an alpha particle.
6.* It is the angle between the original and final directions of motion of a scattered particle.
7. (a) 8 in 10^4;
   (b) See discussion on page 32–11.
8. (a) $\Delta v = 8 \times 10^5$ m/sec;
   (b) $\Delta t = 1.6 \times 10^{-17}$ sec;
   (c) $\vec{\mathbf{a}} = 5 \times 10^{22}$ m/sec^2 at right angles to the original velocity.
9.* We cannot measure b directly because we do not know where each nucleus is, nor do we know the exact trajectories of the incident alpha particles.
10.* N_θ is proportional to the square of the aiming error.
11.* It is a Coulomb field; that is, it varies as $1/r^2$.
12.* Through angles between 90° and 180°.
13. 160.
14. (a) 40 turned back; 120 scattered sideways;
    (b) and (c) See graphs on pages 32–12 and 32–13.
15. See discussion on page 32–13.
16. About 2 : 1.
17.* About 6 times as close.
18.* $r_0 = 7 \times 10^{-15}$ meter.
19.* The law of conservation of energy.
20. (a) Graph required;
    (b) Silver, gold.
21. (a) Gravitational force is very small compared to coulomb force.
    (b) 2.5×10^{35} for any distance.
22. (a) 3.6×10^{-14} meter;
    (b) Zero;
    (c) 1.8×10^{-14} meter.
23. The same for both beams.
24. (a) 1/1800;
    (b) 18.
25. (a) 6.6×10^{-20} kg-m/sec;
    (b) 2.0×10^5 m/sec;
    (c) 2% of the initial kinetic energy;
    (d) Yes; accurate to about 1%.
26. (a) 5.80×10^6 m/sec;
    (b) Zero.
27. (a) 10^{-7} newton;
    (b) Need 10^7 times as many batteries.
28.* By Kepler's Third Law, R^3/T^2 is constant, so the ratio of the frequencies is $\sqrt{8} = 2.8$.

COMMENTS AND SOLUTIONS

PROBLEM 1

Suppose you have a large collection of small steel balls which you can fire at different targets in turn. You cannot see the targets directly, but you can observe the paths of the balls before and after striking. In each case below, tell what you know about the target from the behavior of the balls. The balls are all fired at the target in the same direction.

(a) All the balls bounce off the target with angle of reflection equal to angle of incidence.

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1	4, 5	2	1, 3		1
2, 3	6, 9, 10, 11, 12	7, 13, 16	8, 14	15	
4, 5	17, 18, 19, 28	21, 23#, 27	20, 22, 25#	24, 26	20, 25#

(b) All the balls bounce back, but with no definite relation between angle of reflection and angle of incidence.

(c) Practically all the balls go through the target, but a few are lost in it.

(d) All the balls are lost in the target.

(e) The paths of all the balls cross after reflection at a point in front of the target.

This problem is intended as the basis for a class discussion to show how "scattering" of particles from an unseen target can be used to learn some of the properties of the target.

(a) A flat plate extending over the whole target area would give specular reflection of all "bullets," provided only that the bullets collide elastically with the plate (see Fig. 14–1, page 242 of the text). We cannot determine exactly the composition of the plate, or how thick it is. Perhaps we could learn more by firing bullets of different speeds and sizes; very fast bullets may not reflect specularly, very small bullets may perhaps go through.

Another possibility is that there are several plates in the target, all parallel, but lying in different planes. This could work for a particular direction of incidence but not for all directions; if the direction of incidence is varied, some bullets would get through, either directly or after several reflections.

(b) A rough plate extending over the whole target area would give rise to diffuse reflection (see Fig. 14–2, page 242 of the text). The plate must be made of a material that reflects the bullets elastically, and the irregularities must be both sufficiently random and of larger size than the bullets. (We can also think of more complicated arrangements to give diffuse reflection.)

(c) If most of the bullets go through without appreciable change in direction or speed, most of the target must be more or less empty. Since some bullets do not get through, a small fraction of the target area must be occupied by something that can stop, or destroy bullets. One or more chunks of something like mud or putty would do, provided the chunks are thick enough to stop our bullets. By counting the number of bullets that get through and the number of bullets that stop we can determine the total cross-sectional area of the "mud balls" but not their number or their shape. We could find the shapes of the mud balls if we were able to aim the bullets very carefully, something which cannot be done with atomic bullets fired at atomic targets. In our example the "mud balls" must somehow be tied down, or we should be able to see them recoil out of the target area (if they are visible).

(d) A wall sufficiently thick and soft to stop the bullets apparently extends over the whole target area. The experiment does not disclose whether the wall is made of mud, snow, molasses, or what have you. Nor do we learn anything about the size and shape of the wall.

(e) A curved "mirror," concave toward the direction from which the bullets arrive, could focus the bullets. In order to bring all particles in a beam to a focus, the mirror should be parabolic. As in part (a), the mirror must be made of a material that can reflect the bullets elastically. It should perhaps be said that this is one experiment we could not carry out with atomic bullets fired on atomic targets; in such an experiment we can only observe the changes in direction of the particles.

PROBLEM 2

To help visualize the thickness of the gold foil used in the alpha-particle deflection experiment described in Section 32–1, find the number of sheets of this foil which, when stacked together, are equal to one leaf of your textbook.

Two hundred pages, or 100 leaves, of the text are 0.8 cm thick. Hence, each leaf is 8×10^{-3} cm or 8×10^{-5} m thick. Since each sheet of gold foil is 1×10^{-7} m thick, we need a stack of 800 sheets of gold foil to equal the thickness of one leaf of the textbook.

PROBLEM 3

One square meter of silver foil has a mass of 5×10^{-3} kg.

(a) What is its thickness? (The density of silver is 1×10^{4} kg/m^3.)

(b) The mass of a silver atom is 1.8×10^{-25} kg. What is the length of the side of a "box" representing a single atom?

(c) What is the thickness of the foil expressed in atomic "boxes"?

(a) The volume of the foil equals its area times its thickness. Since the mass is the volume times the density, then

$$\begin{aligned} \text{mass} &= \text{volume} \times \text{density} \\ &= \text{area} \times \text{thickness} \times \text{density}, \end{aligned}$$

and

$$\begin{aligned} \text{thickness} &= \frac{\text{mass}}{\text{area} \times \text{density}} \\ &= \frac{5 \times 10^{-3}\ \text{kg}}{(1\ \text{m}^2)(1 \times 10^{4}\ \text{kg/m}^3)} \\ &= \underline{5 \times 10^{-7}\ \text{meter}}. \end{aligned}$$

(b) The volume of the "box" representing the

atom equals the mass of the atom divided by the density of silver. Hence, the length of one side is the cube root of this volume:

$$\text{length of side} = \left(\frac{\text{mass}}{\text{density}}\right)^{1/3} = \left(\frac{1.8 \times 10^{-25}\,\text{kg}}{1 \times 10^{4}\,\text{kg/m}^3}\right)^{1/3} = 3 \times 10^{-10}\,\text{meter}.$$

(c) The thickness of the foil in units of atomic "boxes" is

$$\frac{5 \times 10^{-7}\,\text{m}}{3 \times 10^{-10}\,\text{m}} = 2 \times 10^{3}.$$

PROBLEM 4* (See Short Answers.)

PROBLEM 5* (See Short Answers.)

PROBLEM 6* (See Short Answers.)

PROBLEM 7

In a scattering experiment using gold foil, 4 alpha particles in 10,000 are scattered through an angle of more than 5°.

(a) How many particles would be scattered through more than 5° if we doubled the thickness of the foil?

(b) If we made the foil a thousand times thicker?

This problem shows that the probability of scattering through any given angle depends only on the effective cross-sectional area of the target. In this case, since the nuclei all have the same radius, the scattering probability must depend only on the total number of nuclei per unit of cross-sectional area, hence directly on the thickness of the target.

(a) If the target thickness is doubled, the number of nuclei per unit of cross-sectional area is doubled, hence twice as many alpha particles would be scattered into the same angular interval, or 8 in 10^4.

(b) This part of the problem serves to review the reasons why very thin foils must be used in the alpha-particle scattering experiments. With the original foil, 4 alpha particles out of 10^4 were deflected more than 5°. With such a small probability of deflection the chances are overwhelming that the deflections are due to single collisions with a scattering angle of more than 5°. Therefore the average number of collisions with $\theta > 5°$ made by an alpha particle in traversing such a thin foil is 4×10^{-4}. What happens if we stack 10^3 such foils one after the other? There are several difficulties. The first difficulty is that the alpha particles would probably not get through the stack, but would lose their kinetic energy by collisions with the electrons in the foils and would come to a stop inside the stack. Suppose that this does not happen to be true for the foil thickness and alpha-particle energy used in this experiment, so that the particles do get through without much change in energy. We still have troubles.

In traversing 10^3 foils, an alpha particle will, on the average, make (4×10^{-4} collision per foil)(10^3 foils) = 0.4 collision with $\theta > 5°$. This does not mean, however, that 4 alpha particles out of 10 suffer deflections of more than 5°. The average number of collisions is sufficiently high that the probability of an alpha particle making more than one collision is not negligible. If some alpha particles make more than one collision, the fraction that makes at least one collision must be less than 4 out of 10 (the actual fraction is $1 - e^{-0.4} = 0.33$). Furthermore some of the particles that make more than one collision with $\theta > 5°$ end up with a net deflection of less than 5°. This would seem to reduce the fraction of particles deflected more than 5° to less than 4/10. There is, however, an effect which tends to raise the number of particles deflected more than 5°. A 5° deflection could result from a series of collisions all smaller than 5°. From a crude argument we can see that this effect must be quite important in our example: a particle could be deflected 5° by two 2.5° collisions. The probability of two such collisions is appreciable. For small angles N_θ is proportional to $1/\theta^2$; therefore the average number of collisions with $\theta > 2.5°$ is $4 \times 0.4 = 1.6$. If the average number of collisions is 1.6, the probability of two or more such collisions must be sizeable (the actual probability turns out to be $1 - e^{-1.6} - 1.6e^{-1.6} = 0.48$. It seems clear that in our experiment multiple collisions contribute as many, or even more, deflections as single collisions. This is the other reason why very thin foils are normally used. With thin foils multiple collisions are negligible.

PROBLEM 8

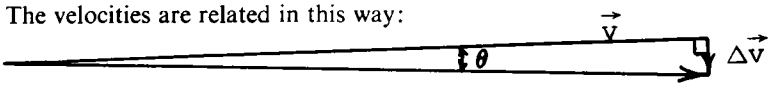
A beam of alpha particles from polonium is shot perpendicularly into the gold foil described in Section 1. We observe that some of the particles are scattered through an angle of about 3° or 5×10^{-2} radian. The alpha particles have a kinetic energy of 8.5×10^{-13} joule, which corresponds to a speed of 1.6×10^7 m/sec.

(a) What velocity $\Delta\vec{v}$ must be added at right angles to their original velocity to produce this change in direction?

(b) How long does it take an alpha particle to pass through a single gold atom in the foil?

(c) What is the average acceleration of one of these alpha particles if it gets its whole deflection in passing through a single gold atom?

(a) The velocities are related in this way:



Since θ is so small, its value in radians is just $\theta = \Delta v/v$, so

$$\Delta v = (1.6 \times 10^7 \text{ m/sec})(5 \times 10^{-2})$$
$$= \underline{8 \times 10^5 \text{ m/sec}}.$$

(b) From Section 32–1, we have a diameter of 2.5×10^{-10} meters for each gold atom. Then

$$\Delta t = \frac{2.5 \times 10^{-10} \text{ m}}{1.6 \times 10^7 \text{ m/sec}} = \underline{1.6 \times 10^{-17} \text{ sec}}.$$

(c)
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{8 \times 10^5 \text{ m/sec}}{1.6 \times 10^{-17} \text{ sec}}$$
$$= 5 \times 10^{22} \text{ m/sec}^2,$$

at right angles to the original velocity.

An acceleration of this magnitude requires a force extraordinarily large compared to forces we can exert on an alpha particle in the laboratory. In spite of the approximations made in this problem, we see that even to be scattered through small angles, the alpha particle must approach the charged nucleus very closely.

PROBLEM 9* (See Short Answers.)

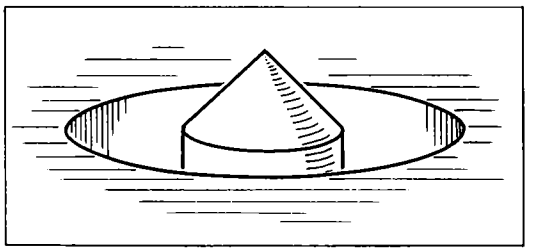
PROBLEM 10* (See Short Answers.)

PROBLEM 11* (See Short Answers.)

PROBLEM 12* (See Short Answers.)

PROBLEM 13

Imagine a device which can drop 1000 steel balls through a circular hole 5 cm in radius and spread them evenly over the area of the circle (Fig. 32–19). In the center of the hole we fix a steel cone whose sides make an angle of 45° with its base, and whose base has a radius of 2 cm. In a group of 1000 balls, how many are scattered sideways before they reach the hole?



As seen from above, the cone covers a circle whose radius is 2 cm. The steel balls are spread evenly over the whole area, and since the area of a circle is proportional to the square of its radius, the number of balls which strike the cone and are scattered before they reach the hole is

$$\frac{(2 \text{ cm})^2}{(5 \text{ cm})^2}(1000) = \underline{160}.$$

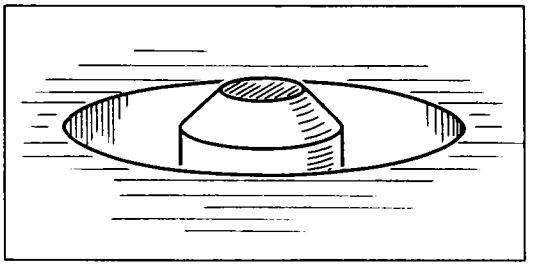
PROBLEM 14

The point of the cone in Problem 13 is sawed off to leave a smooth plane parallel to the base and 1 cm in radius (Fig. 32–20).

(a) In each group of 1000 balls, how many are turned back and how many are scattered sideways by it?

(b) Draw a graph of the scattering angle θ versus the aiming error b, measured from the center of the hole.

(c) Draw a graph of N_θ (the number of balls scattered through an angle greater than θ) versus θ.



(a) The balls which strike the central plane of the cone are turned back, and those which strike the sloping sides are scattered sideways. As seen from above, these cover a circle of 1-cm radius and a ring beyond that whose outer radius is 2 cm. Then,

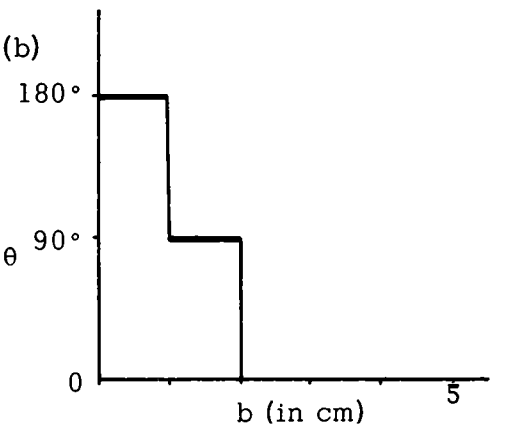
No. of balls turned back

$$= \frac{(1 \text{ cm})^2}{(5 \text{ cm})^2} \times 1000 = \underline{40}.$$

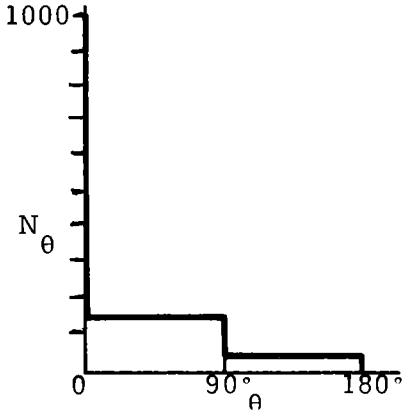
No. of balls turned sideways

$$= \frac{(2 \text{ cm})^2 - (1 \text{ cm})^2}{(5 \text{ cm})^2} \times 1000 = \underline{120}.$$

(b)



(c)



PROBLEM 15

(a) To construct the curve of b vs. θ for an abrupt force, draw a fairly large circle to represent a plane cut through a rigid sphere and measure the scattering angles for various aiming errors. (Assume that the angle of reflection of a ball hitting the surface equals the angle of incidence. Why?)

(b) From your curve of b vs. θ construct a graph of N_θ vs. θ and compare it with Fig. 32–16.

(c) Explain why this relationship is independent of the energy of the particles fired at the sphere.

(a) As the particle reaches the surface of the sphere it experiences a sudden change in momentum. We assume two things:

(1) The collision is elastic because the force is not only abrupt (i.e., acts almost instantaneously), but also is the same when the particle comes in and when it goes out;

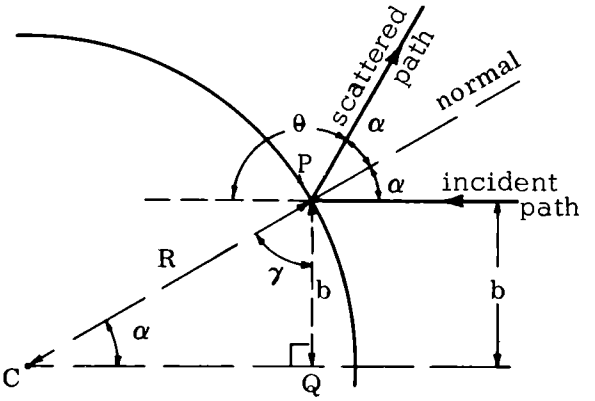
(2) The mass of the particle is very small compared to the mass of the target sphere.

Assumption 2 implies that the kinetic energy transferred to the target sphere in the collision is negligible. Assumption 1 then requires that the kinetic energy of the particle before and after collision be the same (nearly). Thus the *magnitude* of the momentum of the particle is the same before and after collision: $p_1 = p_2$.

Assumption 1 also requires that the force on the particle depend only on its position relative to the sphere. In particular, the *direction* of the force must depend only on the position, not on its velocity (no friction). By symmetry the force must be normal to the surface of the target sphere. It follows that there is no change in the tangential component of momentum of the particle during the collision: $\vec{p}_{t1} = \vec{p}_{t2}$.

We now draw a triangle to represent the momentum of the particle, and its normal and tangential components, before collision. We also draw the corresponding triangle after collision.

Since $p_1 = p_2$ and $p_{t1} = p_{t2}$, the two right triangles are equal (one is the mirror image of the other). Therefore $\alpha = \beta$.



We now draw a cross section through the sphere on the plane defined by the center of the sphere and the straight-line path of the incident particle:

C = center of sphere

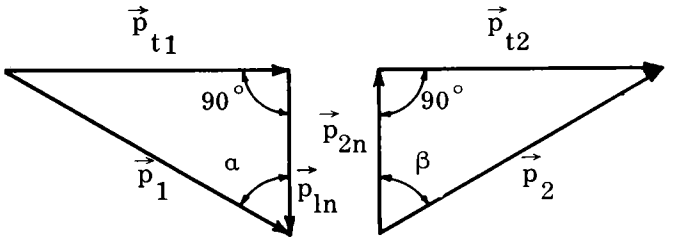
b = aiming error

θ = angle of scattering = $180° - 2\alpha$

$\theta/2 = 90° - \alpha = \gamma$.

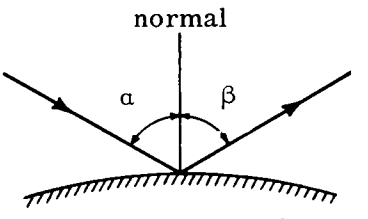
We can draw a number of paths for different values of the angle α and measure the corresponding aiming error. Alternatively we can look up b/R in a table of cosines. From the triangle CPQ we see that:

$$b/R = \cos\gamma = \cos(\theta/2).$$



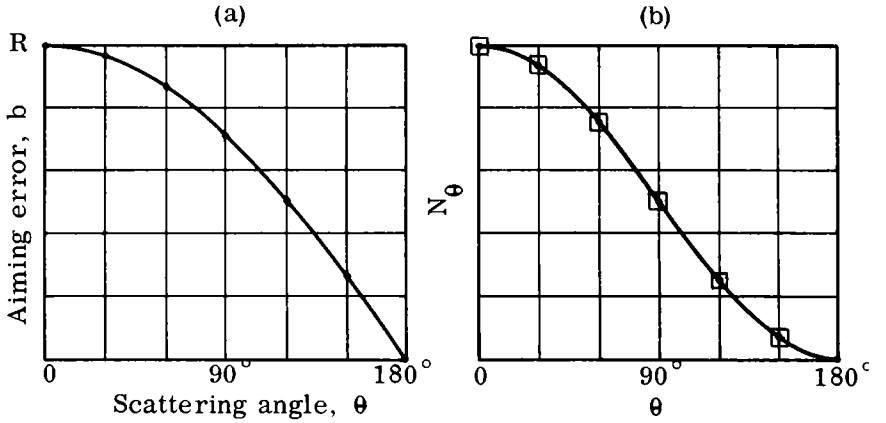
Before collision

After collision



Path of particle

θ	(a) $\frac{b}{R}$	(b) $(b/R)^2$
0	1.000	1.000
30°	0.966	0.933
60°	0.866	0.750
90°	0.707	0.500
120°	0.500	0.250
150°	0.259	0.067
180°	0.000	0.000



By either method we find the results shown in column (a) of the table above and plotted on the graph (a).

(b) With n targets per unit area in the beam, the probability of scattering through an angle greater than θ is given by $n\pi b^2$. The number of scatters, N_θ, through an angle greater than θ is therefore proportional to b^2 and hence to $(b/R)^2$. The calculated values of $(b/R)^2$ are shown in the preceding table, column (b), and were used to draw the graph (b) of N_θ vs. θ. Points read from Fig. 32–16 are shown on the same graph as well as in the table. They agree when multiplied by some constant so as to make them agree at one point, say $\theta = 0$.

(c) The result in (a) was obtained without mention of the energy. The scattering is independent of energy as long as the assumptions are valid. The original result is a very good approximation as long as the assumptions are valid.

PROBLEM 16

From the graph in Fig. 32–15, determine the ratio of the number of particles scattered through an angle greater than 15° to those scattered through an angle greater than 22.5°.

Because the graph is small, only an approximate answer is possible. The ratio of

$$\frac{N_{15^\circ}}{N_{22.5^\circ}} \approx \frac{1.2}{0.6} \approx 2.$$

PROBLEM 17* (See Short Answers.)

PROBLEM 18* (See Short Answers.)

PROBLEM 19* (See Short Answers.)

PROBLEM 20

Three experiments were done with the same alpha particle beam, the same counters at the same positions, and different foil targets. Five counters were arranged with respect to each target and the beam to count particles scattered by the angles given in the table. The time interval of counting was the same for all three experiments. The foils were designed so that the beam passed through the same number of atoms per square centimeter regardless of which target was used. The results presented in the table below give the numbers of particles scattered into the same range of scattering angles. For example, the count at 30° includes all particles scattered in any direction in the range between 29° and 31°; the count at 60°, all those scattered in any direction in the range between 59° and 61°, etc.

SCATTERING ANGLE	NUMBER COUNTED		
	Target 1	Target 2	Target 3
30°	2790	35,920	102,810
60°	346	4451	12,760
90°	100	1288	3685
120°	39	496	1423
150°	14	187	532

(a) Plot the number scattered versus the scattering angle for each target. (Plot all three graphs on the same axes.)

(b) Target 1 was aluminum. What were the other targets?

This problem shows the connection between the angular distribution of scattered particles and the atomic number of the scattering nucleus. Note that the experimental conditions presented in the problem (identical counters, identical geometry, identical numbers of atoms in the beam path, etc.) ensure that the observed differences can result only from differences in the nuclei in the target material.

(a) A plot of the data (which we omit) shows, qualitatively, the same angular dependence for each target. For a quantitative evaluation it is simpler to compare the counting rates numerically.

(b) The probability of scattering into a given range of angles should be proportional to the square of the nuclear charge (see Section 32–4). We calculate the ratio of the number of counts for targets 2 and 3 to the number of counts for target 1. The results are:

θ	Target 2/Target 1	Target 3/Target 1
30°	12.9	36.8
60°	12.9	36.9
90°	12.9	36.8
120°	12.7	36.5
150°	13.4	38.0
Average:	13.0	37.0

We note that the ratios are the same at all angles, as we would expect from theory. Since the ratios of counts should be equal to the square of the ratio of nuclear charges, we have:

$$\left(\frac{q_2}{q_1}\right)^2 = 13.0, \quad \frac{q_2}{q_1} = 3.60;$$
$$\left(\frac{q_3}{q_1}\right)^2 = 37.0, \quad \frac{q_3}{q_1} = 6.08.$$

Assuming the nuclear charge of aluminum to be 13 elementary charges, we have

$$q_2 = q_1(3.60) = (13)(3.60) = 46.8 \text{ elem. ch.}$$
$$q_3 = q_1(6.08) = (13)(6.08) = 79.0 \text{ elem. ch.}$$

Target 2 was probably silver (Z = 47) and target 3 was probably gold (Z = 79). The consistency of the data given is not quite good enough to guarantee that we are right, but we are certainly within one unit of atomic number. In fact, the data given are the results of experiments using aluminum, silver, and gold.

PROBLEM 21

(a) Explain why the gravitational force between an alpha particle and a gold nucleus has little effect on the scattering of alpha particles by gold.

(b) What is the ratio of the electric force on an alpha particle to the gravitational force when the separation of an alpha particle and a gold nucleus is 10^{-14} m? When it is 10^{-13} m?

This problem illustrates the enormous magnitude of electrical forces relative to gravitational forces.

(a) The gravitational force between an alpha particle and a nucleus is much smaller than the coulomb force at any separation distance. For a quantitative comparison, see part (b).

(b) The electrical force between two charged objects separated by a distance r, from Coulomb's law, is

$$F_c = \frac{kq_1q_2}{r^2},$$

while the gravitational force, from Newton's law (Chapter 21), is

$$F_g = \frac{Gm_1m_2}{r^2}.$$

The ratio F_c/F_g at *any* given distance is then

$$\frac{F_c}{F_g} = \frac{kq_1q_2}{Gm_1m_2}.$$

Substituting the correct values for gold and helium, we get

$$\frac{F_c}{F_g} \underline{(2.3 \times 10^{-28} \text{ nt-m}^2/(\text{elem.ch.})^2)} \times \underline{(79 \text{ elem.ch.})(2 \text{ elem.ch.})} \over (6.7 \times 10^{-11} \text{ nt-m}^2/\text{kg}^2) \times (3.3 \times 10^{-25} \text{ kg})(6.6 \times 10^{-27} \text{ kg})$$
$$= \underline{2.5 \times 10^{35}} \text{ for any distance.}$$

PROBLEM 22

An alpha particle of kinetic energy 1.0×10^{-12} joule is scattered by a gold nucleus.

(a) What is the minimum possible distance of closest approach?

(b) What must be the aiming error if the alpha particle is to get this close?

(c) How close could a proton of the same energy approach? A neutron?

In solving this problem, we follow the argument used in the text on page 606.

(a) and (b) The potential energy at the minimum separation distance, r_0, is

$$U = \frac{kq_1q_2}{r_0}.$$

If the collision is head-on (the aiming error is zero), the alpha particles will come to rest at a minimum distance r_0 such that $U(r_0)$ is equal to the initial kinetic energy of the system. Solving,

$$r_0 = \frac{kq_1q_2}{E_K} = \frac{(2.3 \times 10^{-28} \text{ nt-m}^2/(\text{elem.ch.})^2) \times (79 \text{ elem.ch.})(2 \text{ elem.ch.})}{1.0 \times 10^{-12} \text{ joule}}$$
$$= \underline{3.6 \times 10^{-14} \text{ meter}}.$$

(c) For the proton, $q_2 = 1$ elem. ch.; hence r_0 is half as large as in (a), or $\underline{1.8 \times 10^{-14} \text{ m}}$.

Since the neutron has no charge, it is not repelled by the Coulomb barrier and can get as close at it pleases to the nucleus for any energy. Actually, there are important short-

range nuclear forces which come into play for distances of the order of the nuclear radius. These forces also act on charged particles in the nucleus and those fired at the nucleus with enough energy to bring them through the Coulomb field into the region of the nucleus.

PROBLEM 23

In one scattering experiment a proton beam strikes a gold target. In a second experiment a beam of deuterons (atomic mass = 2, atomic number = 1) is used. The kinetic energy of the particles is the same in both beams. How will the minimum distance of closest approach to gold nuclei of the two kinds of beams compare?

This simple problem illustrates that the minimum distance of closest approach (for Coulomb scattering) depends only on the kinetic energy and *charge* of the bombarding particles and scattering nucleus. Following the arguments on page 606, the minimum distance of closest approach, r_o, is determined by the point at which the bombarding particle is brought to rest in a head-on collision. At that point the particle's potential energy is equal to its initial kinetic energy,

$$\frac{kq_1q_2}{r_o} = \text{original } E_K.$$

Since the two particles, proton and deuteron, have the same charge and kinetic energy, the minimum distance of closest approach must be the same in both cases.

As an extension of this problem, students might be asked to discuss the case in which the two particles have the same speed, rather than the same kinetic energy. In this case, the kinetic energy, $\frac{1}{2}mv^2$, of the deuteron (mass twice that of the proton) is twice the proton kinetic energy, and the minimum distance of closest approach for the deuteron will be half that for the proton.

PROBLEM 24

(a) What fraction of its energy is lost by an alpha particle in a head-on collision with an electron? (The mass of an alpha particle is about 7200 times that of an electron.)

(b) Approximately how many such collisions must occur before the energy of the alpha particle is reduced by 1 percent?

(a) We can find the speed, v'_e, of the electron and the speed, v'_α, of the alpha particle after the collision by applying conservation of energy and momentum. The formulas are given in Section 23–8; they are:

$$v'_e = \frac{2\,m_\alpha}{m_\alpha + m_e} v_\alpha; \quad v'_\alpha = \frac{m_\alpha - m_e}{m_\alpha + m_e} v_\alpha.$$

Since the mass, m_α, of the alpha particle is much greater than the mass, m_e, of the electron, the results show that the speed of the alpha particle is only slightly smaller after collision than before, as we would have expected. The speed acquired by the electron is given with sufficient accuracy by:

$$v'_e = 2\,v_\alpha.$$

This result is so simple that some students may want to find a simpler explanation. Here is one. Suppose that there is an observer moving along with the alpha particle at the same speed v_α and in the same direction, say North. What does he see? Relative to this speedy observer the alpha particle is initially at rest; when the electron collides head-on with the stationary, and much more massive, alpha particle, it will bounce back, due North, with nearly the same speed v_α. What is the motion of the same electron after collision relative to the laboratory? The electron is moving North at the speed v_α relative to the moving observer who, in turn, is moving North with speed v_α relative to the laboratory. Clearly the speed of the electron relative to the laboratory must be $2\,v_\alpha$.

To find the kinetic energy lost by the alpha particle, we calculate the kinetic energy gained by the electron:

$$E'_e = \tfrac{1}{2}m_e(v'_e)^2 = \tfrac{1}{2}(m_e/m_\alpha)m_\alpha(2\,v_\alpha)^2$$
$$= \frac{4m_e}{m_\alpha}\left(\frac{1}{2}m_\alpha v_\alpha{}^2\right)$$
$$= \frac{4\,m_e}{m}E_\alpha = \frac{4}{7200}E_\alpha = \underline{E_\alpha/1800}.$$

The energy lost by the alpha particle is equal to this by energy conservation.

(b) Eighteen head-on collisions would be required to make the alpha particle lose 1/100 of its energy. It may be interesting to note that 1 percent is approximately the energy lost by an alpha particle from a polonium source in traversing a gold foil 10^{-7} meter, or 400 atoms, thick. The way in which the alpha particle loses energy is by collisions with atomic electrons. Not all collisions are head-on, so that more collisions are involved than we have actually calculated here.

PROBLEM 25

An alpha particle with speed 1.00×10^7 m/sec is scattered by a gold nucleus at rest through an angle of 60°. Assume that so little kinetic energy is transferred to the gold nucleus that the speed of the alpha

particle after the scattering is the same as it was before.

(a) What is the momentum transferred to the gold atom?

(b) What is the velocity given to the gold nucleus?

(c) What is the kinetic energy given to the gold nucleus?

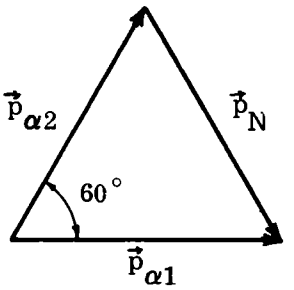
(d) Was your initial assumption a good one? Explain.

In a collision between an alpha particle and a heavy nucleus only a small fraction of the initial kinetic energy is transferred to the nucleus. To determine the actual amount we apply conservation of momentum and energy to the interaction.

(a) The total momentum of the system (alpha particle plus nucleus) does not change. Since the nucleus is initially at rest, the initial momentum is just the momentum of the alpha particle, $\vec{p}_{\alpha 1}$. After the collision the alpha particle has momentum $\vec{p}_{\alpha 2}$, and the nucleus has momentum $\vec{p}_N$. The vector sum of the final momenta must equal the initial momentum:

$$\vec{p}_{\alpha 2} + \vec{p}_N = \vec{p}_{\alpha 1}.$$

We construct a vector triangle showing the addition of $\vec{p}_{\alpha 2}$ and $\vec{p}_N$ to give $\vec{p}_{\alpha 1}$. We know the angle between $\vec{p}_{\alpha 2}$ and $\vec{p}_{\alpha 1}$: this is just the angle of scattering, 60° in this example.



If we assume that the final kinetic energy of the alpha particle is almost equal to the initial kinetic energy, then the magnitude, $p_{\alpha 2}$, of the final momentum of the alpha particle must be almost equal to the magnitude $p_{\alpha 1}$ of the initial momentum. With this assumption the triangle is very nearly equilateral and p_N is nearly equal to $p_{\alpha 1}$. If a numerical answer is wanted:

$$\begin{aligned} p_{\alpha 1} &= m_\alpha v_{\alpha 1} \\ &= 6.62 \times 10^{-27}\ \text{kg} \times 1.00 \times 10^{7}\ \text{m/sec} \\ &= \underline{6.6 \times 10^{-20}\ \text{kg-m/sec.}} \end{aligned}$$

The momenta transferred to the gold atom, p_N, are slightly smaller than this.

(b) From the momentum $m_N v_N$ of the nucleus we can calculate its speed. We have approximately:

$$\begin{aligned} m_N v_N &= m_\alpha v_{\alpha 1} \\ v_N &= \frac{m_\alpha}{m_N} v_{\alpha 1} = \frac{4.00\ \text{amu}}{197\ \text{amu}} v_{\alpha 1} \\ &= 0.020 \times 10^{7} = 2.0 \times 10^{5}\ \text{m/sec.} \end{aligned}$$

As noted before, the actual speed will be a little less. The direction of $\vec{v}_N$ is the same as that of $\vec{p}_N$ and is shown in the diagram.

(c) We can now calculate the kinetic energy, E_N, acquired by the nucleus:

$$\begin{aligned} E_N &= \tfrac{1}{2} m_N v_N^2 = \tfrac{1}{2} m_N \left(\frac{m_\alpha}{m_N} v_{\alpha 1}\right)^2 \\ &= \frac{m_\alpha}{m_N} \left(\tfrac{1}{2} m_\alpha v_{\alpha 1}^3\right) = \frac{m_\alpha}{m_N} E_{\alpha 1} = \underline{0.02\ E_{\alpha 1}.} \end{aligned}$$

By the conservation of energy the final kinetic energy of the alpha particle will be:

$$\begin{aligned} E_{\alpha 2} &= E_{\alpha 1} - E_N = E_{\alpha 1}(1 - 0.020) \\ &= 0.98\ E_{\alpha 1}. \end{aligned}$$

(d) We can now check the accuracy of our approximation. We have:

$$\frac{p_{\alpha 2}}{p_{\alpha 1}} = \frac{v_{\alpha 2}}{v_{\alpha 1}} = \sqrt{\frac{E_{\alpha 2}}{E_{\alpha 1}}} = \sqrt{0.980} = 0.99.$$

Our answers to parts (a), (b), and (c) are accurate to about 1%.

While this energy loss is small, it is quite large compared to the energy lost in a collision with an electron (see HDL 24). Why have we said that the energy loss of an alpha particle in traversing matter is mainly due to collision with electrons? The rather obvious answer is that collisions with nuclei are exceedingly rare. Suppose a polonium alpha particle traverses a gold foil 10^7 meters thick. The probability of a collision in which it is deflected more than 60° is only 1/50,000.

PROBLEM 26

(a) If the distance of closest approach of the alpha particle in Problem 25 was 1.65×10^{-13} m, what was its speed at this point?

(b) At what speed was it moving toward the nucleus at that point?

(a) Since energy is conserved the sum of the kinetic and potential energies remains constant:

$$\underset{\text{kinetic energy}}{\tfrac{1}{2}mv^2} + \underset{\text{potential energy}}{\frac{kq_\alpha q_N}{r}} = E, \text{ the total energy.}$$

When the alpha particle is a long way from the nucleus and moving with speed v_o, r is so large that the potential energy is negligible, and the total energy is

$$\begin{aligned} E &= \tfrac{1}{2}mv_o^2 \\ &= \tfrac{1}{2} \times 6.62 \times 10^{-27} \times (1.00 \times 10^7) \\ &= 3.31 \times 10^{-13} \text{ joule.} \end{aligned}$$

When $r = 1.65 \times 10^{-13}$ meter the potential energy is

$$\begin{aligned} k\frac{q_\alpha q_N}{r} &= 2.31 \times 10^{-28} \text{ joule-m/(elem. ch.)}^2 \\ &\quad \times \frac{(2 \text{ elem. ch.})(79 \text{ elem. ch.})}{1.65 = 10^{-13} \text{ m}} \\ &= 2.21 \times 10^{-13} \text{ joule.} \end{aligned}$$

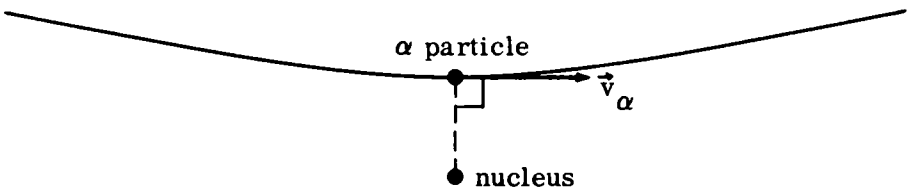
Hence, the kinetic energy at the distance of closest approach is

$$\begin{aligned} E_K = \tfrac{1}{2}mv^2 &= (3.31 - 2.21) \times 10^{-13} \text{ joule} \\ &= 1.10 \times 10^{-13} \text{ joule.} \end{aligned}$$

Then

$$\begin{aligned} v &= \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \times 1.10 \times 10^{-13}}{6.62 \times 10^{-27}}} \\ &= \underline{5.80 \times 10^6 \text{ meters/sec}}. \end{aligned}$$

(b) At the distance of closest approach the component of velocity along the line to the nucleus is <u>zero</u>.



PROBLEM 27

In the hydrogen atom, the electron is on the average 5×10^{-13} m from the positively charged proton.

(a) Estimate the average force between the electron and proton.

(b) If the atom were placed between the plates used in the Millikan experiment of Section 27-7, how many of the 90-volt batteries would we need to put in series with the plates to tear the atom apart?

(a) $$F = k\frac{q_1 q_2}{r^2} = \frac{2.3 \times 10^{-28}(1 \times 1)}{(5 \times 10^{-11})^2}$$
$$= 9 \times 10^{-8} \approx 10^{-7} \text{ newton.}$$

(b) The plates of the Millikan experiment described in Section 27-7 exerted a force of $\approx 10^{-14}$ newton on each elementary charge when three 90-volt batteries were connected in series with them. In order to increase the force to 10^{-7} newton, we would <u>need 10^7 times as many 90-volt batteries</u>.

Obviously, this is not the mechanism of ionization in a gas, because ionization occurs long before such large voltages are applied. A more effective process uses collisions with charged particles, where the necessary electric field is produced by a single elementary charge close by ($\sim 10^{-10}$ m) instead of an enormous number of charges at much larger distances ($\sim 10^{-3}$ m).

PROBLEM 28* (See Short Answers.)

chapter
33

Photons and Matter Waves

Chapter 33 introduces two basic facts that are essential to a further understanding of atoms and matter: (1) Light exhibits both wave and particle properties. (2) Particles of matter exhibit wave properties as well as particle properties. The chapter can be considered in three groups of sections.

CHAPTER SUMMARY

SECTIONS 1 THROUGH 3. Section 1 describes an experiment in which a light beam is aimed at a group of oil drops. The manner in which electric charges are ejected from the drops requires the assumption that light consists of particles rather than a smooth wave. If light does come in particles, this raises a question as to how to understand interference and diffraction phenomena which are characteristic of waves. An answer is that although light comes as a random volley of particles, the pattern that builds up, as more and more of these particles arrive, is determined by the wave aspect of the light.

A few experiments in probability are suggested in order to illustrate how random events can build up an orderly pattern. Finally Taylor's classic experiment is described. This experiment shows that individual particles of light, or photons, exhibit wave behavior even though there are no other photons nearby to "interfere" with them.

SECTIONS 4 THROUGH 7. In these sections more of the properties of photons are discussed. The photoelectric effect is described, and it is established that each photon of a given wavelength carries a definite energy. This energy is proportional to the frequency ν, of the wave associated with the photon and equals $h\nu$ where h is Planck's constant. The Compton effect is described. A photon collides with an electron, bouncing off with reduced frequency. Analysis of this process shows that each photon of a given wavelength also carries a definite momentum, $h\nu/c$, or h/λ. The relationship between energy and momentum for a photon is shown to be different from the relationship between energy and momentum for a slow particle. Our ideas of photons are shown to be compatible with our ideas of electromagnetic waves.

SECTIONS 8 THROUGH 11. Now that light has been shown to have wave and particle properties, de Broglie's hypothesis is described. De Broglie thought that particles of matter might have wave properties and that the wavelength associated with a particle would be h/p. This imaginative hypothesis was confirmed by a number of beautiful experiments which are described. Beams of electrons, neutrons, and helium atoms all show interference patterns when they reflect from, or pass through, regularly spaced arrays of atoms or other obstacles. The wave nature of a particle becomes important when it moves among or encounters objects whose spacing or size is comparable to the particle's wavelength. Under these circumstances the laws of Newtonian mechanics, which can be applied successfully to problems involving large particles, apparently failed. A new formulation must be developed to describe the wavelike behavior of very small particles. This new mechanics is called wave mechanics or quantum mechanics. Wave mechanics becomes identical to Newtonian mechanics where the wavelength is much shorter than the other distances involved in the problem.

The chapter closes with a discussion of the similarities and differences of the waveparticle nature of light and of matter.

SCHEDULING CHAPTER 33

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the volume summary section. Sections which can be deemphasized or abbreviated without loss in continuity are designated with brackets [ ].

Chapter 33	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTIONS	CLASS PERIODS	LAB PERIODS	EXPERIMENT	CLASS PERIODS	LAB PERIODS	EXPERIMENT
1, [2], [3]	1	0	—	2	0	—
4, 5, 6, 7	2	1	IV–14	3	1	IV–14
8, 9, [10], [11]	2	0	—	3	0	—

RELATED MATERIAL FOR CHAPTER 33

LABORATORY. Experiment IV-14, *Randomness in Radioactive Decay*, should be done with this chapter. See the Lab Notes for suggestions.

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILMS. "Photons," by John G. King of M.I.T., demonstrates the particle nature of light with a photomultiplier and an oscilloscope. Short bursts of very weak light are passed through an intermittent shutter and fall on the photo-multiplier. The output is examined with the oscilloscope showing that light does not come as an "even stream," but in "packages" at random intervals. Running time: 18 minutes.

"Interference of Photons," by John G. King of M.I.T. Using the same detection apparatus as in the film "Photons," a movable double slit is used to move an interference pattern across the photo-multiplier. The light intensity is reduced until calculation shows that at any given time, there is rarely more than one photon between the light source and the photomultiplier. The resultant observation of the characteristic interference pattern shows the coexistence of wave and particle behavior in light. Running time: 13 minutes.

"Photoelectric Effect," by John Strong of the Johns Hopkins University (in color), shows a number of qualitative demonstrations of the photoelectric effect and performs a quantitative experiment in which the kinetic energy of the photoelectrons is measured as a function of the incident photon energy. This film can carry the discussion of Sections 4 and 5 very nicely. Running time: 28 minutes.

"Matter Waves," by Alan Holden and Lester Germer of the Bell Telephone Laboratories, actually exhibits the wave nature of electrons in a modern version of the original Davisson-Germer experiment. The electron diffraction patterns are explained in terms of wave behavior and an optical analog experiment is performed to show that a nearly identical pattern can be obtained with light. This film presents about the only direct experience that a student is apt to have with the wave nature of matter and therefore is highly recommended. Running time: 28 minutes.

"Pressure of Light," by Jerrold R. Zacharias of M.I.T. could be used with Section 6 of this chapter if it has not been shown in connection with the work of Chapter 14. The film gives a remarkable demonstration of light pressure. Running time: 21 minutes.

33-1 The Graininess of Light

PURPOSE. To describe some experimental observations which cannot be readily understood from the point of view of a wave model and thus to introduce a basis for reconsideration of a particle model.

CONTENT. Electrons are ejected from specks of matter by a beam of X rays. The average number of electrons ejected is proportional to intensity, as would be expected, but individual events occur randomly in time, in a way that is completely at variance with the predictions of a wave model. This can be explained, however, by regarding the beam of X rays as a volley of *particles*. Thus, one of the intriguing facts of modern physics is introduced: There are some situations in which light displays particlelike behavior; although we have already seen that in other situations it behaves like a wave.

EMPHASIS. This section presents the first qualitative evidence for the quantum nature of light. It must be thoroughly understood.

DEVELOPMENT. Students who have seen the film on the Millikan oil drop experiment should have little trouble picturing the experiment described in this section. The interpretation, however, may present some problems. The most difficult idea is that all the drops should lose electrons at the same time *if* the X rays were behaving like waves. It will be well to go through a calculation of such a process in a very simplified way. It is reasonable to assume that the same amount of energy will remove an electron from any drop. A beam of electromagnetic radiation (behaving as waves) delivers the same energy per second (P joules/sec) to each drop. Then if it takes a certain energy (E joules) to remove an electron from a drop, radiation will have to be absorbed for E/P seconds before an electron is ejected. This time should be roughly the same for all drops. As an example, calculate the time that would be required for a wave disturbance from a 100-watt light bulb to eject an electron from an atom one meter away from the light bulb. Suppose it takes 2×10^{-19} joule to eject an electron from an atom and the atom has a radius of 10^{-10} m. If we imagine a sphere of one meter radius centered on the bulb, the area of this sphere over which the light is spread is $4\pi r^2 = 4\pi$ (meters)^2. The atom covers a fraction $4r^2/4\pi r^2 = 10^{-20}/4 = 2.5 \times 10^{-21}$ of the area, so it captures only that fraction of the 100 watts. This is 100 watts $\times$ 2.5×10^{-21} = 2.5×10^{-19} joule/sec. So the atom must absorb energy for 2×10^{-19} joule/2.5×10^{-19} joule per sec, or 0.8 second, in order to eject an electron. After working out this calculation, you can point out that under these identical conditions, electrons have been observed to be ejected in 10^{-8} second. (See the caption for Figure 33-2.)

As a homely illustration of the phenomenon, you could consider corn popping! If you put corn into a hot skillet and jiggle it in the approved way, nothing much happens for a few minutes. Then one kernel will pop, and suddenly they all start to pop. They

don't pop at exactly the same time because differences in the rate at which heat is transmitted to the individual kernels, but you have to wait a certain time before anything happens at all. Imagine the surprise if, as soon as you poured in the corn, a kernel would pop! This doesn't happen because corn popping isn't a one-quantum process. With the X-ray beam regarded as a wave, the photoelectric effect is just as surprising.

The photon model of light which is now invoked to understand this surprising result uses arguments very similar to those for scattering in the previous chapter, Sections 32–1 to 4. The frequency of occurrence of the encounters, that they occur randomly in time, and that they may occur very promptly, are all understood in exactly the same way as in that earlier work, although we now have the incident "particle" (the photon) absorbed by the atom, followed by ejection of an electron. (The individual process is more complicated than simple scattering, of course.)

A more detailed examination of the photoelectric process is made in Section 33–4. Defer discussion of details till then. Do not become embroiled in detailed discussion of exactly what the classical model predicts. For the most part the explanations are well beyond even the best classes. In the light of further evidence to be presented in Sections 4 and 5, the deficiencies in the classical wave model will become apparent enough to convince students of the need for a new theory.

COMMENT. On page 613, right-hand column, the text suggests that the student may want to figure out why the *magnetic* field is not important for electron ejection. The argument goes as follows: In a plane electromagnetic wave, E and B are related: $B = E/c$. (See Chapter 31, page 587.) The force exerted on an electron, moving with a velocity v, by an electric field is just $F_e = E$. Now the force exerted on such an electron by a magnetic field is $F_m = Bv = (E/c)v = F_e v/c$. So F_m is smaller than F_e by the factor v/c. The velocities of electrons in atoms are considerably less than c, so the magnetic field is correspondingly less effective than the electric field.

FILMS. The two films "Photons" and "Interference of Photons" contain quite detailed descriptions of a photoelectric effect experiment, and of the photon interference experiment (Section 33–3), respectively. If you are able to use these films, an effective presentation consists of a reasonably fast treatment of the sections on photons (33–1 through 33–7), followed by the films. In this way your class discussion can be mainly concerned with the questions of principle — for example, the material of the foregoing development. The films, particularly the first, "Photons," dramatically convey the main point of this work — the random nature of the individual events, which leads us to a model for light which has particlelike properties. The technical aspects of these rather complicated experiments are also easily taken care of in this way.

33–2 The Orderliness of Chance

PURPOSE. To show how individually random events, when observed in sufficiently large numbers, will develop a regular and predictable pattern. The motivation is that we must understand how photons can individually behave in a random way, but, when large numbers are studied, seem to follow the regular patterns (interference and diffraction) predicted by the wave theory.

CONTENT. An experiment is proposed to illustrate the statistical behavior of a group of random events. Instead of individual grains of light, a set of marked cards is used. In any one trial, it is not possible to predict which card will be drawn. But after many such drawings, the average number of appearances of each kind of card tends toward a pattern which can be predicted from the original distribution of numbers of each kind of card in the pack.

EMPHASIS. A general understanding of probability is important to the thorough understanding of photon interference patterns, but may be skipped if you are *hard* pressed for time. Probability is fascinating in its own right. Do not get bogged down here.

DEVELOPMENT. An experiment of the type described in the text is illuminating and worth doing. If the experiment described in the text is done each student could make a set of 3 "L" cards, one "C" card and 6 "R" cards. Then each member of the class could draw, record the draw, shuffle, draw, etc., for about ten minutes. Then the individual results could be tabulated. The individual results will show the random fluctuations typical of chance, but the results of the class as a whole should show a pattern much closer to the average of 3 : 1 : 6.

You will not want to carry on the experiment too long especially since very many trials are needed — more than you have time for — to obtain results very close to the predicted ratios. When the main point has been made, you may wish to remind students that they really are familiar with this situation. They toss a coin when a random choice between two alternatives is required, and they all believe that this is a fair way to make the choice. If they tossed many times, heads would come up about as frequently as tails.

33–3 Graininess and Interference

PURPOSE. To discover whether individual photons trace out the interference patterns produced by strong light.

CONTENT. Our knowledge of interference phe-

nomena was obtained with intense beams of light, involving many photons all going through the apparatus simultaneously. Is interference a result of the interaction of photons with each other, or is it a property of individual photons? The question is answered by describing G. I. Taylor's experiment in which he photographed the diffraction of light around the shadow of a needle. Even when the intensity of light was so low that only one photon at a time was in the space between the lamp and the photographic plate, a long exposure showed the same diffraction pattern as when the light was intense.

EMPHASIS. This section need not be discussed; it should be assigned as reading material.

DEVELOPMENT. In the rest of the chapter a ready understanding of wave concepts and properties is assumed. You may need to review the various topics that arise by referring back to Chapters 16, 17, and 18. A little time spent in reminding students of diffraction and interference, and their explanation in terms of the superposition of wave amplitudes, may be necessary. The test could be that, presented with Figure 33–11 (c) or (d), they should immediately think of *waves* as the explanation.

The film "Interference of Photons" illustrates a modern version of the Taylor experiment. A photo-multiplier is used rather than Taylor's photographic plate so the arrival of individual photons can be heard and seen. One hears and sees very few photons at the known positions of the interference minima, and very many at the maxima. It is clearly demonstrated that "interference" effects are *not* the result of interactions between two or more photons. Note when the light strikes the photo-multiplier, it is not spread out over the interference pattern.

What is the significance of this new experimental observation in our study of photons and light waves? Had the result been just the opposite, we would still have to contend with the particle properties of light as evidenced by the experiment of Section 1. This new observation is something extra, and it appears to be somewhat paradoxical. How can a single photon interfere with itself? For interference fringes to occur, light must travel coherently and simultaneously through both slits of a two-slit apparatus, but the individual photons arrive finally, each at only one definite place; and the interference pattern is a statistical accumulation. Can a particle travel through two slits simultaneously? Certainly not a particle such as we have used in Part III. On the other hand, could you find out which slit the photon actually went through? If you cover one slit, to ensure that the photon goes through the other one, then the interference pattern disappears, and you will only see the diffraction pattern of a single slit. In fact, it can be shown that any supplementary apparatus which will detect the passage of the photon through *one* of the slits will also destroy the coherence, and hence the interference pattern. The situation is certainly paradoxical. What is clear is that, with this new result, no detailed discussion in terms of classical waves (as with the ripple tank) or classical properties can tell us what is happening. A theory unlike any we have met before is required. The individual photons, each a particle in some sense, have wave properties. We shall meet an analogous situation with particles of matter in the later sections of this chapter. This dual nature of matter and light is one of the important facts that must be faced by any theory of modern physics, yet it is most difficult to appreciate. There are some situations in which light behaves as waves. One example is a beam of light passing through a pair of slits and being detected by its heating effect on a thermocouple. There are other situations in which light behaves as particles. An example is the way the X rays knocked charge out of bits of matter in the Millikan experiment.

One approach to this "paradox" which is often effective is the following. Light is light. It is not a wave, and it is not a particle. Our concepts of waves and particles are idealizations of possible kinds of behavior. There is no reason to believe that light will fit in either of these categories uniquely; in fact it is fortunate that any analogy at all is possible. We try to force all new phenomena that we observe into some familiar category, so that we are capable of some kind of description and some degree of understanding. Nevertheless, we should not get in the habit of giving these efforts toward categorization an independent reality. Our ultimate goal is a consistent description of things as they are.

FILM. "Interference of Photons." See Section 1, Films.

33–4 The Photoelectric Effect

PURPOSE. To learn about some of the properties of photons, by examining several manifestations of the photoelectric effect.

CONTENT. (a) When monoenergetic photons are absorbed in a cloud chamber, the ejected electrons all have the same energy.

(b) The ejection of electrons from a metallic surface has a significant and sharp dependence on the wavelength of the light; it must be shorter than a certain critical value called the photoelectric threshold. The actual value of this threshold wavelength depends on the particular surface.

Both of these experiments show that the photons, which constitute the beam of monochromatic light or X rays, have a *unique* energy, not just an energy spread with the appropriate average value. The second experiment also shows that this unique energy depends

on the wavelength, shorter wavelengths corresponding to higher energy.

EMPHASIS. Treat thoroughly but briefly. The next section builds directly on this one by using the same, simple apparatus in more complicated experiments.

DEMONSTRATION: THE PHOTOELECTRIC EFFECT

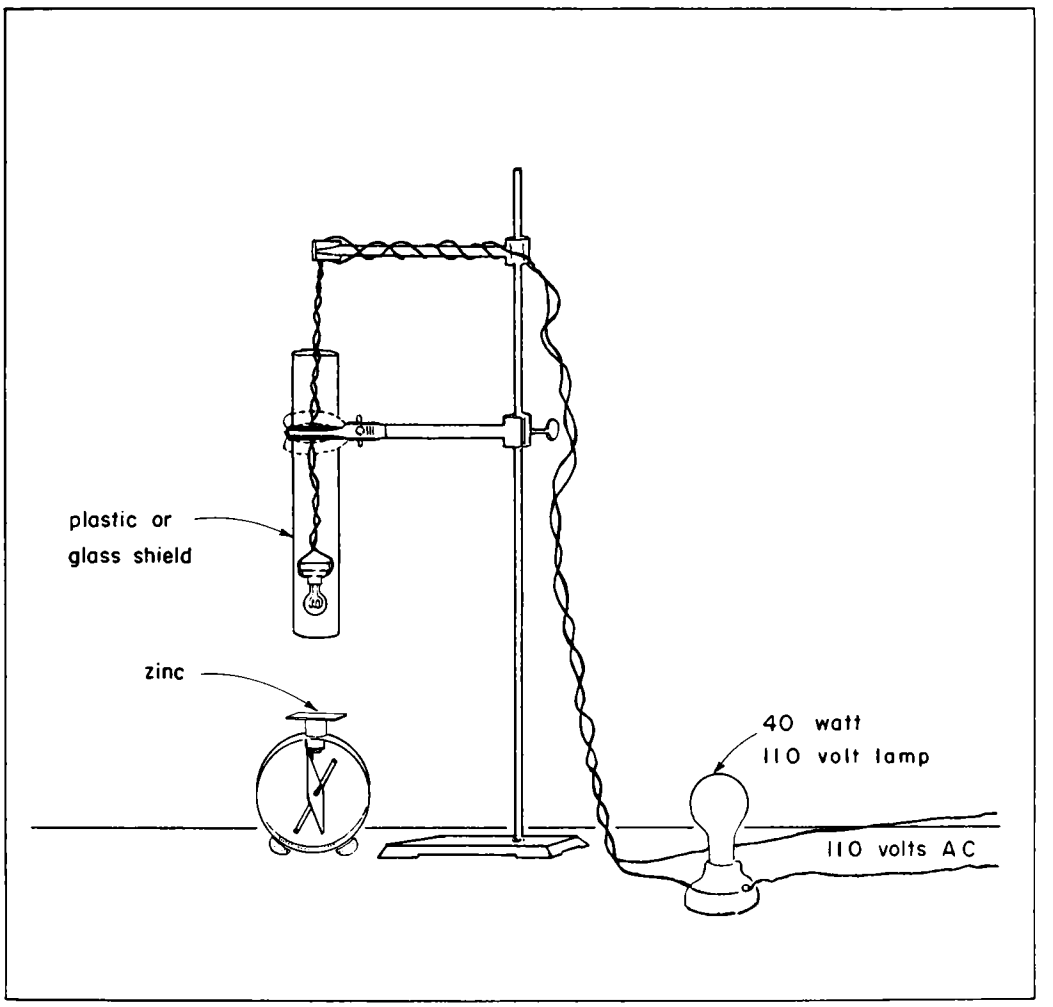
This demonstration shows that ultraviolet light ejects electrons from zinc, whereas visible light does not. An electroscope with a zinc or galvanized-iron top is used to show that ultraviolet light from a mercury-vapor sterilizing lamp will eject electrons from zinc.

The ultraviolet lamp must be connected in series with a 40-watt, 110-volt incandescent bulb to limit the current when operated from 110 volts A.C. The bulb of the ultraviolet lamp, made of special glass, transmits the near-ultraviolet emitted by mercury vapor. It must be surrounded, except at one end, by an ordinary glass tube or by a thin sheet or tube of plastic, to prevent harmful ultraviolet light from reaching the eyes. The zinc plate of the electroscope should be rubbed with steel wool until shiny to remove any oxide or other impurities from the surface.

Charge the electroscope negatively and show that it does not discharge when moved close to a 100-watt incandescent light. Now move the electroscope beneath the ultraviolet lamp. The negatively charged electroscope will discharge rapidly. If, while it is discharging, a piece of glass or plastic is held between the ultraviolet light and the electroscope, the discharge will cease because the glass acts as a filter which absorbs the ultraviolet light.

Now repeat the experiment with the electroscope charged positively. No discharge takes place when the zinc is illuminated by the incandescent light. When illuminated by ultraviolet, the electroscope discharges considerably more slowly than when it was charged negatively.

An entirely different mechanism is responsible for this behavior. When the electroscope is negative, the electrons ejected from the zinc are pushed away. When the electroscope is positively charged, they are quickly pulled back to the plate and do not contribute to the discharge. Since they have only about 1 electron volt of kinetic energy, they cannot escape. The slow discharge from a positively charged plate is the result



of ionization of the air by the ultraviolet light. This ionization is, of course, also present when the electroscope is negatively charged, but then its contribution to the total discharge is negligible.

The near-ultraviolet from the sterilizing lamp will not release electrons from copper or aluminum.

A General Electric OZ4S11 ozone lamp that takes a 40-watt, 110-volt incandescent lamp as a current limiter and fits an intermediate screw-type socket can be obtained from home-appliance stores. It is used in electric dryers and washing machines.

COMMENT. The photograph of the droplets in the cloud chamber, Figure 33–6, shows beautifully the randomness in space of the absorption process. That the drops formed are identical in size, i.e., that the ejected electrons all have the same energy, requires a somewhat trusting interpretation of the figure. Avoid detailed discussion of this point. We are putting together a series of clues whose total import is overwhelmingly in favor of the photon hypothesis of Einstein.

The mechanics of the photoelectric effect with the metallic surface can be effectively left to the first "Photons" film. The point of this part of the section is to introduce photoelectric emission from a surface, and to give its significant dependence on wavelength.

When available, the film on the photoelectric effect, which shows and interprets photoelectric phenomena, will be a help in teaching Sections 4 and 5.

33–5 Einstein's Interpretation of the Photoelectric Effect

PURPOSE. To establish the fact that the energy of a photon is proportional to is frequency.

CONTENT. An experiment is described in which the kinetic energy of photoelectrons is measured as a function of the wavelength (or frequency) of the photons that ejected them. It is found that the maximum kinetic energy of the electron, E_{c1}, is related to the frequency as follows:

$$E_{c1} = h\nu - B.$$

The quantity B is the amount of energy required to pull the electron away from its atom or from the surface of the piece of material of which it is a part. The quantity $h\nu$ is interpreted as being the energy of the photon. When $h\nu$ exceeds B, the photon can not only pull the electron out of the atom or of the metal surface, but it can also give it kinetic energy. This explanation of the photoelectric effect was proposed by Einstein in 1905.

EMPHASIS. This is the quantitative climax of the first part of this chapter. It must be understood, not only for itself, but for any understanding of Chapter 34.

DEVELOPMENT. As with a mystery story, the whole problem of the photoelectric effect is understood in a simple way once an adequate interpretation has been discovered. This is especially true of the energy *distribution* of photoelectrons from a metallic surface, compared with the constant electron energy seen in a cloud chamber. In your development of the material of this section it will therefore be better to get to the conclusion quickly, and then to understand the various intermediate steps in the light of the photon picture.

COMMENT. The photoelectric effect from a metallic surface is sufficiently complicated that it is necessary to keep reminding students just exactly which functional relationships are being investigated in each part of the experiment.

(1) The photoelectric current (the rate at which electrons are ejected) is proportional to the *intensity* of the light under all normal conditions. No experiment has been described here to check this fact, since it tells nothing new about light. It would be expected with the classical wave model, and it is also an immediate consequence of the photon hypothesis.

(2) For constant light intensity, the total photoelectric current stops abruptly for wavelengths longer than the threshold wavelength. The implications of this fact were discussed in the previous section.

(3) For constant light intensity, and fixed wavelength, the ejected electrons have a distribution in energy (Figure 33–8). As discussed in the text, this complicating feature arises because the target is a solid, which can absorb some of the electron's kinetic energy as the electron escapes from its surface. The distribution in energy under these conditions thus tells us something about the solid material. It is only the maximum electron energy, however, which is simple to interpret, and from which the photon energy properties can be deduced.

(4) For constant light intensity, the *maximum kinetic energy* of the ejected electrons has a simple straight-line dependence when plotted against the *frequency* of the light. Moreover, the straight-line dependence has the same *slope* independent of the material used, i.e., the slope is a property of the photons themselves. The interpretation of these results is that for light of a frequency ν (where $\nu\lambda = c$, the velocity of light), the photons have energy $h\nu$, where h is a universal constant (of dimensions energy times time). This is then the relationship we have been searching for — the relation between the wave characteristics of a light beam and the unique energy carried by its photons.

33–6 The Mechanics of the Photon; Photon Momentum

PURPOSE. To consider the order of magnitude of the energies and frequencies involved in the photoelectric effect, and to show that photons carry momentum as well as energy.

CONTENT. The numerical value of h, the slope of

the straight-line graph, Figure 33–9, is obtained, and the energy carried by a photon of visible red light is calculated. Energy relationships become simpler when expressed in electron volts.

Detailed analysis either of the particle model or of the electromagnetic wave theory of light leads through the analysis of collisions of high-energy photons with single electrons, as seen in a cloud chamber. The momentum acquired by the recoiling electron is vectorially just the negative of the change in momentum suffered by the photon deflected in the collision.

EMPHASIS. This section shows concretely the need for the momentum carried by radiation, particularly photons, and how it preserves the law of conservation of momentum first introduced in Newtonian mechanics. The relation between the momentum and the wavelength will later be generalized to apply to the motion of any particle.

DEVELOPMENT. Having in the previous section established the qualitative features of the revolutionary changes in our thinking about light, it is now necessary to acquire some familiarity with the numerical magnitudes involved. The final chapter will go much more easily if students feel able to handle the relationships of this section, and if they obtain some feeling for electron-volt orders of magnitude. Even if you have not been assigning many of the HDL problems so far, it will be wise to go through Problems 10 and 13, say, in class.

We know from our earlier work with light that it exerts a pressure, i.e., momentum, so it is a logical necessity that photons do so. Here we see the experimental evidence. This will bring alive the whole photon concept; photons really act like particles.

Students may want to know when the Compton effect occurs and when the photo effect occurs. Baldly speaking, for low-energy photons (but those with enough energy so that a photoelectric process can go on), the probability of the occurrence of a photoelectric process is far greater than the probability of occurrence of Compton scattering. For photons of higher energy, the probability of a photoelectric process decreases rapidly and becomes smaller than the probability for Compton scattering. Therefore if we look at photons of relatively low energy, we are likely to see photoelectric processes. If we look at photons of higher energy, we are likely to see Compton scattering.

Here is a plausibility argument that may help you understand the general behavior. Photon absorption (leading to photoelectric ejection of an electron) is a very improbable event. Compton *scattering*, in which a photon is absorbed and then reemitted, involves two such events each of about the same degree of improbability. Hence its occurrence is much less probable even than photon absorption. It does happen at the energies of the earlier experiments. Unless apparatus is devised to look for it especially, however, the much more probable photon *absorption* process will mask it completely. As is described in the text, the situation changes as we go to higher and higher photon energies. The atomic binding of the electron is, in comparison, smaller and smaller, so that to the photon the electron appears to be almost free, and a photon cannot be *absorbed* by a free electron. A photon can only be scattered by a free electron. Thus at the higher energies scattering becomes the more probable process. (For us, then, it is something like looking for the needles where there isn't a lot of hay lying around.)

The statement made above, that a free electron cannot absorb a photon, will be immediately clear to the students if they attempt the graphical solution of the process in the context of Problem 15. An electron with momentum $p = h\nu/c$ has a kinetic energy $p^2/2m$, whereas the photon with this momentum had energy $h\nu = pc$. With the numerical data of Problem 16, $h\nu = 1.2 \times 10^4$ electron volts, whereas $p^2/2m = 3.4$ electron volts. It is therefore *not* possible for an electron to consume *both* the energy and the momentum carried by a photon. In photoelectric absorption the rest of the system (atom or solid) absorbs most of the momentum, and all the electron worries about is energy conservation. At the energies for which the Compton effect is actually observed one must use relativistic mechanics, as is mentioned in the text, but a similar result is obtained.

COMMENT. Since light pressure is so small as to be extremely difficult to detect, the Compton effect is the experiment which is commonly used to demonstrate the momentum of light. The "so-called" light-pressure motors often seen in jewelers' windows actually operate by the heating of the residual gas in the evacuated jar. If you showed the film "Pressure of Light" in your study of Part 2, students can be reminded that they saw a demonstration of photon momentum in this film.

In this section the electron volt is introduced as the "natural" unit of energy for talking about the photoelectric effect. When visible light is used, the energies of the photons turn out to be a few electron volts. This is not pure coincidence. The volt was chosen as the "practical" unit of electric potential difference because it was the right size for talking about the potentials in a chemical cell — flashlight cell, 1.5 volts; lead automobile cell, 2.0 volts. These potentials are generated in chemical reactions which involve the exchange between atoms and molecules of one or two of the outermost electrons. Similarly visible light is generated in the same sort of reaction — the movement of the outermost electrons of an atom or molecule. It is not surprising that the energies are the same. Why this is *visible* light is, of course, a different question; but it is probable that light, to be visible, must make much the same kind of electrochemical reaction in our eyes.

The expression $p = h\nu/c$ is easy to remember, but the form $p = h/\lambda$ will have more relevance to the remainder of the chapter.

33-7 Photons and Electromagnetic Waves

PURPOSE. To show how the wave model and the particle model of light fit together.

EMPHASIS. Descriptive material. A reading assignment should suffice.

COMMENT. Electromagnetic waves of long wavelength (e.g., radio waves) are constituted of streams of photons whose individual energies are so minute that detecting apparatus is insensitive to anything less than enormous numbers of photons. All that is seen is the average effect, and this is what the wave model describes correctly. The experiments with light which established the wave model were also of this kind. The graininess of light will usually not be seen, although special experiments prove that it is there, even for very long wavelengths.

On the other hand, at extremely high energies, where individual photons are easily detected, there is usually no apparatus fine enough to examine their wave nature. A particle model gives most of the features. Our fundamental theory of photons must include both extremes, and neither the wave model nor the particle model is adequate.

DEVELOPMENT. If you have been able to show the photon films after going through the first part of this chapter, this section is the place to summarize the new knowledge. The reconciliation of the rather complex, mysterious new picture of photons with the wave model, so laboriously justified in an earlier part of the course, is very necessary to maintain confidence! Problem 12 of HDL will help to put the point across.

It will probably be best, however, not to go so far as to list all of the experiments performed, and precisely the information obtained from each. The main points to be retained are just that light sometimes behaves like waves, and sometimes like particles, but that the picture is more complicated than just either/or waves or particles. Do not belabor this point. We cannot explain it any more than we can explain why gravity exists. We learn that it does exist, how it behaves, and how to calculate what it will do in all events. The same should hold for light. Learn the facts experimentally; make a simple (!) theory that describes them; learn how to predict what light will do in all circumstances. This we have done.

33-8 Matter Waves

PURPOSE. To present experimental evidence that particles (electron, helium atoms, neutrons) possess wave properties.

CONTENT. The section consists of the following sequence of ideas:

(1) de Broglie's speculation that particles have properties of waves whose wavelength, λ, is related to the particle momentum, p, by the equation $\lambda = h/p$. This is the same relation that holds for light, with the same constant, h (Planck's constant).

(2a) Detailed figures and explanations of experiments involving diffraction (Figure 32-11) and interference (Figure 33-12) of light and electrons. Although the wavelengths are not the same for light and for electrons, the patterns are closely similar, except for scale factors. De Broglie's hypothesis is thus tested and *proved* qualitatively and quantitatively for electrons.

(2b) Experiments with a crystal spectroscope show that helium atoms and neutrons also exhibit interference properties characteristic of de Broglie waves.

EMPHASIS. Very important for this and the last chapter. Treat thoroughly.

DEVELOPMENT. The rest of this chapter now presents the other half of the foundation for modern physics. That matter, indeed *any* particle, possesses wave properties will be rather harder to accept than the particle behavior of light, since the idea of a particle of matter is deeply rooted in our common sense experience of the world. There is nothing wrong with that idea, but there are circumstances in which we see that Newtonian mechanics is inadequate, and probabilities and wave effects take the place of the definite Newtonian orbits to which they reduce in ordinary experience. It should be possible to offset this feeling that particles cannot have wave behavior by pointing out the symmetry that this new knowledge brings to our picture of the physical world.

The apparatus required for obtaining a suitable beam of electrons and investigating the wave properties is very complicated and expensive (beyond most college laboratory equipment). The film on "Matter Waves" uses a piece of research quality equipment in the hands of Lester Germer (one of the three persons who first gave experimental evidence for the existence of matter waves) to display the effect of electron diffraction in a convincing way. Alan Holden carries out the discussion and provides an optical analog experiment, producing the same pattern with light.

The compelling line of argument adopted in the figures in the text follows from juxtaposing results of light-wave experiments with the results of analogous experiments with electron beams. The results are closely similar, so we are led to conclude that particles of matter, in motion, exhibit wavelike characteristics. For most students this argument will be sufficient. A commentary in the figures, together with information about some of the detailed features of the processes involved in each case, is included in the comments. Your students should leave this section *convinced* that

in some situations particles (all particles) show wave-like properties. They will spend the rest of the volume in seeing how this wave-nature of particles enables us to understand some of the hitherto puzzling features of atomic physics.

COMMENT. As with the photon arguments of the previous sections, the whole picture of matter-waves is much more convincing and compelling than any one small part. Avoid becoming involved in arguments about small details, but persuade the students to wait, and judge the evidence as a whole.

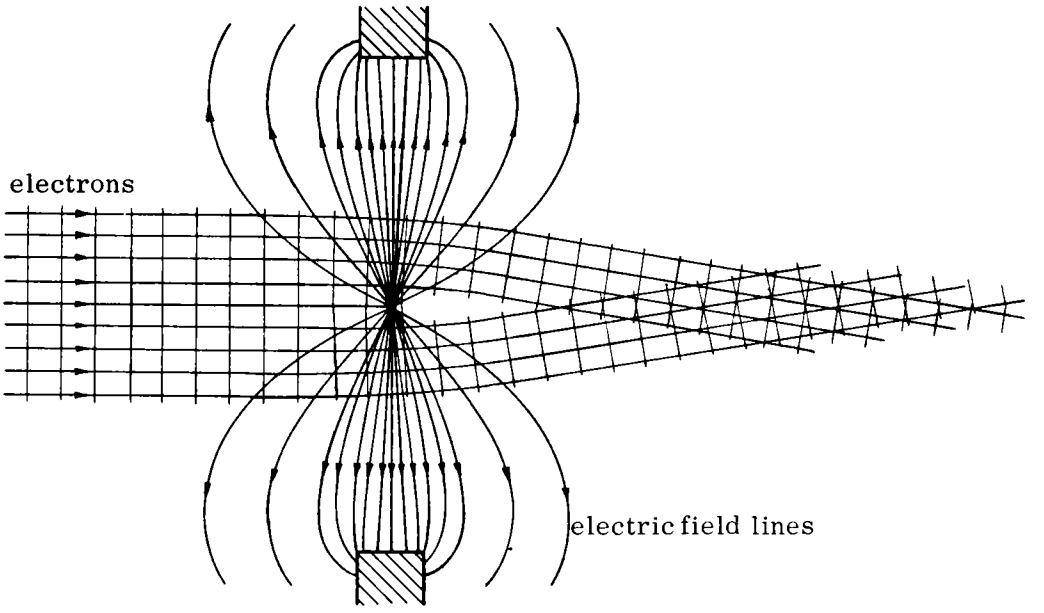
Comment on Figure 33–11. This experiment involves diffraction by an *edge*. The physics of this situation is no more complicated than that of diffraction by a slit (Section 19–6). The "expected" pattern of light intensity (Figure 33–11b) may look surprising but is what results from a careful analysis. The waves tend to bend around the edge somewhat (as you can see in a ripple tank), so that the light intensity does not drop abruptly to zero. Interference effects cause the "wiggles" in light intensity, which gradually disappear as the distance from the edge of the shadow increases. The *calculation* of the light intensity, however, is much more difficult than in the case of the slit. We do not need to calculate it however. [Figure 33–11c demonstrates experimentally that light, whose behavior in simpler situations we understand, does just what is predicted by Figure 33–11b. The light pattern of Figure 33–11c could be reproduced in your laboratory with very little more trouble than for the Part II diffraction experiments. With a line source of the kind used there, a razor blade for an edge, and a ground glass screen, the diffraction pattern can be readily seen.]

The real point of the experiment as presented here is contained in the very close similarity between Figures 33–11c and 33–11d. In this situation, electrons behave similarly to light. Note that the intensity patterns appear almost identical because the scales of the photographs have been adjusted suitably. The whole scale of the electron experiment must thus be altered. The edge must be much finer: the almost perfect edge of a crystal is used rather than the edge of a razor blade; and the distance from edge to screen increased. By varying the *energy* of the electrons, one can show that the scale of the pattern of Figure 33–11d changes in just the way suggested by de Broglie.

Comment on Figure 33–12. This illustrates the result of an experiment which shows the *interference* of matter waves. As with the diffraction experiment, the very short wavelength of the electron waves necessitates using apparatus that is more complicated than that discussed in Part 2. The wavelength of the electrons is 0.086×10^{-10} m, which is about 10^{-5} of the wavelength of visible light.

The fact that one sees interference fringes after light has passed through a biprism should be acceptable to the students without much argument. The familiar conditions under which interference arises are reproduced by this apparatus: two trains of waves from the same source are made to cross. A screen placed in their path should show dark lines where the waves annul each other, and bright lines where they add to each other's effect. In any case, experimentally we do see the interference fringes with light (Figure 33–12a).

It may be necessary to justify that the electrical apparatus of Figure 33–12b acts on electron waves in a manner which is similar to that in which a biprism acts on light. A more detailed picture of the electric field lines in the neighborhood of the wire shows this in a qualitative way. The electrons have negative charge, so they will experience a force toward the wire. Particle trajectories are as shown, and the wave-trains which we now associate with these trajectories will bend so as to remain coincident with the trajectories.



Thus by ray tracing we expect the device to act like a converging cylindrical lens. The unexpected result is that we see interference fringes, proving that there is a wave property associated with the electrons.

It is difficult to think of any "particle" explanation for this experiment. Variation of the electron energy will produce a change in the fringe separation, so the phenomenon is a property of the electrons rather than of the apparatus. Even your most doubting students should begin to feel convinced.

Note on Intensity Variation in Photographs. The photographs in Figure 33–12 show broad variations in intensity of the interference fringes. This is presumably due to *diffraction* of the whole beam by the defining aperture which, in each case, is the half of the biprism through which the beam passes. The superposition of a diffraction pattern on a series of interference fringes is exactly the same phenomenon as was met in Section 19–7 (double-slit interference). The distance of separation of the diffraction bands depends on λ/d, where d is the size of the aperture, whereas the spacing of the interference fringe depends on the angle of the biprism. (The distance of separation of the virtual objects in Figure 13–12a depends on the angle of the biprism.) The fact that there are fewer *bands* in the case of the electron photograph would mean that, after allowing for the change in scale of the whole apparatus due to the difference in wavelength of electron and visible light, the electron's "biprism" is considerably narrower than that for visible light.

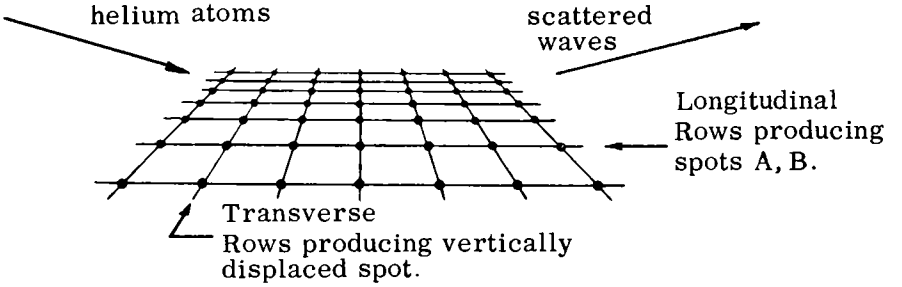
Problems 20 and 21 involve a more detailed discussion of this experiment. Your better students will benefit from an attempt at those problems, and a class discussion of Problem 20b will be useful in making sure that the matter-wave idea is really going home.

Comments on Figures 33–13 *and* 33–14. The purpose of these figures is to show that particles other than electrons (helium atoms, neutrons) also exhibit wave characteristics. For helium atoms (Figure 33–13) the important piece of apparatus is the *surface* of a crystal. (Helium atoms cannot easily pass through into the volume of the crystal and re-emerge.) This surface is a two-dimensional regular arrangement of atoms, each of which will be a source of scattered matter-waves. For neutrons the apparatus which scatters the wave is the whole volume of the crystal, since neutrons easily penetrate this particular crystal. The neutron thus encounters atoms arranged in a three-dimensional array. It seems clear in both cases that if the particles in the beam behaved in accord with Newtonian mechanics, they would be scattered by the atoms in more or less random directions. If the students will, on the basis of their experience with Part II, accept the fact that emergence of particles at only very special angles must indicate an interference of waves, then little more need be said. The results of each experiment (Figures 33–13c and 33–14c) show beautifully sharp variations of scattered intensity with angle. The conclusion is inescapable. Helium atoms and neutrons behave like waves also; the chances are that the wave description is fundamental in describing the motion of anything. Newtonian mechanics is an approximation.

Details of the Interference Process. Students who accept the above account will only be side-tracked by going into more detail about the actual methods used, and the interference process involved. A close examination of the captions for Figures 33–13 and 33–14 may stimulate awkward questions, however, and some of these points are discussed in what follows.

A diffraction grating for visible light is made by ruling closely-spaced valleys and ridges on a metal surface (for light, a few thousand lines per cm). When a wave strikes the grating, there will be *interference* between the wavelets scattered from each ridge. The scattered light will be a maximum at a point on the screen where the difference in path length to each ridge is just equal to an integral multiple of λ, the wavelength. The maxima will occur at the same angles as the maxima from interference due to two *slits* whose distance apart is equal to the spacing of the ridges. The maxima for a grating are very much sharper than for two slits, however. It is as though the sinusoidal intensity variation of two slits had superposed on it the very sharp *diffraction* pattern due to a slit as wide as the whole grating. Thus all of the light that strikes the grating is *concentrated* in very narrow regions whose angular position depends on the wavelength. That is why gratings are used in spectroscopes for investigating the constitution of light which contains a mixture of wavelengths.

For the experiment of Figure 33–13b, regularly spaced rows of atoms fill the role of the rulings on an ordinary grating. (This is the only way to get ridges comparable in spacing to the wavelength of the helium atoms.) In Figure 33–13b, the surface layer of atoms


helium atoms
scattered waves
Longitudinal Rows producing spots A, B.
Transverse Rows producing vertically displaced spot.

produces the central spot. This spot corresponds to the condition of no difference in path length to and from any of the scattering atoms. It is so located that the angles of incidence and reflection are equal. (As matter of fact, the well-known law of reflection of geometrical optics can be viewed as the result of just such an interference effect.)

Constructive interference at A (and B) arises from superposition of waves scattered to A (and B) by atoms in adjacent longitudinal rows of atoms. The path difference to A (or B) from two such rows is λ. The array of atoms, however, has another set of regularly spaced rows, those marked transverse in the figure. The spot above the central spot corresponds to a path difference of λ from the source, via successive *transverse* rows. The pattern on the screen is thus a *two-dimensional* one of spots of light, rather than one-dimensional bands of light, as with a ruled grating.

With helium atoms, which cannot penetrate the volume of the crystal, the only interaction is with the surface layer of atoms. With *neutrons*, however, which easily travel through this particular crystal, a very special kind of constructive interference involving the whole volume of the crystal is needed to get any scattering at all. The maximum amplitude of the wave scattered from any particular *plane* of atoms will be in the direction of the central spot of Figure 33–13b, but even it will not be very big, because the individual atoms do not scatter neutrons strongly. For very special angles of incidence, illustrated in Figure 33–14b, the amplitudes corresponding to the central spot, arising from all planes of atoms, will be in phase. The scattering will be the coherent sum of scattering from *all of the atoms in the crystal.* The experimental result is illustrated in Figure 33–14c. There will, for certain wavelengths, be other angles at which constructive interference from the whole crystal can occur. The situation is just an elaboration of the one we have here. The angles of the emerging beams give important information about the structure and dimensions of the crystal.

33–9 When Is the Wave Nature of Matter Important?

PURPOSE. To see, quantitatively, when the wave nature of matter is important, and to reconcile it with our everyday experience.

EMPHASIS. It is important that students catch their breath here and digest this material. Much of the significance of the chapter would be lost if this section were skimped.

CONTENT. For various ordinary motion problems the appropriate numerical values for λ, h, and p are inserted into de Broglie's relation, $\lambda = h/p$. In these situations, the masses are large, so that even for the small velocities usually encountered, p is tremendous on an atomic scale. Thus λ will almost always be so minute that any easy observation of the wave properties is impossible.

Electrons, with velocities characteristic of atomic phenomena (about 10^6 m/sec) have wavelengths of the same order of magnitude as atomic dimensions. (This is, of course, not an accident. See COMMENT on this section.) Thus wave properties of *electrons* are important in *atomic physics.* A ripple-tank picture of waves encountering obstacles (Figure 33–15) illustrates how the relative sizes of wave and obstacle determine how the wave will be affected by the obstacle.

Similarly protons and neutrons with energies characteristic of nuclear phenomena have wave properties which are important in encounters with *nuclei.*

Neither of these kinds of phenomena are readily observable in a direct way. One has to perform experiments of great complexity to see the wave properties.

DEVELOPMENT. After exposure to the wealth of fresh factual information that has been presented in Section 8, this section will be useful as an aid to assimilation. Applying de Broglie's relation to familiar situations, as is done in the text, will bring home the order of magnitude of distances, energies, etc., involved in quantum mechanical systems. For example, what is the wavelength of an automobile traveling as slowly as can be measured? Can a batter blame the wave-nature of the baseball when he fails to connect? How important are the wave effects of atoms in a gas? The students will soon see why the wave-nature of matter was so hard to detect.

Any theory of the wave-nature of particles must have the property that when the particle wavelength λ becomes very small compared with the significant dimensions of the problem (the size of the scatterer, for example) the results predicted must be exactly the same as those obtained by classical mechanics. We know this must be so, because experiment shows that ordinary-sized objects do obey Newtonian mechanics. The requirement that the new wave theory must correspond to classical mechanics where λ is negligible is a very important check on any new theory that is suggested, and in quantum mechanics it has been very helpful in deciding which avenues to follow.

For those of your students who have forgotten some of the characteristics of wave phenomena as discussed in Part 2, and for those who by now would like to see something "real," some ripple-tank demonstrations on the scattering of bodies of various size may be refreshing. Students might better understand the importance of the ratio of wavelength to size if you could reproduce the results illustrated in Figure 33–15, and the intermediate cases. Demonstrations illustrating some of the other phenomena of Section 8 are possible with the ripple tank, but they are rather difficult, and are likely to take more time than they are worth at this stage.

COMMENT. That electrons with energies characteristic of atomic phenomena — a few electron volts —

have wavelengths comparable to atomic size is no accident. In Chapter 34, the properties of atoms are examined using the hypothesis that the atomic electrons move in a way governed entirely by wave dynamics. Atoms are the size that they are because they contain electrons, and electrons with energies of a few electron volts have just that wavelength. The same is true, with altered numerical values, for protons and neutrons interacting with nuclei.

33-10 Light and Matter

PURPOSE. To tie together the new picture of matter and the earlier work on light.

CONTENT. For both material particles and photons:

(a) Energy and momentum are conserved in every individual event.

(b) Waves determine the probable location.

(c) When λ is small compared to other relevant dimensions, a Newtonian approximation can be used.

(d) When λ is comparable to other relevant dimensions, a Newtonian approximation is not adequate.

Despite their similarities, photons are distinguished from material particles by the ease with which they can be created and destroyed.

EMPHASIS. Easy, descriptive material. Reading should suffice.

COMMENT. We are now at the stage where the symmetry between the modern pictures of light and matter has been shown. These classically completely distinct entities both behave in similar ways. For example, all of the results and implications of the Taylor experiment in photons (Section 3) apply equally to electrons. The interference pattern of Figure 33–12 is painted by even the lowest intensity electron beam. We can never tell, when we fire an electron into some apparatus, exactly where it will appear at the other end. The shaded pattern of light and dark bands calculated by the matter-wave theory only predicts where we shall *probably* find it, (on the average).

All of this is true for light, too. But photons themselves were a new phenomenon, anyway, so it is hardly surprising that they do not behave in an accurately determinable way. With electrons — particles of matter — this is a radical departure from the comfortably certain Newtonian mechanics of Part 3. It has been, and still is, a cause for discontent and speculation among philosophers and a few physicists. At present, however, the only mechanics we have — quantum mechanics — predicts the probability pattern of any given experiment, but can say no more about any individual trial and its outcome.

The following remarks are more for the teacher's information than for the student's instruction. It is fortunate that some problems involving the wave properties of electrons are amenable to wave arguments of the kind used in Part 2. Chapter 34 is concerned with these, and a good plan at this stage is to minimize the philosophy, and move on to useful calculations which will reinforce the belief in the matter-wave theory. These comments have been included for use in answering searching questions as and when they arise.

Particles and Photons. We have seen that some of the properties of particles are the same as some of the properties of light. The similarity is in some cases remarkable. It is difficult to show the particle properties of electromagnetic radiation, and difficult to show the wave-nature of particles, at least with ordinary laboratory apparatus. But let us be careful to make a fair comparison: between photons and particles of the *same wavelength.* For the electrons which made the interference pattern of Figure 33–12, $\lambda = 8.6 \times 10^{-12}$ m. A photon with that wavelength has an energy of 20,000 electron volts, and X rays with this energy exhibit quite marked particlelike properties in atomic collisions. But the photon and the electron, each with $\lambda = 8.6 \times 10^{-12}$ m, *are* distinguishable. For one thing, an electric field will deflect the electron, but not the photon. The electron has a charge, the photon is electrically neutral. They are different. So let us compare photons and *neutrons*. . . . We can go on in this way, comparing the behavior of photons with that of all the other particles we know about. There will always be some experiment which shows a difference between them. A photon is one kind of entity; we can call it a particle, provided we do not take that word in its classical sense. There are many other kinds of particles whose properties have been ascertained by experiments, and they all differ in some respect, although sometimes the differences are very small and hard to detect. One marked way in which photons differ from all other particles is in the ease with which the less energetic ones can be created. Compare a light bulb, which is creating many photons with wavelengths in the visible region, with an electrical circuit. In the circuit, passage of electrons is observed. You can never observe effects unless you make a continuous circuit for current to *flow* around. Electrons cannot be created at one end of a wire and destroyed at the other, producing a current. Their number stays constant. On the basis of what we have seen in the lab, this is then a very marked difference between photons and electrons. The other particles we have met — neutrons, protons, helium atoms — share the electron's property of being conserved in ordinary situations.

It is in fact possible to create electrons, neutrons, protons, etc. Readers of popular science magazines may have encountered antimatter; antielectrons (called positrons), antiprotons, antineutrons. It is not possible to create a single electron, for example, but a pair consisting of an electron and a positron can be created, provided the right conditions exist. A posi-

tron is a quite different particle from an electron — it has the opposite electric charge. More important, on meeting any other electron, both of the particles disappear, leaving behind energy in the form of photons. To understand the situation one needs Einstein's theory of relativity, but the facts are as follows. For the creation of a pair, an amount of energy equal to their combined mass times the square of the velocity of light is needed. Even for such light particles as electrons, an electron-positron pair requires an energy of 10^6 electron volts. So it is not *easy* to create particles. On the other hand, photons can be created *singly*, and the creation of a photon with wavelength in the visible region requires only about two electron volts of energy. With the right experimental arrangement, such photons are quite easily created; a light bulb does quite well. So in the distinguishing feature mentioned in the text, the emphasis is on the *ease* with which photons can be created. The antiparticles, by the way, are not only hard to create, they are very easy to destroy. That is why one does not encounter them in ordinary situations. The particles with which they annihilate are present in great abundance in ordinary matter.

33–11 What It Is That Waves

EMPHASIS. Reading only.

DEVELOPMENT. We have met waves, in the ripple tank for example, as a type of disturbance of a system. The system, or substance, has many other measurable properties. We now know about electromagnetic waves and matter waves, both of which are propagated in a vacuum. It is natural to ask about other properties of the vacuum, and to think that perhaps it has properties like those of our well-known wave carriers (the water surface). But there is no need for such properties to exist. The vacuum has the property that it can carry electromagnetic and matter waves, and that is sufficient.

COMMENT. The attitude among most present-day physicists toward this question is described in the text. It can be elaborated as follows: The theories of light (electrodynamics) and of the atomic particles (quantum mechanics) are complete mathematical and physical schemes, so far as they go. The result of any proposed experiment can be predicted by them, and that is what we want of any theory. The quantities in terms of which the theory is formulated — electric field strengths, say, for electrodynamics, or "probability amplitudes" for quantum mechanics — exist (i.e., can be nonzero) throughout all space. These are the quantities we manipulate — the field strength or probability amplitude — and the "vacuum" is just the name we give to all of empty space. It is not an entity in our theory. Its properties are solely that it can support these field strengths or probability amplitudes. Nothing more *need* be said of it in order to use electrodynamics or the quantum theory of particles. Whether or not the "vacuum" has other properties is at present not known. Are there other kinds of waves which can be propagated through a vacuum, waves associated with phenomena which have been so far undetectable? Are there measurements which can be made which involve the vacuum in a more active way than those we know how to make? There could be. Future scientists may find such things. We don't know.

Nonetheless, students will be puzzled by the question "What is it that waves?" The only examples of wave motion that they have encountered occur in systems with many other properties, and it is natural to wish to transfer all attributes of the models to the actual system under study. Clearly, in using a model you have to know where the similarity between model and actual system stops. Any answer of this kind is somewhat unsatisfying, unfortunately, until the students understand and can work with the actual theory of electrodynamics. They will then no longer need models and will gradually realize that the question "What is it that waves?" does not have to be answered in order to understand all that we know at present about the universe.

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The table below classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1.* The time at which a first jump occurred would always be later for weak light.
2. Use a long exposure time.
3.* (a) No; (b) Yes.
4. The photons of red light do not have enough energy to produce changes in this photographic emulsion.
5.* The points corresponding to frequencies of 6.8 and 7.4×10^{14} cycles/sec.
6.* 1.4 ev.
7. (a) 4.0×10^{-19} joule;
   (b) 8.0×10^{-7} sec;
   (c) 240 meters;
   (d) 200 trials.
8. See graph and discussion on page 33–16.
9. Measure the wavelength of the reflected light.
10. (a) 4.6×10^{14} cycles/sec;
    (b) 3.0×10^{-19} joule;
    (c) 1.9 ev;
    (d) 4000 to 7000 Å; 3.1 and 1.8 ev, respectively.
11. 0.5 volt.
12. (a) 6.6×10^{-26} joule;
    (b) 10^{29} photons/sec;
    (c) 10^{14} photons;
    (d) No.
13. (a) 4.0×10^{-19} joule; 1.3×10^{-27} kg-m/sec;
    (b) 2.5×10^{18} photons/sec;
    (c) 6.5×10^{-11} newton/m^2.
14. (a) The minimum wavelength corresponds to a full-energy electron being stopped and giving up its energy to a single photon.
    (b) 0.620 angstrom.
15. (a) See discussion on page 33–19;
    (b) 6.62×10^{-24} kg-m/sec;
    (c) 4.73×10^{-17} joule;
    (d) 1.94×10^{-15} joule;
    (e) By 2.4 percent.
16. (a) $\Delta p = 2h\nu/c$;
    (b) $N(2h\nu/c)$;
    (c) $2E/c$; No difference;
    (d) 0.7×10^{-5} newton/m^2.
17. 10^4 ev.
18.* $\lambda = 0.73$ angstrom.
19.* $2d \sin\theta = 2\lambda$.
20. (a) 0.36 m;
    (b) No difference, only there are more of them.
21. Spacing reduced to 1/10 of its original value.
22. (a) 2.9×10^{-13} m, 2.9×10^{-14} m, 2.9×10^{-15} m;
    (b) Larger, about the same, smaller;
    (c) (i) 10^8-ev neutrons, (ii) 10^4-ev neutrons;
    (d) 10^8-ev neutrons.
23. (a) 0.31 A;
    (b) 1600 ev;
    (c) 1.0×10^{-26} ev;
    (d) 4.7×10^{-24} A.
24. (a) $\lambda = h/(2mE_K)^{1/2}$;
    (b) $\lambda = 12/(V)^{1/2}$ angstrom;
    (c) 4×10^{-6} volt.
25. (a) 250 angstrom;
    (b) diameter/λ = 1/20; disturbance dies out after only a few wavelengths;
    (c) See discussion on page 33–23.

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1, 2, 3, 4	1, 3	2, 4			
5, 6, 7	5, 6	9	7, 8#, 10, 11	12, 13, 14, 15, 16#, 17	8#, 10
8	18, 19		21	20#	20#
9			23, 25	22#, 24	

PROBLEM 1* (See Short Answers.)

PROBLEM 2

In a certain camera, at dusk, the light that gets through the lens is so dim that in 1/100 second one photon on the average lands in 10 mm^2 of the film. If you want to take a finely detailed, sharp picture, what should you do?

To take a finely detailed picture we need more photons. To get them we must use a long exposure time.

PROBLEM 3* (See Short Answers.)

PROBLEM 4

Some photographic materials can be handled safely in red light but are spoiled instantly when white light is turned on. How do you account for this?

Light of shorter wavelength is more effective in spoiling photographic materials, just as it is in ejecting electrons from metals. White light contains a mixture of colors from red to violet. Hence, it contains shorter wavelengths than red light alone.

In terms of photons, the white light contains photons of higher energy than red light. When these are absorbed, they can produce changes in materials which cannot be produced by photons of red light.

PROBLEM 5* (See Short Answers.)

PROBLEM 6* (See Short Answers.)

PROBLEM 7

In the three-month experiment described in Section 33–3, G. I. Taylor estimated the energy of the light reaching his photographic plate. He found it was 5×10^{-13} joule per second. (He obtained that value by comparing the average blackening of the plate with that produced in 10 seconds by a candle 2 yards away, without any absorbing screens of smoked glass.) From that estimate, calculate the average distance between photons, as follows:

(a) Assume the wavelength of the useful light was about 5000 angstrom units. What energy did each photon carry?

(b) Calculate from the given flow of energy the average time that elapsed between the arrival of one photon and the next.

(c) From the average time, calculate the average distance between one photon and the next in the beam of light.

(d) If you were asked, "How many photons are in the box at any chosen instant," you would have to answer "None" most of the time, but on looking again and again and again, you should expect to "see" one photon in the box about once in so many trials. How many? Assume the box was 1.2 meters long.

(a) If $\lambda = 5000$ A, the energy of a simple photon is $E = h\nu = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{5000 \times 10^{-10}} = \underline{4.0 \times 10^{-19} \text{ joule}}$.

(b) The average number of photons arriving per second, or N, equals the energy arriving per second divided by the energy of a single photon. The average time between the arrival of each photon is

$$\frac{1}{N} = \frac{4.0 \times 10^{-19} \text{ joule}}{5 \times 10^{-13} \text{ joule/sec}} = \underline{8 \times 10^{-7} \text{ sec}}.$$

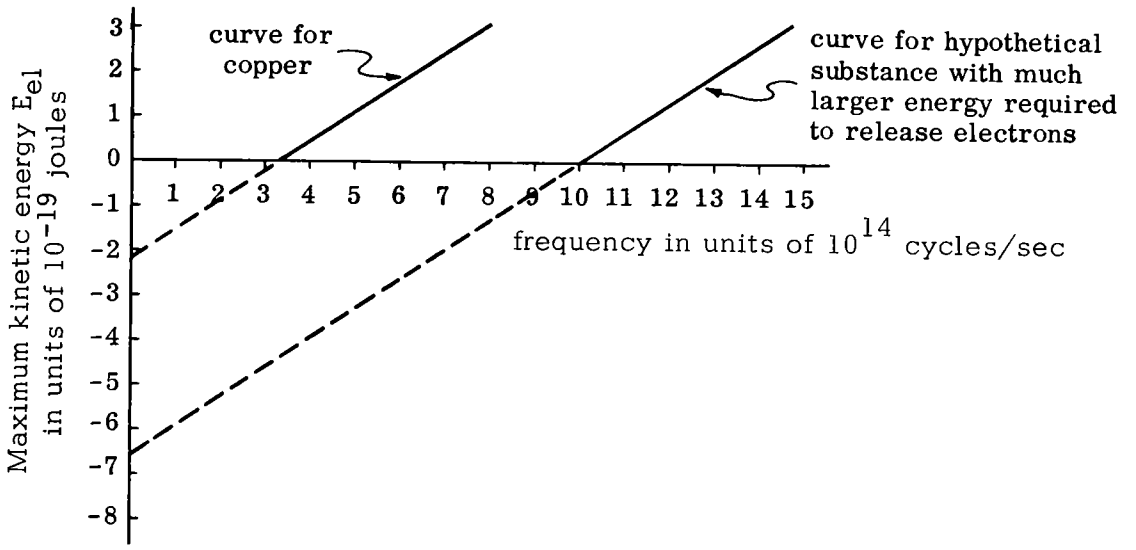
(c) Since photons in air travel at 3×10^{8} m/sec and they arrive, on the average, 8.0×10^{-7} sec apart, if the beam were in space, the photons would be separated by 8.0×10^{-7} sec $\times$ 3×10^{8} m/sec = <u>240 meters</u>.

(d) To answer this question, it is helpful to picture a beam in space with an average spacing between photons of 240 meters, but with any individual photon likely to be anywhere along the beam at any instant of time because of the random nature of the emission process. Then if you make the further assumption that the longitudinal extent of a photon is very short, you can make the following argument. A box 1.2 meters long placed in the beam extends for 1/200 of the average distance between photons. Therefore, when you look in the box at any instant of time you will see a photon only once in every <u>200 times</u> that you look.

PROBLEM 8

Copy Fig. 33–9 in a rough sketch, with any scale you like. Draw another graph with the same scale for a different photoelectric surface, for example, a metal that requires two or three times as much energy to remove an electron.

All of the photoelectric straight-line graphs have the same slope. If three times as much energy is required to remove an electron from some given metal as is required for copper, then the frequency of light which will just remove electrons from that metal is three times higher than the corresponding frequency for copper. The threshold frequency for copper was shown in Figure 33–9 of the text as 3.3×10^{14} cycles/sec. We have drawn a curve for which the threshold frequency is 10×10^{14} cycles/sec.



PROBLEM 9

A good mirror reflects about 80 percent of the light which hits it. How could you determine whether 20 percent of the photons were not reflected, or whether all of the photons were reflected but each with 20 percent less energy?

If each photon lost 20% of its energy, the color of the reflected light would be shifted to that of a longer wavelength than the incident light. Since the reflected light has the same color as the incident light, we conclude that 20% of the photons are absorbed completely.

PROBLEM 10

Red light has a wavelength of about 6500 angstroms.

(a) Estimate its frequency.

(b) Estimate the energy in joules of one photon of red light.

(c) Estimate the energy in electron volts of one such photon.

(d) Find out approximately the longest and shortest wavelengths you can see, and compute the energy of the corresponding photons.

(a) $\nu = \dfrac{c}{\lambda} = \dfrac{3 \times 10^8 \text{ m/sec}}{6.5 \times 10^{-7} \text{ m}}$

$= \underline{4.6 \times 10^{14} \text{ cycles/sec.}}$

(b) $E = h\nu = 6.6 \times 10^{-34} \text{ joule-sec} \times 4.6 \times 10^{14} \text{ cycles/sec} = \underline{3.0 \times 10^{-19} \text{ joule.}}$

(c) $E = 3.0 \times 10^{-19} \text{ joule} \times \dfrac{1 \text{ ev}}{1.6 \times 10^{-19} \text{ joule}}$

$= \underline{1.9 \text{ ev.}}$

Or $E = \dfrac{12{,}397}{\lambda} = \dfrac{12{,}397}{6{,}500} = 1.9$ ev (see the box on page 622 of the text).

(d) You can see light of wavelengths from 4000 A (violet) to 7000 A (red). Using the fact that $E = h\nu = hc/\lambda$, we can set up a proportion to find the energies of the 4000-A and 7000-A photons from the energy of the 6500-A photons computed above.

$$\frac{E_{7000}}{E_{6500}} = \frac{6500}{7000};$$

$$E_{7000} = 1.9 \times \frac{6500}{7000} = \underline{1.8 \text{ ev.}}$$

$$\frac{E_{4000}}{E_{6500}} = \frac{6500}{4000};$$

$$E_{4000} = 1.9 \times \frac{6500}{4000} = \underline{3.1 \text{ ev.}}$$

PROBLEM 11

Light of 5000-angstrom wavelength illuminates a surface. What voltage is needed to stop all the photoelectrons emitted from the surface if their binding energy to the surface is 2.0 electron volts?

The energy of the 5000-A photons is $E = \dfrac{12{,}397}{5{,}000} =$ 2.5 ev. (See Problem 10 or the box on page 622.) Part of this energy, 2.0 ev, is used up getting the electron out of the metal, while the remainder, 0.5 ev, remains with the fastest electrons as kinetic energy. It therefore will take a retarding potential of <u>0.5 volt</u> to stop these electrons. (Most of the electrons which

emerge lose some of this 0.5 ev of energy in collisions inside the solid.)

PROBLEM 12

A typical FM radio station broadcasts with a wavelength of 3 meters. Estimate:

(a) the energy in one photon of this radiation.

(b) the number of photons broadcast per second if the radiated power is 10 kilowatts.

(c) roughly the number of photons received by a receiving set during one vibration of the letter "s" in the audio pattern, if the receiver is 100 km from the station. (Make the following simplifying assumptions:

(i) the radiation spreads uniformly in all directions along radii from the source so that an inverse-square law holds for energy flow — this is not true for any ordinary antenna.

(ii) the receiving antenna collects radiation from an area of 1 meter^2 perpendicular to the line of travel of waves.

(iii) the sound of the letter "s" involves air oscillations of frequency about 4000 cycles per second.)

(d) Will you ordinarily notice the individual photons in receiving the letter "s"?

(a) $E = h\nu = h(c/\lambda) = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{3}$

$= \underline{6.6 \times 10^{-26}\ \text{joule}}.$

(b) If each photon carries about 10^{-25} joule, the number of photons required to produce 10^4 joules is

$$\frac{10^4\ \text{joules}}{10^{-25}\ \text{joule}} = 10^{29}.$$

Consequently, $\underline{10^{29}\ \text{photons/sec}}$ are required to produce 10^4 joules/sec (or 10 kilowatts).

(c) When the radio waves get out to a radius of 100 km, their total energy is spread over an area A equal to the surface of a sphere of 100-km radius:

$$A = 4\pi R^2 = 4\pi \times 10^{10}\ \text{m}^2.$$

A portion of this surface with an area of 1 m^2 will capture a fraction equal to 1 m^2/($4\pi \times 10^{10}$ m^2) of the total energy. The receiver gets this fraction of the 10^{29} photons/sec emitted by the transmitter, or $1/(4\pi \times 10^{10}) \times 10^{29} \approx 10^{18}$ photons/sec.

One vibration of the sound "s" lasts 1/4000 sec; so 1/4000 sec $\times\ 10^{18}$ photons/sec $\approx$ $\underline{10^{14}\ \text{photons}}$ arrive at the receiver during one vibration of the letter "s."

(d) You would *not* notice one photon in 10^{14}!

PROBLEM 13

(a) How much energy is carried by an "average" photon of visible light with wavelength of about 5000 angstroms? How much momentum?

(b) Estimate the number of photons of visible light emitted per second from a 100-watt light bulb emitting 1 percent of its power in the visible region.

(c) What is the light pressure exerted by the photons from that bulb when they strike a black body 2 meters away head on?

This problem asks for the calculation of the pressure of light, which was discussed in Part II and in the film "Pressure of Light."

(a) $E = h\nu = h(c/\lambda) = 6.6 \times 10^{-34} \times (3 \times 10^{8})/(5000 \times 10^{-10}) = \underline{4.0 \times 10^{-19}\ \text{joule}}.$

$$p = \frac{E}{c} = \frac{4.0 \times 10^{-19}}{3.0 \times 10^{8}} = \underline{1.3 \times 10^{-27}\ \text{kg-m/sec}}.$$

Alternatively,

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{5000 \times 10^{-10}} = 1.3 \times 10^{-27}\ \text{kg-m/sec}.$$

(b) One percent of 100 watts, or one watt, is emitted in the visible region. One watt is one joule/sec, so the number of visible light photons emitted per second is

$$\frac{1\ \text{joule/sec}}{4.0 \times 10^{-19}\ \text{joule/photon}} = \underline{2.5 \times 10^{18}\ \text{photons/sec}}.$$

(c) The fraction of these photons that strike an area A at a distance of 2 meters is equal to the fraction of the area of a sphere of 2 meters radius that the area A will cover, or $A/4\pi R^2 = A/16\pi$. Hence $A/16\pi \times 2.5 \times 10^{18}$ photons per second strike the area A. Each photon carries 1.3×10^{-27} kg-m/sec of momentum, so the total momentum imparted to the area A each second is $A/16\pi \times 2.5 \times 10^{18} \times 1.3 \times 10^{-27}$ kg-m/sec^2. Now the momentum gained by an object in a given time is called the impulse (see Part III), and equals the average force acting on the object multiplied by the time: $\Delta p = F\Delta t$. Hence, in this case, $6.5 \times 10^{-11} A = F \times 1$. The pressure, P, is the force per unit area, or $P = F/A$, so

$$P = \frac{F}{A} = \underline{6.5 \times 10^{-11}\ \text{newton/m}^2}.$$

(Compare this with the atmospheric pressure, about 10^5 newtons/m^2.)

PROBLEM 14

In a particular X-ray tube electrons hit a target after they have been accelerated by a potential difference of 20,000 volts. As they decelerate to a stop in the target, a few of them emit X-ray photons.

(a) Why is there a definite minimum wavelength of photons observed from this tube? (This limit is known as the Duane-Hunt limit.)

(b) Calculate that minimum wavelength.

(a) Since each X ray is generated by a single electron, the maximum energy that a single X ray can have is equal to the kinetic energy that the electron gives up as it is decelerated to a stop in the target. The wavelength corresponding to this maximum energy is

$$\lambda = \frac{c}{\nu} = \frac{hc}{E_{max}}.$$

Any smaller wavelength would give a larger photon energy than is available.

(b) $$\lambda = \frac{1.24 \times 10^4 \text{ ev} - \text{A}}{2.00 \times 10^4 \text{ ev}} = \underline{0.620 \text{ angstrom}}.$$

PROBLEM 15

A photon of wavelength $\lambda = 1.00$ A hits an electron "at rest." The photon bounces away at right angles to its original path.

(a) Draw a vector diagram of the photon momentum before the collision and the photon and electron momentum after the collision.

(*Note:* For such a collision in the fractional loss of energy is small: ν' is nearly equal to ν, so in drawing a vector diagram for momenta you can take the momentum of the photon after collision to have practically the same magnitude as the momentum of the photon before collision when you are finding the direction of the electron's recoil. Taking the electron recoil given by that assumption, use conservation of momentum and conservation of energy to compute the answers asked for.)

(b) Calculate the momentum of the photon after the collision.

(c) Calculate the kinetic energy of the electron after the collision.

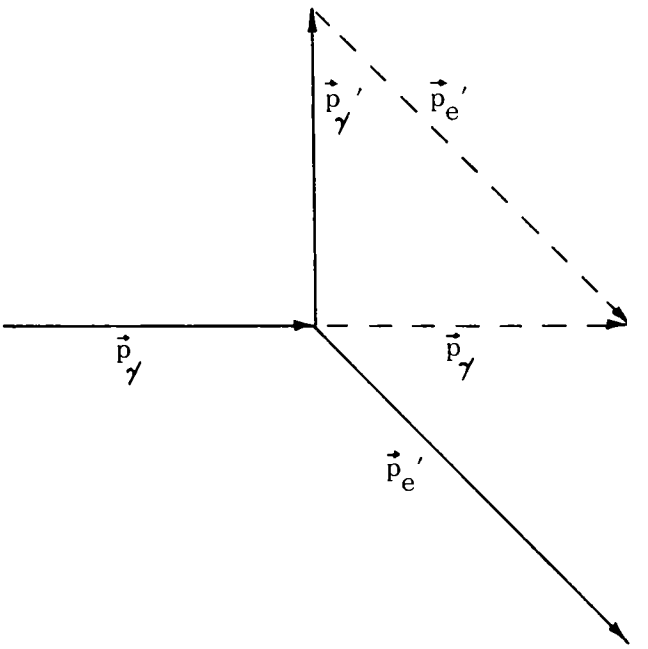
(d) Calculate the energy of the photon after the collision.

(e) Is ν' very different from ν?

(a) Let $_\gamma\vec{\mathbf{p}}$ be the photon momentum before the collision. It is stated that the photon bounces away from the collision at right angles to its original trajectory and with a final momentum whose magnitude can be taken to be the same as the initial momentum. Thus $\vec{\mathbf{p}}'_\gamma$, below, represents the final photon momentum. To conserve momentum, the electron must acquire momentum $\vec{\mathbf{p}}'_e$ such that $\vec{\mathbf{p}}'_\gamma + \vec{\mathbf{p}}'_e = \vec{\mathbf{p}}_\gamma$ as shown in the vector diagram below.

(b) The magnitudes of the initial and final momenta of the photon are obtained from the wavelength λ:

$$p'_\gamma \approx p_\gamma = \frac{h}{\lambda} = 6.62 \times 10^{-34}/1.00 \times 10^{-10}$$
$$= \underline{6.62 \times 10^{-24} \text{ kg-m/sec}}.$$



(c) From the sketch in the answer to part (a), we see by simple geometry that $p'_e \approx \sqrt{2}\, p_\gamma$. Then

$$E'_e = p'^2_e/2m_e = (\sqrt{2}\, p_\gamma)^2/2m_e$$
$$= \frac{2 \times (6.62 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} \text{ joule}$$
$$= \underline{4.73 \times 10^{-17} \text{ joule}}.$$

(d) By conservation of energy $E_\gamma = E'_\gamma + E'_e$,

$$E_\gamma = h\nu = \frac{hc}{\lambda} = cp_\gamma$$
$$= 3.00 \times 10^8 \times 6.62 \times 10^{-24} \text{ joule}$$
$$= 1.99 \times 10^{-15} \text{ joule}.$$
$$E'_\gamma = E_\gamma - E_e$$
$$= (1.99 - 0.05) \times 10^{-15} \text{ joule}$$
$$= \underline{1.94 \times 10^{-15} \text{ joule}}.$$

The change in photon energy is very small (2.4 percent).

(e) Since $E_\gamma = h\nu$ and $E'_\gamma = h\nu'$,

$$\frac{\nu - \nu'}{\nu} = \frac{E_\gamma - E'_\gamma}{E_\gamma}.$$

The relative change in frequency is equal to the relative change in energy of the photon, i.e., 2.4 percent. This justifies the initial assumption in (a) that the change in the photon's frequency is so small that it can be neglected in calculating p'_e.

PROBLEM 16

A plane mirror is illuminated perpendicularly to its surface by light of frequency ν. The surface of the mirror is hit by N photons per unit time and per unit area.

(a) What is the change in momentum of a single photon when it is reflected by the mirror?

(b) What is the pressure of light on this mirror?

(c) Express the pressure in terms of the total energy E falling on the mirror per unit time and per unit area. Would it make any difference if the light, instead of being monochromatic, were spread over a whole range of frequencies?

(d) What is the numerical value of this pressure, assuming that the intensity of illumination is 10^3 joules/m^2-sec? (This intensity corresponds roughly to direct sunlight illumination at noon in summertime.)

(a) For one photon with initial momentum p reflected perpendicularly from the mirror, the magnitude of the change in momentum is $\underline{\Delta p = 2p = 2h\nu/c}$.

(b) The pressure exerted on the mirror is equal to the total change in momentum per unit time and unit area. Pressure $= N\,\Delta p = \underline{N(2h\nu/c)}$.

(c) Total energy per unit time and unit area is $E = Nh\nu$, so the pressure $= \underline{2E/c}$. Hence, the light pressure due to N photons per unit time and unit area, whose frequencies are spread over a range, can be the same as the light pressure from N photons per unit time and unit area of a single frequency ν. All that is necessary is that the average frequency of the non-monochromatic photons be equal to ν.

(d) Pressure $= (2 \times 10^3)/(3 \times 10^8)$
$= \underline{0.7 \times 10^{-5} \text{ newton/m}^2}$.

PROBLEM 17

The layers of atoms in crystals are known (from density and Avogadro's number) to be a few angstroms apart, say 3×10^{-10} m. X rays from ordinary commercial X-ray tubes are found to be diffracted at quite large angles, such as 10°, by such crystals acting as interference gratings. Make a rough estimate of the energy of photons in such X rays.

From Figure 33–14(b) of the text we see that $\lambda = 2d \sin\theta$. Since $\sin 10° \approx 0.17$, then $\lambda = 2(3 \times 10^{-10}\text{ m})(0.17) = 1 \times 10^{-10}$ m $\approx$ 1 angstrom unit. The photon energy is given by $E = h(c/\lambda) = 1.24 \times 10^4$ ev-A/1 A $\approx \underline{10^4 \text{ ev}}$.

PROBLEM 18* (See Short Answers.)

PROBLEM 19* (See Short Answers.)

PROBLEM 20

A narrow stream of 100-ev electrons is fired at two parallel slits very close together. The distance between the slits is estimated to be 10 angstrom units. The electrons passing through the slits reach a screen 3 meters away, and form a pattern of interference fringes.

[It seems almost impossible to make such a pair of slits in any real sheet of matter, made of atoms which are themselves 1 angstrom or more in diameter, and even more impossible to measure their separation, since light has a wavelength of thousands of angstroms. Yet the equivalent of this pair of slits can be made — has been made and used. It is the "biprism" of Fig. 13–12(b).]

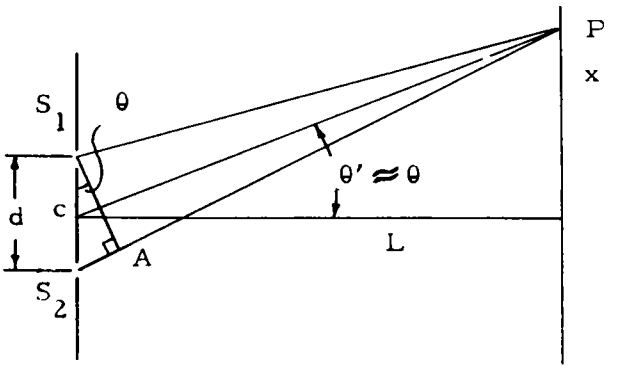
(a) Estimate the distance between one bright fringe and the next.

(b) The bright fringes would appear as bright marks on a fluorescent screen sufficiently sensitive to glow when bombarded with 100-ev electrons (or as dark marks on a photographic film, or as large pointer readings on some meter attached to an electron collector). What is the essential difference between the bright-fringe electrons and those that reach the screen in a dark fringe: Are they bigger? More massive? Of better quality? Of greater charge, or what?

(a) The analysis of two-slit interference has been given in Section 17–4. Referring to the figure

at the right, bright fringes occur at P whenever the length of AS_2 is equal to a whole number of de Broglie wavelengths of the 100-ev electrons, i.e., $AS_2 = n\lambda$, where $n = 0, 1, 2, 3$, etc. So, from Section 17–4, we have $x = n\lambda L/d$, and the distance of separation on the screen between fringes is $\Delta x = \lambda L/d$. For the electrons,

$$\lambda = h/p = h/\sqrt{2mE}.$$



Hence,

$$\Delta x = \frac{hL}{d\sqrt{2mE}} = \frac{(6.6 \times 10^{-34}\ \text{joule-sec})(3\ \text{m})}{(10 \times 10^{-10}\ \text{m})\sqrt{2(9.1 \times 10^{-31}\ \text{kg})(100\ \text{ev})(1.6 \times 10^{-19}\ \text{joule/ev})}}$$
$$= \underline{0.36\ \text{m}}.$$

(b) There is no difference whatever between the bright fringe electrons and those that reach the screen in the dark fringe. It is only that more electrons arrive at the bright fringes (this is what makes the fringes bright) than arrive at the dark fringes. The fact that the dimensions of the diffracting slits are comparable to the de Broglie wavelength of the electron means that the behavior of the electrons is dominated by their wavelike nature. This means that the position at which the electrons will arrive is governed by a complicated probability pattern. The particular destination, within this pattern, of any individual electron is unpredictable; its mass, charge, energy, and size, are, however, identical with those of all of the rest of the 100-ev electrons.

PROBLEM 21

Electrons passing through a double slit (fantastic but possible, as discussed in the previous question) make an interference pattern on a remote screen. If the gun voltage used to provide the original stream of electrons with kinetic energy is changed from 50 volts to 5000, what does this do to the spacing of the fringes?

In the discussion of Problem 20 we showed that the spacing, Δx, between bright fringes was given by $\Delta x = hL/d\sqrt{2mE}$, so the spacing Δx is inversely proportional to $\sqrt{E}$. Changing the energy of a beam of electrons from 50 to 5000 electron volts increases E by a factor of 100, $\sqrt{E}$ by a factor of 10, and therefore reduces Δx tenfold.

PROBLEM 22

The sizes of atomic nuclei are hard to measure with alpha particles, because we have to look for small differences from Coulomb scattering. Neutrons are often used.

(a) What are the de Broglie wavelengths of neutrons with kinetic energy 10^4 electron volts? 10^6 electron volts? 10^8 electron volts?

(b) Compare these wavelengths with the diameter of a gold nucleus (15×10^{-15} m).

(c) Look back at Fig. 33–15 to see how waves behave when they meet an obstacle. Suppose we fire a stream of neutrons at a small target of gold. For which of the neutron waves in (a) do you expect to find

(i) most of the neutrons pass by the nuclei undisturbed while some are absorbed and some bounce off, all like balls in Newtonian mechanics?

(ii) most of the neutrons pass the nucleus apparently undisturbed and the rest come out of the gold target equally in all directions, regardless of the shape of the nucleus?

This problem introduces some ideas and orders of magnitude of nuclear physics.

(a) The mass of a neutron is approximately equal to the mass of the proton, 1.7×10^{-27} kg. Consider the case $E = 10^4$ electron volts = 1.6×10^{-15} joule. Then,

$$p = \sqrt{2m_nE}$$
$$= \sqrt{2 \times 1.7 \times 10^{-27} \times 1.6 \times 10^{-15}}\ \text{kg-m/sec}$$
$$= 2.3 \times 10^{-21}\ \text{kg-m/sec}.$$

The de Broglie relationship gives us that

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}\ \text{joule-sec}}{2.3 \times 10^{-21}\ \text{kg-m/sec}}$$
$$= \underline{2.9 \times 10^{-13}\ \text{m}}.$$

The other values of E increase by factors of 10^2 and 10^4, and since $\lambda \propto p^{-1}$, i.e. $\lambda \propto$

$E^{-1/2}$, λ will be decreased by factors of 10 and 100, respectively.

(b) In tabular form, the results are as follows:

E	λ	λ/diameter of gold nucleus
10^4 ev	2.85×10^{-13} m	19
10^6 ev	2.85×10^{-14} m	1.9
10^8 ev	2.85×10^{-15} m	0.19

(c) (i) When the wavelength of a "particle" is small compared with the objects it encounters, it tends to behave like a classical particle. In this case, the 10^8-ev neutrons would behave in this way. Since the nucleus is not another billiard ball, but a complicated congregation of nucleons, the incident neutron may sometimes be absorbed into the nuclear motion, instead of scattering off the surface like a billiard ball.

(ii) It is at the other extreme, where λ is much greater than the size of the object, that the incident particle will not "see" the detailed structure of the scatterer, but for all cases comes out as a spherical wave. Here the 10^4-ev neutrons would do this.

In both cases most of the neutrons will not be affected by the scatterer, but for different reasons: in (i), the geometric area of the nuclei is such a tiny part of the target area. For (ii), the overall influence of the scattering object on the wave vanishes as its size becomes small — there just isn't much surface area for the waves to slap against.

(d) Since the 10^8-ev neutrons behave somewhat like billiard balls in colliding with gold nuclei, they would be better for measuring nuclear size than the other two groups. By counting the number of neutrons per m^2 in the neutron beam and the number absorbed or deflected from the beam by a thin sheet of gold nuclei, we could calculate the fraction of the square meter covered by the gold nuclei. Then if we computed the number of nuclei by weighing the sheet of gold, we could get the area per nucleus.

PROBLEM 23

(a) What is the wavelength of X rays whose photons each carry 40,000 electron volts of energy?

(b) At about what energy do electrons have a de Broglie wavelength equal to that of 40,000-volt X rays? (Give your answer in electron volts.)

(c) What energy baseballs?

(d) What is the wavelength of a baseball moving at 10 meters per second?

(a) Using the information in the box on page 622 that $\lambda = 1.24 \times 10^4/E$, we find that 40,000-ev rays have wavelengths of $(1.24 \times 10^4)/40{,}000 = \underline{0.31\ \text{A}}$.

(b) Using the de Broglie relationship, the momentum p of *any* particle with this wavelength is given by

$$p = h/\lambda = 6.62 \times 10^{-34}\ \text{joule-sec}/0.31 \times 10^{-10}\ m = 2.13 \times 10^{-23}\ \text{kg-m/sec}.$$

For an electron, $m_e = 9.11 \times 10^{-31}$ kg,

$$E_e = p^2/2m_e = \frac{(2.13 \times 10^{-23})^2}{2 \times 9.11 \times 10^{-31}} = 2.49 \times 10^{-16}\ \text{joule} = 1.56 \times 10^3\ \text{ev}.$$

Thus electrons with this wavelength must have an energy of about 1600 ev.

(c) For a baseball, the momentum, p, is the same. The mass of a standard baseball, m_b, is five ounces or 0.14 kg. Since, with constant p, $E \propto 1/m$, we can obtain the baseball energy, E_b, by proportion:

$$\frac{E_b}{E_e} = \frac{m_e}{m_b};$$

$$E_b = 2.5 \times 10^{-16}\ \text{joule} \times \frac{9.1 \times 10^{-31}}{0.14} = \underline{1.6 \times 10^{-45}\ \text{joule, or } 1.0 \times 10^{-26}\ \text{ev}}.$$

The velocity of this baseball can be obtained from p:

$$v_b = p/m_b = \frac{2.13 \times 10^{-23}}{0.14}\ \text{m/sec} = 1.5 \times 10^{-22}\ \text{m/sec}.$$

(d) If $v_b = 10$ m/sec, then $p = mv_b = 1.4$ kg-m/sec. Then,

$$\lambda = \frac{h}{p} = \frac{6.60 \times 10^{-34}}{1.4} = 4.7 \times 10^{-34}\ \text{m} = \underline{4.7 \times 10^{-24}\ \text{A}}.$$

The moral is clear: we never have to worry about the wave-nature of baseballs.

PROBLEM 24

The relation between photon energy in electron volts and the wavelength of light in angstroms is shown in the box on page 622.

(a) Work out a similar relation between the kinetic energy of a particle and its de Broglie wavelength.

(b) If the particle is an electron and its kinetic energy is generated by accelerating it through a potential difference, relate the potential difference in

volts to the de Broglie wavelength of the electron in angstroms.

(c) What is the energy of electrons whose wavelength is the same as that of yellow light?

(a) The relation between the kinetic energy E_K of a particle and its momentum p is

$$p = (2mE_K)^{1/2}.$$

Since $\lambda = h/p$, the desired relation is

$$\underline{\lambda = \frac{h}{(2mE_K)^{1/2}}}.$$

(b) The kinetic energy of an electron which is accelerated through a potential difference V is $E_K = qV$, where q is the charge on the electron. Hence, if V is in volts, the wavelength in meters is

$$\lambda = \frac{h}{\sqrt{2mqV}}$$
$$= \frac{6.6 \times 10^{-34}\ \text{joule-sec}}{\sqrt{2(9.1 \times 10^{-31}\ \text{kg})(1.6 \times 10^{-19}\ \text{coul})V}}$$
$$= \frac{1.2 \times 10^{-9}}{\sqrt{V}}\ \text{meter},$$

and λ in angstroms is thus

$$\underline{\lambda = \frac{12}{\sqrt{V}}\ \text{angstroms.}}$$

This formula will give a good approximation of the wavelengths for electrons of energies up to 2500 electron volts, where the electron has then a velocity close to 1/10 the velocity of light; above this energy, relativity becomes important.

(c) From part (b), if λ is in angstroms then

$$V = \frac{(12)^2}{\lambda^2} = \frac{144}{(5900)^2} = \underline{4 \times 10^{-6}\ \text{volt.}}$$

Note: This extremely small voltage explains why we cannot produce electron waves of wavelengths comparable to those of visible light. Any beam of electrons from which we may start has a spread in energy large compared to 4×10^{-6} ev; electrons "at rest" at room temperature have a mean kinetic energy of 2.5×10^{-2} ev.

PROBLEM 25

Figure 8-2 shows an electron-micrograph of protein molecules.

(a) From that picture calculate the size of the molecules.

(b) Find the ratio of the diameter of a molecule to the wavelength of visible light. Why cannot a molecule be seen with visible light?

(c) Why is it possible to photograph the protein molecules by using electrons instead of light?

(a) In the picture each molecule has a diameter of about 2 mm. The magnification is about 8×10^4, so the diameter of a molecule is

$$\frac{2 \times 10^{-3}\ \text{m}}{8 \times 10^4} = 2.5 \times 10^{-8}\ \text{m} = \underline{250\ \text{angstroms.}}$$

(b) Diameter/wavelength

$$= (2.5 \times 10^{-8})/(5 \times 10^{-7}) = 1/20.$$

The effect of the molecule on visible light would be similar to that of the small object in the ripple tank shown in the left-hand photograph of Figure 33-15, where the disturbance vanishes within a couple of wavelengths from the object.

(c) Electrons in the electron microscope have wavelengths which are small compared with the size of the molecule. For example, from the preceding problem we find for $V = 100$ volts that $\lambda = 1.2$ angstroms. This is a sufficiently short wavelength to resolve points closer than a tenth the diameter of a protein molecule, and reveal its spherical shape. However, electrons of 100 ev have very little penetrating power. To get the needed penetration, electron microscopes operate from a few thousand volts up to as high as 300,000 volts for examining thick objects.

chapter
34

Quantum Systems and the Structure of Atoms

This chapter is the culmination of the work of the course. In it, ideas from the fields of mechanics, physical optics, and electricity are brought to bear on some of the paradoxes that were faced by physicists at the beginning of the twentieth century. From the successes of treatments of this type, modern physics was born, the quantum theory and wave mechanics were developed.

CHAPTER SUMMARY

INTRODUCTION. The facts that atoms are stable, and (for those of the same chemical species) identical in all respects, are two central problems which classical theory cannot explain.

SECTIONS 1 THROUGH 3. The Franck-Hertz experiment, described in Section 1, presents direct evidence that energy exchange between mercury atoms and colliding electrons involves only certain discrete amounts of energy. A simple interpretation of the observations is that the internal structure of atoms only allows certain discrete values of the energy. On this basis we construct an energy-level diagram for an atom, and examine the photon spectra which should arise when an atom changes its energy state. Energy conservation requires that the photons have only certain discrete energies. Also, the result of the last chapter relating the energy of a photon to its frequencies ($E = h\nu$) therefore requires that the emitted light have only discrete frequencies. These frequencies correspond exactly with those of the observed line spectra.

SECTIONS 4 THROUGH 6. We seek a detailed explanation of discrete energy states in terms of the wave properties of matter. It is suggested that the phenomenon of standing waves, which involves discrete frequencies, occurs with the electron waves in the atom. After a discussion of standing waves on a string, the corresponding problem for wave-motion of a particle in a box is solved as an introduction. Then, in Section 6, a mathematical solution to the problem of electron waves in a hydrogen atom is sketched, and excellent agreement with the observed energy levels is obtained. We can also quantitatively account for the spectra of other hydrogenlike atoms, such as ionized helium.

SECTION 7. A valediction, summarizing the understanding of the subatomic world that we have gained in this course.

SCHEDULING CHAPTER 34

The table given below suggests schedules for this chapter.

RELATED MATERIALS FOR CHAPTER 34

LABORATORY. Experiment IV–15, *The Spectrum of Hydrogen and Planck's Constant*, allows students to measure accurately the wavelengths of the three visible spectral lines of hydrogen. Using these measured values with the theory of Section 6 provides confirmation of the validity of the predictions of the wave theory of hydrogen energy levels and also allows a determination of the value of Planck's Constant by a method quite independent of the photoelectric determination of Chapter 33.

Chapter 34	*10-week schedule for Part IV*			*15-week schedule for Part IV*		
SECTIONS	CLASS PERIODS	LAB PERIODS	EXPERIMENT	CLASS PERIODS	LAB PERIODS	EXPERIMENT
1	1	0	—	2	0	—
2, 3	1	0	—	2	0	—
4, 5	2	0	—	2	0	—
6, 7	1	1	IV–15	1	1	IV–15
Problems	1	0	—	1	0	—

HOME, DESK, AND LAB. See the HDL Notes for answers, solutions, and a table which classifies problems according to their estimated level of difficulty.

FILM. "The Franck-Hertz Experiment," by Byron L. Youtz of Reed College with an epilogue by James Franck. This film provides a useful illumination of the text material, although the apparatus used is somewhat different from that described in the text (see Development of Section 1). Running time: 30 minutes.

Introductory Section

The introduction to the chapter reviews two difficulties which confronted physicists prior to the development of the quantum theory. These problems, resolved later in the chapter, are: (1) Electrons, orbiting around a nucleus, do not radiate energy and spiral in toward the nucleus, as classical physics would predict. (2) All atoms of the same chemical species are found to be identical in all respects. For instance they all have the same size. Classical theory permits orbiting systems to have an equilibrium at any radius and thus an arbitrary size. In the development of Section 1, a short review of the classical picture of an atomic system is outlined to cover these points.

You may want to point out that, in the first printing of the first edition of the text, the names of Rutherford and Thomson under the photographs on the frontispiece to the chapter were inadvertently interchanged. This has a negligible effect on the chapter arguments that follow.

34–1 The Experiments of Franck and Hertz; Atomic Energy Levels

PURPOSE. To present the experimental evidence for the fact that an atom can only accept energy from a colliding electron in discrete amounts, the magnitudes of which are characteristic of the particular atom.

CONTENT. A full description of the essential parts of the Franck-Hertz experimental apparatus is given. Electrons of known energy pass through a vessel containing atoms of mercury or some other element. The energies of the emerging electrons are measured. It is found that the atoms only accept energy from the electrons in fixed, discrete amounts.

The outcome of this experiment suggests that atoms exist only in states with certain characteristic energies. Atoms are usually in their lowest energy state — the "ground state." On collision with an electron of sufficient energy, the energy of the atom is raised to another of these states of fixed energy — an "excited state" — accepting from the electron just the fixed energy difference between the excited state and the ground state.

EMPHASIS. These ideas are basic to the chapter. Treat fully.

DEVELOPMENT. A short review of the properties of a classical atom, compiled from material of some of the previous chapters, is a useful way to begin. The concept of *internal energy*, and what it means in this case, is crucial for an intuitive understanding of the atomic energy level picture which is developed in this section. The wave picture (Section 6) uses many of the characteristics of the classical picture.

REVIEW. Consider the classical model of the hydrogen atom.

(1) The proton has 1800 times the mass of the electron. As with a satellite around the earth, or the planets around the sun, we can neglect to a good approximation the motion of the heavy body, the proton, and regard it as fixed. (See Chapter 22 in general, and Chapter 24, Section 3.)

(2) The Coulomb force between the positive proton and negative electron is a force of attraction. The potential energy of the system when the electron is a distance r from the proton is $-ke^2/r$, if the potential energy is taken as zero when the electron is far from the proton. (Chapter 28, Section 6.)

(3) The total energy of the system is:

$$E = \tfrac{1}{2} m_e v^2 - \frac{ke^2}{r}.$$

This expression is similar to that for planetary motion, since the gravitational force and the Coulomb force have the same $\frac{1}{r^2}$ dependence. (See Chapter 24, Section 4.)

(4) We can have a circular orbit for *any* value of r. The velocity of the electron must conform to Newton's law:

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}; \quad v^2 = \frac{ke^2}{mr}.$$

(5) This last equation gives the surprisingly simple result that

$$\frac{1}{2} mv^2 = \frac{1}{2} \frac{ke^2}{r};$$

the kinetic energy is equal to just half the magnitude of the potential energy. Then, for a circular orbit, the value of r thus determines the total energy:

$$E = \frac{1}{2} mv^2 - \frac{ke^2}{r} = -\frac{1}{2} \frac{ke^2}{r}.$$

The total energy is negative as a result of our choice of the zero of the potential energy. $E < 0$ means that energy must be supplied to the electron to remove it completely from its atom. (See Section 24–5 on earth satellites and escape velocity.) By (4) above, the radius r can take any value. Then E, the internal

energy of the atom, can have *any* (negative) value we want. The fact that we have restricted this part of the discussion to circular orbits need not be stressed. The results are more complicated in the case of elliptic orbits, but they are qualitatively the same.

The above points are all either familiar, or simple developments of earlier results. Only a little more time will be needed to expound them than is required to make them. One does not want to overemphasize the results of the classical model, since we know that it does not work, and we know vaguely that waves must be brought in. But it is important that the students have some concrete place from which to start. An atom is a dynamical system. It therefore has internal energy, connected in some way with the motion of the constituent parts. But we have not yet learned any details about this motion.

THE FRANCK-HERTZ EXPERIMENT: The PSSC film on this experiment will be useful. It helps get a good general picture of the experiment and will answer some of the likely questions concerning experimental techniques. Note, however, a point of difference between the filmed experiment and the one described in the text: The apparatus differs in that the gas occupies the region across which the potential V_A is applied. When the applied potential is increased to, say, 10 volts, only a few electrons ever acquire enough energy to exicte the 8.84 ev level, as described in the text. Instead, almost all undergo an inelastic collision as soon as they have more than 4.86 ev of kinetic energy, thus exciting the first state. They are thereby brought almost to rest and accelerate again over the remaining part of the gap, once again gaining enough energy to excite another atom to its first excited state. Thus what is detected is excitation to the first excited state. The processes described in the text do occur, of course. We see that from the emission spectrum (Section 2). But the higher excited states are not detected directly in a Franck-Hertz experiment of the type illustrated in the movie.

* * *

A discussion of the results of the experiment from a classical point of view, considering solely circular orbits for the atomic electrons, may be helpful. Classically one can see that it would be nearly impossible to hit an atom without producing changes in internal energy, since the bombarding electron would surely disturb the orbits of the atomic electrons. The fact that no change in internal energy occurs until the energy of the bombarding electrons exceeds some threshold value has no classical explanation. It is as if an atom possessed a "stiffness." The natural explanation is that the atom can exist only in certain states of internal energy, and we can change the energy of the atom from one to another only by providing a large enough amount of energy.

Great pains are taken in the text to avoid saying that the modern picture of an atom is that of an electron moving in a well-defined circular orbit. The early attempts at quantum theory by Bohr were in terms of well-defined paths, the so-called quantized orbits. This picture has been superseded by the wave model. Therefore, any references to electrons moving in circular or elliptical orbits should be firmly linked to a classical view. It is useful to emphasize to the student that we do not, at this point in the text, understand exactly what is happening within an atom, and the Franck-Hertz experiment shows us that, whatever is taking place, the atom is able to accept energies only in certain amounts. Our natural conclusion is therefore that the atom can exist only in states of certain energy. In some manner which has not yet been discussed, these states of energy must be related to the state of motion of the electrons about the nucleus of the atom.

Several of the early problems in HDL involve considerations such as the above. In order to keep class discussion from getting too abstract and to aid the students' understanding of the exact process, you might ask such questions as:

(a) An electron with an energy of 6.20 electron volts collides with a mercury atom. With what energy might it bounce off? Answer: It might bounce elastically, retaining all 6.20 electron volts. It might bounce inelastically leaving the mercury atom excited to 4.86 electron volts and retaining $6.20 - 4.86 = 1.34$ electron volts of energy.

(b) A beam of 10.00-ev electrons are shot through mercury gas. With what energies could they come out? Answer: $10.00 - 8.84 = 1.16$ ev; $10.00 - 6.67 = 3.33$ ev; $10.00 - 4.86 = 5.14$ ev; and don't forget an electron that collides twice, $10.00 - 2(4.86) = 0.28$ ev.

(c) Students might rightly ask, "If one electron bounced off a mercury atom and left it excited by 4.86 ev, could another electron hit it and raise it to 6.67 ev, thus, in example (b) above, giving rise to electrons of $10.00 - (6.67 - 4.86) = 8.19$ ev?" In principle this could happen, but in practice it does not. Once the mercury atom is excited to 4.86 ev it radiates this energy away as a photon of light in about 10^{-8} second. Therefore very few atoms are in any state but the ground state at any one time; and the electrons can only raise them to excited states from the ground state.

QUIZ PROBLEMS. You can repeat the previous class discussion questions with different energy electrons and, say, cesium atoms.

HDL Problems 2 and 3 are simple exercises which emphasize energy conservation in the process of atomic excitation. If students have understood the text material, Problem 2 should present no difficulty. Problem 3 requires a result from Chapter 25, on molecular motion and heat.

34–2 Dissecting Atomic Spectra: Excitation and Emission

PURPOSE. To examine the emission spectrum in a Franck-Hertz experiment as additional evidence for the presence of atomic energy levels.

CONTENT. We assume that when an atom changes its energy state, going to a state with less energy, a photon is emitted. The photon must have just the right energy to conserve energy. Since $E = h\nu$ this means that the frequency also must be just right. Thus only certain discrete frequencies will be found, corresponding to the differences between the discrete energy levels that the bombarding electrons are able to excite. The appearance of lines of *lower* frequency as the incident electron energy is *increased* can be understood in terms of the reduced spacing of the higher energy levels.

Other excitation mechanisms can also produce emission spectra.

EMPHASIS. Should go easily. Combine with Section 3.

DEVELOPMENT. Once the idea of atomic energy levels has been accepted, the hypothesis of this section should appear to the students to be a very reasonable application of photon properties. In Section 33–5 it was seen that an electron acquired energy to escape from a metallic surface by absorbing a photon. It is then reasonable to suppose that an electron may *lose* energy by *emitting* a photon. We expect the photon's frequency to be that given by energy conservation. The detailed results of observations on the emission spectra from mercury atoms are displayed in Fig. 34–4. The energy-level diagrams show what levels may be excited at the particular energy of the incident electrons, and the photon frequencies expected. The beautiful agreement with the observed line spectra should be completely convincing.

It may be helpful to remind your class of the experimental arrangement for observing spectra. One needs a line source, a prism or grating to spread the colors, and some means of observing the position of the lines. A photographic film is frequently used as a detector. The vertical lines on the film result from the fact that a line source is used. In the photographs in the text, lines of longer wavelength appear on the right-hand side. Note that it is the photographic *negative* which is shown — the black line represents the light, seen against the gray background.

DEMONSTRATION. It will be desirable for your students to observe some line spectra. If you wish to demonstrate to an entire class at once, give each student a piece of the inexpensive grating material which is now available (PSSC laboratory kits, and other sources) and let them observe a colored flame set up in front of the class. An electric discharge tube rather than a flame spectrum perhaps brings the demonstration closer to the material of the chapter. Tubes of various gases are available from scientific supply houses, or perhaps a local neon sign shop will make you several. Tubes which will show the hydrogen spectrum are now available from several scientific suppliers, though they tend to be rather expensive. Ordinary hydrogen gas will *not* work because what will be seen is the molecular spectrum of H_2, not the atomic spectrum. A small high voltage transformer will be needed to excite the tubes. You should not try to reproduce the effects of Fig. 34–4. When the voltage applied is low, the spectra are present, but *very* faint.

QUIZ PROBLEM

Ten-volt electrons pass through a gas and are observed to retain 6 ev and 3 ev of energy. What frequencies of light do we expect to see emitted from this gas? Answer: Since 3 or 6 ev are retained by the electrons, the gas atoms were excited to 4 or 7 ev of energy. Photons of 4 and 7 ev should be emitted, as well as one of 3 ev where an atom excited to 7 ev emits a photon of 3 ev and remains excited to 4 ev. The frequencies $\nu = \frac{E}{h} = \frac{7 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.7 \times 10^{15}$ cycles/sec; also 0.97×10^{15} and 0.73×10^{15}.

34–3 Absorption Spectra

PURPOSE. To present additional evidence for the picture of photons and the transitions between atomic energy states by a consideration of dark-line (absorption) spectra.

DEVELOPMENT. Photon absorption, producing excitation of an atom, is in a sense the same process as that discussed in Section 2, reversed in time. The photon just passes by the atom unless its energy is exactly that which is required to excite one of the atomic levels. You can illustrate a point made in Section 33–10 by comparing this situation with the electron-induced excitation, Fig. 34–3. One of the fundamental ways in which photons differ from particles, such as electrons, is that photons are absorbed (and emitted), but particles are not. Thus the photons may excite atoms only if they have *exactly* the energy required. The photon vanishes completely, its energy (and momentum) being taken up by the atom. Excitation of an atom by an electron, on the other hand, can occur whenever the electron's kinetic energy is greater than the minimum required. The electron is slowed down by just the amount required by energy conservation, and the electron itself carries off any "extra" energy.

Note the information in the caption for Fig. 34–5. Since the illustration is of a photographic negative, the light lines correspond to an *absence* of photons of that wavelength.

One question which frequently arises in discussions of absorption spectra is the following: Since an atom absorbs energy by being raised to an excited state, and then must decay back to the ground state by emission of a photon or series of photons of the same energy, why do we see dark lines? Wouldn't the re-emitted photon just make up for the one which was absorbed? The answer lies in the fact that the re-emitted photon goes off in a completely arbitrary direction, but our optical system is sensitive only to light traveling in one direction, that of the incident photons. Thus only very rarely do we see the re-emitted photons. Most of the re-emitted photons go off in some direction other than the one in which we are looking. If we could look in all directions at once and count each photon, it would appear that the incident photon was scattered by the atom.

DEMONSTRATION. If you have a fairly good spectroscope, you can illustrate absorption spectra for your class. By means of mirrors, reflect the sun's light into your instrument. Using a fairly narrow slit, you should see dark lines crossing the otherwise continuous spectrum. These are the absorption lines. There is a very prominent dark line in the orange-yellow. It is caused by absorption by sodium atoms. You can prove this by looking at a sodium flame spectrum at the same time. Reflect it in with another mirror. The bright yellow line will appear at the same place. This also proves, incidently, that the yellow sodium line is caused by a transition from an excited state directly to the ground state, otherwise it would not show up in absorption. See the quiz problem.

QUIZ PROBLEM

A certain atom has energy levels at 4 electron volts and 7 electron volts above the ground state.

(a) What energy light is absorbed? Answer: Photons of 4 and 7 ev energy are absorbed. 3-ev photons are *not* absorbed since the atom will ordinarily be in its ground state. Of course an atom *could* absorb a 4-ev photon and then a 3-ev photon, but this would be extremely improbable for the reasons given near the end of the Guide for Section 1.

(b) What would happen to a 6-ev photon if it "hit" this atom? Answer: It would go on by. It would not excite the atom to 4 ev and come off as a 2-ev photon. How do we know this? If the latter process were at all probable, light of all frequencies would be absorbed and not only a few discrete frequencies. Compare Fig. 34–5.

34–4 The Energy Levels of Hydrogen

PURPOSE. To introduce the numerical results for the energy levels of hydrogen, the simplest atom. These results will be explained with a wave model in the next two sections.

EMPHASIS. The "formulas" need not be memorized. Stress only the $1/n^2$ dependence.

DEVELOPMENT. From now on, the discussion concerns the hydrogen atom. Classically it is the simplest atom. The level scheme of hydrogen turns out to have a relatively simple numerical description unlike that of the mercury levels.

Limitations of space and time force the text to omit the fascinating story of the discovery of the formula for the energy levels of hydrogen. The major breakthrough was made in 1885 by Balmer. Many atomic spectra had been observed by that time, but the frequencies of the lines observed did not appear to obey any simple laws (such as being harmonics of a basic frequency, like the various possible vibrations of a string). Balmer studied the spectrum of hydrogen, which was particularly simple since only 9 lines were known. He found that the wavelengths, in angstroms, were given by the formula

$$\lambda = 3645.6 \frac{n^2}{n^2 - 4} \quad n = 3, 4, 5, \ldots, 10, 11$$

This formula was accurate to better than 1 part in 1000. We now know that it describes the frequencies of hydrogen lines in which the atom drops from the $n = 3, 4, 5, \ldots, 10, 11$ down to the $n = 2$ state. Rydberg, in 1890, found that rather similar formulas described the wavelengths of the spectra of other atoms. In particular, he found it simplest to express the spectra in terms of the *frequency* of the lines, as

$$\nu = \nu_0 - \frac{Rc}{n^2} \qquad n = 3, 4, \ldots$$

Where ν_0 is a constant, c the velocity of light, and R a constant called the Rydberg constant. This tantalizing simplicity could not be exploited for another twenty years. It required an understanding of photons, given by Planck (1903) and Einstein (1905), and the nucleus-plus-planetary-electrons model of the atom proposed by Rutherford (1911), before Bohr (1913) could devise a model for electron motion which would make some physical sense out of this simple expression. An *understanding* of the results in terms of wave properties of electrons was only achieved ten or more years later, starting with de Broglie.

34–5 The Origin of Energy Levels

PURPOSE. To illustrate how wave motion entails the possibility of discrete states of motion.

CONTENT. The wave properties of electrons, discussed in the last chapter, are brought forward as a possible explanation of the observed discrete energy levels. The levels must correspond to quite distinct wave patterns. Stationary or standing waves, which

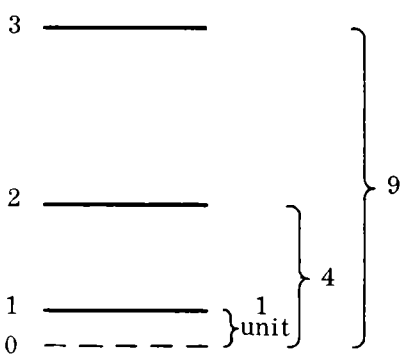
appear on a vibrating string and in many other situations, provide just this behavior. Detailed practical suggestions for their observation are given. To illustrate the analogy which will be drawn in the hydrogen atom, the wave motion of a particle in a box is investigated. The presence of a lowest energy state is noted as a promising sign of the physical correctness of the approach.

EMPHASIS. These ideas are basic to an appreciation of Section 6.

DEVELOPMENT. As emphasized in the text, some actual experiments with tubes or suspended slinkies are essential. The student who has determined for himself the unwillingness of the stretched tube to vibrate at any but its natural frequencies will not require further persuasion.

The connection between standing waves and the traveling waves that have occurred hitherto is necessary in order that λ, the wavelength, may be identified for later use in the de Broglie relationship. The fact that a standing wave may be viewed as the superposition of two equal traveling waves moving in opposite directions is sometimes rather hard to grasp. The experimental approach suggested in the text is sure to be helpful.

The wave mechanics of a particle in a box illustrates the methods we shall use in the next section. Classically, the particle would be reflected at each end of the box without change in speed. Thus its momentum would be constant in magnitude, and the de Broglie relationship can easily be applied. (In the next section the situation is less clear.) The most important point to be emphasized is that there is an energy state below which it is impossible to go. This suggests a possible solution to the puzzling problem of atomic *stability*. It may also be worthwhile to work out the energies of a few of the higher energy levels. The situation is clear-cut here, with no potential energy to worry about. In addition, this will help clarify the point that $1/n^2$ dependence of the energy states of the hydrogen atom is very special. Other systems, such as this one, have different level schemes. To obtain the energies corresponding to the waves shown in Fig. 34–8, we note that the wavelengths are respectively $\lambda_1 = 2d$, $\lambda_2 = \frac{2d}{2}$, $\lambda_3 = \frac{2d}{3}c$, $\lambda_4 = \frac{2d}{4}$, etc. In general, $\lambda_n = \frac{2d}{n}$. Substitution in the de Broglie relationship, $E = \frac{h^2}{2m\lambda^2}$ produces the result that $E_n = \frac{h^2}{8md^2}n^2$.



The expression is mathematically simple, but the relative positions of the energy levels are quite different from those of the hydrogen atom.

34–6 The Wave Theory of Hydrogen Energy Levels

PURPOSE. To present a simplified wave picture of the hydrogen atom and a plausible derivation of the energy level expression.

EMPHASIS. This is the climax of the chapter. Spend enough time to get over the arguments. Remember, however, that to simplify the mathematics, the arguments are made approximate and not completely rigorous. The accuracy of the results is so striking that the qualifying statements may be overlooked.

CONTENT. The problem of finding the correct standing wave patterns is now more complicated; the wave is in three dimensions, and there is no well-defined boundary as there was for a stretched string and a particle in a box. By considering only circular orbits, however, and by assuming that the wave motion is confined to the region surrounding the classical orbit, a standing wave can be found by the methods we have developed. Instead of a wave between two boundaries, we now require that the wave must fit onto itself, i.e., that it goes around the inside with no detectable joint. This determines λ, and, by de Broglie's hypothesis, we can find p and the kinetic energy. The potential energy is calculated from the expression $V_e = \frac{kq_1q_2}{r}$, assuming that the electron is at a distance r for which the wave exactly fits onto itself. This determines the total energy of a set of discrete standing waves which represent the electron's motion about the proton. The expression obtained for the energy levels is correct. The level ordering, $E_n \propto -\frac{1}{n^2}$, is obtained, and the constant factor, 13.6 ev, turns out as a simple combination of e, k, m, and h.

The explanation can be checked against other atoms which should behave similarly. They do. Even the tiny effects of recoil of the nucleus can be accounted for by this simple model.

DEVELOPMENT. Do not stress the difficulties that are expected in trying to find the exact wave-pattern. The illustrations in Fig. 34–9 show that we can expect relatively simple standing waves in these more complicated situations.

If you have gone through the review suggested in the Guide for Section 1, the material in the box should present no great difficulty. It may be useful to state the steps in the argument explicitly, so that you can run through the algebra quickly and obtain the $1/n^2$ dependence.

The steps are

(1) Newton's law (which we use even though the electron is treated as a wave) $F = ma$:

$$\frac{kq_1q_2}{r^2} = +m\left(\frac{v^2}{r}\right).$$

This gives one relationship between v and r, the two unknown quantities in the equation, and we write it as

$$mv^2 = \frac{kq_1q_2}{r}.$$

[Note what this says: the right side is the negative of the potential energy, the left side is twice the kinetic energy. So kinetic energy and potential energy are proportional; they will have the same dependence on n.]

(2) For a standing-wave pattern to be possible, there must be an integral number of wavelengths around the orbit:

$$n\lambda = 2\pi r.$$

(3) By de Broglie's relationship, λ is related to p, hence to v. With (2), this is another relationship between v and r. By combining this with (1), both v and r can be found.

The algebra of this step can be done as follows, if all that you want is the $1/n^2$ dependence of E: (2) and (3) give

$$\frac{nh}{mv} = 2\pi r, \quad vr = n\frac{h}{2\pi m}.$$

(1) can be written (by multiplying both sides by r^2)

$$mv^2r^2 = k\,q_1q_2r.$$

Substituting for vr,

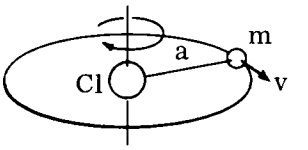
$$mn^2\left(\frac{h}{2\pi m}\right)^2 = kq_1q_2r, \quad \text{or} \quad \frac{1}{r} = \frac{4\pi^2mkq_1q_2}{h^2n^2}.$$

Since $E = -\frac{k\,q_1q_2}{2r}$, as shown in Section 1 of the Guide,

$$E = -\frac{kq_1q_2}{2} \times \frac{4\pi^2mkq_1q_2}{h^2n^2} = -\frac{1}{2}\left(\frac{2\pi kq_1q_2}{nh}\right)^2$$

The result is the same as the boxed material in the text. The algebra has been approached slightly differently. As explained in the text, this very simple result, along with the photon hypothesis, gives the spectrum of hydrogen. It explains, in more basic terms, why an atom cannot radiate energy continuously and also why there is a minimum energy or ground state.

COMMENT. With the necessarily simplified treatment of the hydrogen atom given here it is somewhat fortuitous that exactly the right answers are obtained. The same approach applied to other problems — the energy levels of a rotating diatomic molecule for instance — gives only approximate answers. We can really expect no better, for we have applied a one-dimensional wave model (Fig. 34–10) to a truly three-dimensional problem. The students have not been shortchanged. All the elements of a correct treatment are here: (1) the total energy written as the sum of the classical kinetic and potential energies; (2) introduction of the de Broglie wavelength in classical momentum; (3) the restriction of all possible wave patterns to those which just fit, i.e., make standing waves. The real difference between a more exact treatment and our approximation is in (2) above, where the momentum, instead of being replaced by h/λ, is replaced by a so-called differential operator which allows a full three-dimensional treatment of the wave. The mathematical background for the proper treatment involves the solution of partial differential equations.



There is at least one other system which is simple enough to treat at this level. This is the rigid diatomic molecule, say HCl. Here the chlorine is so massive that, as the molecule rotates, the hydrogen atom simply swings around the chlorine as in the figure. Suppose that the hydrogen atom has a mass m and moves around the chlorine at a fixed distance a with a velocity v. Then $2\pi a = n\lambda$ is the condition required for the wave associated with the hydrogen atom to just fit once around the circle. $n = 1, 2, 3, \ldots$. But $2\pi a = n\lambda = \frac{h}{p}$; $p = \frac{nh}{2\pi a}$; E (the energy of rotation) $= \frac{p^2}{2m} = \frac{n^2h^2}{8\pi^2ma^2}$. The exact answer from wave mechanics replaces our n^2 by $n(n+1)$. Notice that for large n the two answers become nearly the same. If we take $m = 1.7 \times 10^{-27}$ kg and $a = 10^{-10}$ meter (in a molecule the atoms nearly touch), we get $E = 3.2 \times 10^{-22}n^2$ joule. Suppose that n changes from 4 to 3 and a photon is given off. The energy of the photon would be $3.2 \times 10^{-22}(16 - 9) = 22.5 \times 10^{-22}$ joule. Its fre-

quency would be $22.5 \times 10^{-22} \div 6.6 \times 10^{-34} = 3.4 \times 10^{12}$. Its wavelength = 0.88×10^{-4} meter. This is either very long-wavelength infrared or very short-wavelength radio waves. This spectrum has actually been observed. The transition from $n = 4$ to $n = 3$ actually has a wavelength of 1.2×10^{-4} meter.

LABORATORY. Experiment IV–15, *The Spectrum of Hydrogen and Planck's Constant*, should be done in connection with Section 6. Quantitative information is obtained on the first three spectral lines of the Hydrogen Balmer Series with sufficient accuracy to allow an experimental confirmation of the predictions of the wave theory of hydrogen energy levels. Also, a value for Planck's constant can be obtained from this spectroscopic data and compared with the value obtained by quite different means from the photoelectric effect of Chapter 33.

34–7 Conclusion

PURPOSE. To provide a brief recapitulation of the road we have traveled, and to suggest that a great deal lies ahead.

EMPHASIS. Treat as a reading assignment.

FOR HOME, DESK, AND LAB
ANSWERS TO PROBLEMS

The table below classifies problems according to their estimated level of difficulty and the sections with which they are intended to be used. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with the symbol #.

SHORT ANSWERS

1.* 4.2 ev.
2. See discussion on page 34–9.
3. About 10^5 degrees Kelvin.
4. (a) Very close to G_2.
   (b) In the last half of the distance G_1 to G_2; in the last third of the distance G_1 to G_2.
   (c) They can lose either 4.9 ev or 6.7 ev right away; the possible energies are 10 ev, 5.1 ev, 3.3 ev, and 0.2 ev.
   (d) Most electrons gave up their 4.9 ev before reaching the position at which they would have 6.7 ev.
5.* 6.67 ev, 4.86 ev, 1.81 ev.
6.* 1.81 ev.
7. See discussion on page 34–11.
8. (a) 2.1 ev;
   (b) 1.6×10^4 °K;
   (c) See discussion on page 34–12.
9.* The atom absorbs 4.86 ev from the electron. It absorbs no energy from the photon.
10.* −10.4 ev.
11.* The γ-line of the Lyman series.
12.* (a) 13.5 ev;
   (b) −0.136 ev.
13.* (a) Any energy from 10.2 ev up to 12.1 ev;
   (b) 12.1 ev, 10.2 ev.
14. 3600 A.
15. No. The series extends down to 3600 A.
16. (a) 10.2 ev;
   (b) 12.1 ev;
   (c) Lyman α, Lyman β, and H_α.
17. (a) 1.03×10^3 A;
   (b) 1.22×10^3 A;
   (c) Ratio = 5.4.
18.* No; since the only restriction is that 2d equal a whole number of wavelengths, there are an infinite number of energy values which the particle could have.
19.* 10^{-17} joule.
20. (a) 9×10^{-13} joule;
   (b) No, larger.
21. See discussion on page 34–13.
22. $\nu = 3h/8md^2$.
23.* No; the wavelength can be determined from the kinetic energy of the electron, and this different for $n = 1$, $n = 3$.
24. (a) 1.22×10^3 A; 1.03×10^3 A;
   (b) See discussion on page 34–14.

COMMENTS AND SOLUTIONS

PROBLEM 1* (See Short Answers.)

PROBLEM 2

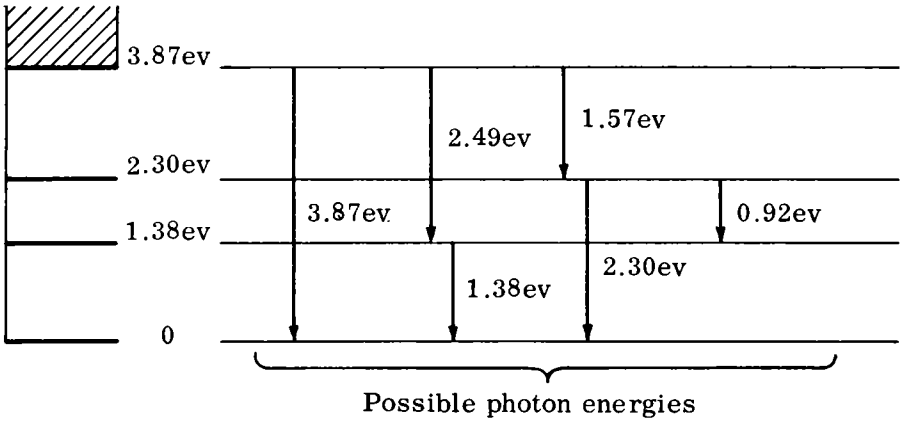
Suppose you are going to bombard a vapor of cesium atoms with 4.00-electron-volt electrons. Use Fig. 34–2 to predict some of the things that you would expect to happen.

This problem in energy conservation, for electron collisions, can also illustrate emission spectra.

Electron Collisions. Since the electrons have greater kinetic energy than that needed to strip away one atomic electron, a number of things can happen. The electron may excite one of the discrete states, in the process losing 1.38 ev or 2.30 ev. It may scatter elastically, losing no kinetic energy. If it ionizes the atom, the atomic electron and the bombarding electron share the remaining kinetic energy. 3.87 ev is required to tear away the atomic electron, so they will have 4.00 − 3.87 ev = 0.13 ev between them.

Emission Spectra. Since some of the atoms are ionized, all possible emission lines should occur. An

SECTION	STARRED	EASY	MEDIUM	HARD	CLASS DISCUSSION
1, 2, 3	1, 5, 6, 9	7, 8	2	3, 4	2
4	10, 11, 12, 13	14	15, 16#, 17		16#
5, 6	18, 19, 23		20#, 22#	21, 24#	20#



ionized atom will encounter some slowly moving electron, moving with a very small kinetic energy of about $3/2\,kT$, where T is the temperature of the cesium gas (see Section 25–2). (For ordinary temperatures, this will be less than 0.1 ev.) The electron is attracted by the net charge on the ion. It can be "captured" into any of the discrete states by emitting a photon to conserve energy. The diagram above shows some possible photons which can be emitted as the atom gradually relaxes to its ground state.

Note: Figure 34–2 shows only a few levels. There are many other levels which lie close together below the ionization energy. Therefore the incoming electron can lose many other possible amounts of energy and also many other spectral lines can be emitted.

PROBLEM 3

If we heat helium gas to a high enough temperature, the average kinetic energy of an atom in its thermal motion will be so high that an inelastic collision between two helium atoms can easily excite one of them to its first excited state at 19.8 electron volts above the ground state. Estimate this temperature, obtaining any data you need from Part III.

This problem illustrates why atoms are usually found in their lowest energy state.

The electron will be excited only when the kinetic energy of a single gas molecule, $3/2kT$ (see Section 25–2), becomes comparable to 19.8 ev. Then, on collision of two atoms, their translational kinetic energy can be used up in exciting one of them.

At this energy the temperature of the gas is

$$T = 2/3(E_K/k)$$
$$= \frac{2(19.8\text{ ev})(1.6 \times 10^{-19}\text{ joule/ev})}{3(1.38 \times 10^{-23}\text{ joule/°K})}$$
$$\approx \underline{10^5\text{ °K}}.$$

The temperature at the surface of the sun is only 6000° K, so even there, helium atoms would be predominantly in their ground state.

PROBLEM 4

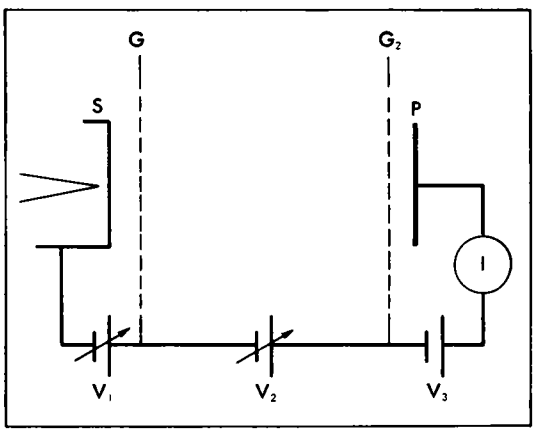
Figure 34–13 is a schematic sketch of a vacuum tube in which electrons emitted by S can be accelerated by either of the potential differences V_1 or V_2. Suppose the tube contains some mercury vapor and is used to perform the Franck-Hertz experiment.

(a) If $V_1 = 0$ volts, $V_2 = 5.0$ volts, and the field between G_1 and G_2 is uniform, in what region of the space between G_1 and G_2 can electrons give up 4.9 ev in inelastic collisions?

(b) If $V_1 = 0$ volts and $V_2 = 10.0$ volts, in what region of the space between G_1 and G_2 can electrons give up 4.9 ev in inelastic collisions? In what region 6.7 ev?

(c) If $V_1 = 10$ volts and $V_2 = 0$ volts, how far do the electrons have to go through the gas before they can lose 4.9 electron volts in inelastic collisions? How far before they can lose 6.7 ev? What electron energies should it be possible to measure to the right of G_2?

(d) The 6.7-ev transfer of energy is observed when the apparatus is operated as in (c), but it is not observed when the apparatus is operated as in (b). Explain this result in terms of your answers to (b) and (c).



(a) The inelastic collisions can occur only after electrons have been accelerated through a potential difference of 4.9 volts. This would happen only very close to G_2.

(b) The 4.9-ev energy is reached just before the electron is half the distance from G_1 to G_2. The mercury vapor can take another 4.9 ev of energy from electrons in the last half of the distance G_1 to G_2. Electrons would give up 6.7 ev in the last third of the distance G_1 to G_2.

(c) Electrons will now have 10 ev of energy when they enter the gas through G_1. Each electron can immediately lose either 4.9 or 6.7 ev. Those that lose 4.9 ev can again lose 4.9 ev. The electron energies which it would be possible to measure to the right of G_2 would be:

$$10.0 - 0.0 = 10.0 \text{ ev}$$
$$10.0 - 4.9 = 5.1 \text{ ev}$$
$$10.0 - 6.7 = 3.3 \text{ ev}$$
$$10.0 - 2 \times 4.9 = 0.2 \text{ ev}.$$

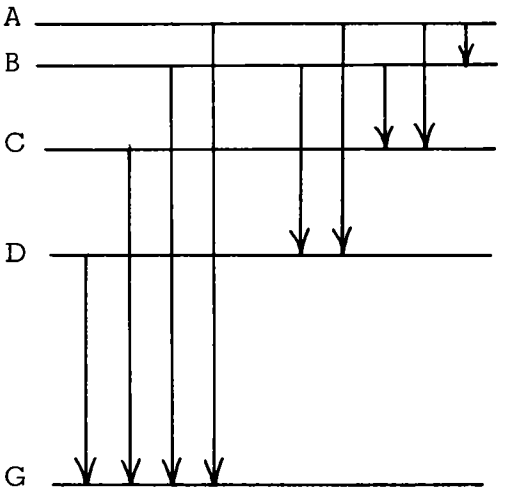
(d) In (c), an electron has sufficient energy to cause either transition when it reaches the gas. However, in (b) each electron reaches 4.9 ev at half the distance G_1 to G_2 and can give up this amount of energy in a collision after this point. If the 6.7 ev is not observed, this means that most electrons have given up the 4.9 ev of energy before they have reached the position where they can gain 6.7 ev, which is about two thirds of the distance.

PROBLEM 5* (See Short Answers.)

PROBLEM 6* (See Short Answers.)

PROBLEM 7

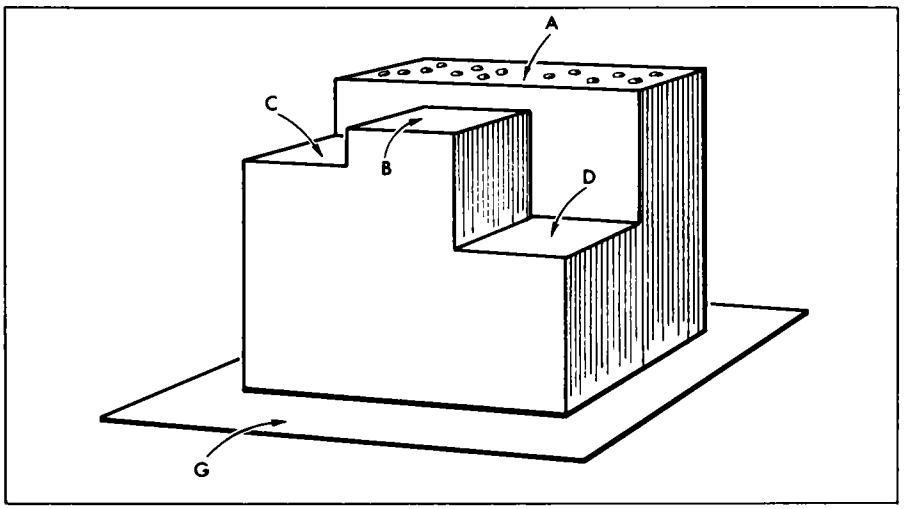
Figure 34–14 shows a mechanical model of energy levels with platforms at four different levels above the ground. A number of uniform putty balls are located on top of *A*. When balls are pushed over the side from *A* and land somewhere below, their temperature will rise in direct proportion to their loss in potential energy. Balls which land on *B*, *C*, and *D* are again pushed until they reach the ground. How many different changes in temperature are possible? Draw the "energy levels" and mark the possible jumps.



As shown in the diagram, there are nine different temperature changes possible. Notice that the transition from *C* to *D* is impossible. This is an illustration of what happens also in atoms. Not all energy levels in an atom are connected through photon emission, although the emission is possible as far as energy is concerned. These missing lines are referred to as "forbidden lines."

PROBLEM 8

If ordinary table salt is introduced into a gas flame or a soft glass rod is heated in it, the flame is colored with the characteristic light from sodium atoms, whose wavelength is 5890 angstroms. This light is emitted by electrons returning to the ground state from the first excited energy level.



(a) What is the energy of this level above the ground state, in electron volts?

(b) What would be the temperature of a gas whose molecules had an average kinetic energy of this amount?

(c) The average temperature of the molecules in a gas flame is about 2100°K. How is it possible for a gas flame to raise sodium atoms to the first excited energy level?

(a) $E = \dfrac{1.24 \times 10^4 \text{ ev-A}}{5890 \text{ A}} = \underline{2.1 \text{ ev}}.$

(b) $E = 3/2(kT)$.

Hence,

$$T = 2/3(E/k)$$
$$= \frac{2(2.1 \text{ ev})(1.6 \times 10^{-19} \text{ joule/ev})}{3(1.4 \times 10^{-23} \text{ joule/°K})}$$
$$= \underline{1.6 \times 10^4 \text{ °K}}.$$

(c) Although the average kinetic energy of the molecules in a gas flame corresponds to a temperature of 2100°K and amounts to 0.27 ev, there are many molecules with greater or less energy, and enough with ten times the average energy to excite the sodium atoms by collisions.

PROBLEM 9* (See Short Answers.)

PROBLEM 10* (See Short Answers.)

PROBLEM 11* (See Short Answers.)

PROBLEM 12* (See Short Answers.)

PROBLEM 13* (See Short Answers.)

PROBLEM 14

In Fig. 34–7(b) and (c), the lines on the right side of the photo approach a limit. What is the wavelength of this limit?

From Fig. 34–7(a) we see that the energies of the photons in the Balmer series range from 12.1 − 10.2 = 1.9 ev to 13.6 − 10.2 = 3.4 ev. The spectral lines crowd around the higher value which corresponds to a wavelength of

$$\lambda = \frac{1.24 \times 10^4 \text{ ev-A}}{3.4 \text{ ev}} \approx \underline{3600 \text{ A}}.$$

PROBLEM 15

The visible spectrum extends from 4000 to 7500 angstroms. Are all lines of the Balmer series in the spectrum of hydrogen visible?

The Balmer series results from energy changes from higher energy levels to the first excited state at 10.2 ev. The final line of the Balmer series is the transition from 13.6 to 10.2 ev or 3.4 ev. This gives a wavelength of

$$\lambda = \frac{12{,}400 \text{ ev-angstroms}}{3.4 \text{ ev}} = 3600 \text{ angstroms},$$

which is outside the visible part of the spectrum in the ultraviolet.

Another approach is to find the energy of the lower limit of the visible spectrum:

$$E = \frac{12{,}400 \text{ ev-angstroms}}{4000 \text{ angstroms}} = 3.1 \text{ ev}.$$

Thus, only transitions from 10.2 + 3.1 = 13.3 ev and below are visible. In the diagram (Fig. 34–7), we see that there are only four lines within that range. In more detail we can set

$$13.3 \text{ ev} \geq 13.6 \text{ ev} - \frac{13.6 \text{ ev}}{n^2_{\max}}$$
$$n^2_{\max} \leq \frac{13.6}{0.3} \approx 45, \quad n_{\max} < 7.$$

Then the visible lines of the Balmer series consist of the following transitions in n: $3 \rightarrow 2$, $4 \rightarrow 2$, $5 \rightarrow 2$, and $6 \rightarrow 2$.

PROBLEM 16

(a) What energy electrons would you have to use in a Franck-Hertz experiment in order to "see" the Lyman α line? (See Fig. 34–7.)

(b) In order to "see" the H_α line?

(c) What other spectral lines would you expect to "see" if you can see the H_α line?

(a) Referring to Fig. 34–7(a), we see that the Lyman α line occurs when hydrogen atoms are in the first excited state. Colliding electrons must have enough kinetic energy to excite the atom to this state, i.e., $\underline{10.2 \text{ ev}}$.

(b) The H_α line occurs when the atom makes a transition from the second excited state to the first excited state. Colliding electrons must now have enough energy to excite the atom to the second excited state, i.e., $\underline{12.1 \text{ ev}}$.

(c) From the second excited state the atom can make a transition to the first excited state and then to the ground state, or it can go directly to the ground state. Thus there will be three spectral lines: Lyman α, Lyman β, and H_α.

PROBLEM 17

(a) What wavelength of light is necessary to excite hydrogen atoms in their ground state so that they emit the H_α line (Fig. 34–7)?

(b) What is the longest wavelength that can be absorbed by the hydrogen atoms in making a transition from the ground state (Fig. 34–7)?

(c) Compare this with the H_α wavelength. (You need only one really significant figure.)

This problem in light absorption helps to see why hydrogen atoms are hard to excite with photons.

(a) The atom must be excited to the second excited state [see Problem 16(b)], and this re-

quires an energy of 12.1 ev (Figure 34–7a). From the box on page 622, we can calculate the wavelength of such a photon:

$$\lambda = \frac{1.24 \times 10^4}{12.1} \text{ angstroms}$$
$$= \underline{1.03 \times 10^3 \text{ angstroms}}.$$

(This is just the energy, and consequently the wavelength, of the Lyman β line, since that is the line resulting from the transition from the second excited state directly to the ground state.)

(b) The longest wavelength will correspond to the least energetic photon which can be absorbed. The transition is to the first excited state, and the energy required is 10.2 ev. The wavelength is then

$$\lambda = \frac{1.24 \times 10^4}{10.2} \text{ angstroms}$$
$$= \underline{1.22 \times 10^3 \text{ angstroms}}.$$

(This is just the Lyman α line.)

(c) The energy of the H_α photon is 12.1 − 10.2 = 1.9 ev. The energy of the Lyman α photon is 10.2 ev. The ratio of their wavelengths is the reciprocal of the ratio of their frequencies and energies:

$$\frac{\lambda(H_\alpha)}{\lambda(\text{Lyman } \alpha)} = \frac{\nu(\text{Lyman } \alpha)}{\nu(H_\alpha)} = \frac{E(\text{Lyman } \alpha)}{E(H_\alpha)}$$
$$= \frac{10.2}{1.9} = \underline{5.4}.$$

(This ratio can also be obtained by simply measuring the energies from the arrows corresponding to these lines on Fig. 34–7a.) Thus the wavelength of the H_α photon is

$$\lambda(H_\alpha) = 5.4 \times \lambda(\text{Lyman } \alpha)$$
$$= 5.4 \times 1.22 \times 10^3 \text{ A}$$
$$= 6.5 \times 10^3 \text{ A}.$$

By remembering that the wavelengths are inversely proportional to energy, you can see that long wavelengths correspond to transitions between the higher excited states and only relatively short wavelengths can be involved in absorption by an atom in the ground state. This has been shown quantitatively in this problem.

PROBLEM 18* (See Short Answers.)

PROBLEM 19* (See Short Answers.)

PROBLEM 20

(a) Estimate the kinetic energy of a neutron in a nucleus of radius

$$R = 6 \times 10^{-15} \text{ m}.$$

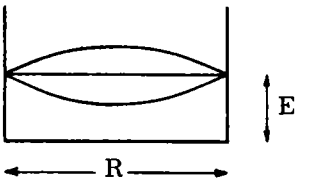
(This is a big nucleus.) Assuming the nucleus is a box of length R will be good enough.

(b) If the nucleus is smaller, should the kinetic energy be smaller?

This problem will help students appreciate the different orders of magnitude of nuclear energies compared with atomic energies.

(a) $\lambda = 2R$; $p = h/\lambda = h/2R$.

$$E = p^2/2m = h^2/8mR^2$$
$$= \frac{(6.6 \times 10^{-34})^2}{8 \times 1.7 \times 10^{-27} \times (6 \times 10^{-15})^2} \text{ joule}$$
$$= \underline{9 \times 10^{-13} \text{ joule}}$$
$$= \frac{9 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ ev} \approx 5.5 \times 10^6 \text{ ev}.$$



This is about a million times as big as atomic energies. This is the characteristic difference between nuclear and atomic processes, and it explains, for example, why fusion and fission nuclear bombs are so much more powerful than ordinary chemical explosives.

(b) The energy depends on $1/R^2$. As R decreases, E must increase, and rapidly. The enormous energies associated with nuclear events are due to the small nuclear size, in just this way. We obtained a factor 10^6, even though a neutron is 2000 times heavier than an electron, in itself an effect which would tend to decrease energies by that factor.

PROBLEM 21

Suppose that an electron is confined within a small cubic crystal about 1 micron (10^{-6} m) on a side. Would quantum effects be detectable if you observed changes in the energy of the electron? (Hint: Calculate the lowest possible energy, considering the electron as a standing wave within the crystal.)

From page 652, the expression for the energy is

$$E = \frac{h^2}{8md^2} = \frac{44 \times 10^{-68}}{8 \times 9.1 \times 10^{-31} \times 10^{-12}}$$
$$= 6.1 \times 10^{-26} \text{ joule}$$
$$= 3.8 \times 10^{-7} \text{ ev}.$$

The largest *changes* in energy will be comparable to the lowest energy — the order of 10^{-7} ev. Such changes would be undetectable at ordinary temperatures, which correspond to average kinetic energies of the order of 10^{-2} ev; even at 10°K the average kinetic energy of an atom is about 10^{-3} ev.

Note that $E = h^2/8md^2$ holds only for a one-dimensional box. For the electron in the crystal, E is three times larger because $p^2 = p^2_x + p^2_y + p^2_z$. Here, of course, we are only interested in the order of magnitude.

PROBLEM 22

Suppose the particle between fixed walls (Section 34–5) is charged and can, therefore, radiate. What is the frequency of its radiation as it goes from the first excited state to the ground state? Express your answer in terms of h, m, and d.

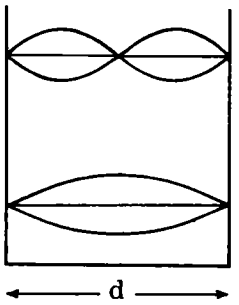
For the two states in question,

$$\lambda_2 = d, \quad p_2 = \frac{h}{\lambda_2} = \frac{h}{d};$$

$$\lambda_1 = 2d, \quad p_1 = \frac{h}{\lambda_1} = \frac{h}{2d}.$$

The two energies are:

$$E_1 = \frac{p_1^2}{2m} = \frac{h^2}{8md^2}; \quad E_2 = \frac{p_2^2}{2m} = \frac{4h^2}{8md^2}.$$



The radiated photon will have energy equal to $E_{\text{photon}} = E_2 - E_1 = 3h^2/8md^2$. Since $E_{\text{photon}} = h\nu$, then $\nu = 3h/8md^2$.

The "frequency" of the charged particle (the classical frequency with which it shoots back and forth across the box) is just $v_1/2d$, where v_1 is the classical velocity, i.e., $v_1 = p_1/m$. Thus, the classical particle frequency for the ground state is

$$\frac{p_1}{2m_1 d} = \frac{2h}{8m_1 d^2}.$$

For the first excited state it will be twice this, since $p_2 = 2p_1$. The average of these two frequencies is just the photon frequency. It is not accidental that the two kinds of frequency are comparable in magnitude, but the fact that the wave mechanical frequency is the arithmetic mean of the two lowest "classical" frequencies is a coincidence.

PROBLEM 23* (See Short Answers.)

PROBLEM 24

(a) Compute the first two wavelengths that you expect for the Lyman series of hydrogen.

(b) Compare these with the spectrum of singly ionized helium (Fig. 34–11). Explain why your comparison indicates that Fig. 34–11 is the Lyman series for ionized He.

(a) The energies involved (see Figure 34–7a) are 10.2 ev (Lyman α) and 12.1 ev (Lyman β). The wavelengths can be found using the formula in the box on page 622:

$$\lambda_\alpha = \frac{1.24 \times 10^4}{10.2} \text{ A} = 1.22 \times 10^3 \text{ A};$$

$$\lambda_\beta = \frac{1.24 \times 10^4}{12.1} \text{ A} = 1.03 \times 10^3 \text{ A}.$$

(These values were obtained in the solution to Problem 17 also.)

(b) The wavelengths involved are of the order of 300 A. In fact, the first two are 303 A and 256 A. Notice that $\lambda_\alpha/4 = 3.0 \times 10^2$ A = 300 A; $\lambda_\beta/4 = 250$ A. So the wavelengths seen in Figure 34–11 are just a quarter of those for hydrogen. This is what we expect for singly ionized helium. The helium nucleus has two elementary charges, and the scale of energy levels of hydrogen-like atoms goes as the square of the nuclear charge. (See the box, page 655, equation 5.) Energies are increased by a factor of 4, as are photon frequencies, so photon wavelengths are decreased by a factor of 4.

APPENDIX 1. THE MAXWELL EXPERIMENT TO VERIFY THE INVERSE SQUARE LAW OF FORCE

The fact that the force in Coulomb's law varies exactly as the inverse square of the separation and not merely some power close to two is most accurately shown by an indirect but simple and beautiful experiment. In this experiment a hollow conducting sphere is highly charged and another smaller conductor touched to its *inside* and then removed. It is found experimentally that the smaller conductor does not pick up *any* charge from the hollow sphere. If there were electric forces acting inside, the sphere charge would be added to or subtracted from the small conductor since it is touched to the large sphere and charge has a chance to transfer between them. Thus this simple experiment proves that the electric force inside a charged conducting sphere is zero. An experiment similar to this and an argument based on such an experiment is given in the film "Coulomb's Law."

Consider the force on a small test charge placed at *any* point, *P*, inside a uniformly charged spherical shell.

Draw the line from *P* through the center, *O*, of the sphere. What possible net force could act on the test charge at *P*? The only possibility is along the line *OP*. The spherical shell has axial symmetry about the diameter through *O* and *P*, so a charge located anywhere on that line could feel no net force in any direction other than along *OP*. The experiment, described above, shows that the resultant force acting on such a test charge is truly zero. It can be easily reasoned that the resultant force along the line *OP* on a charge at *P* can be zero if, and *only* if, the resultant force due to each pair of latitudinal rings, shown as *abdfec* and *ghjlki* in Figure 1, is equal to zero. We will call the forces from these rings F_{ab} and F_{gh} respectively.

Thus our problem is to find out under what conditions the resultant force due to two such rings will be identically zero. Under these conditions, the effects of all such pairs of rings will cancel out and thus the net force on the charge at *P* will be zero.

To calculate the force on a charge, *q*, at *P* due to the ring *abdfec*, we note that all points of the ring are at the same distance, *r*, from *P* (if we take a very narrow ring). Thus the force will be

$$F_{ab} = \frac{Qq}{r_1^{\,n}} \cos \phi$$

where n is the exponent in Coulomb's law.

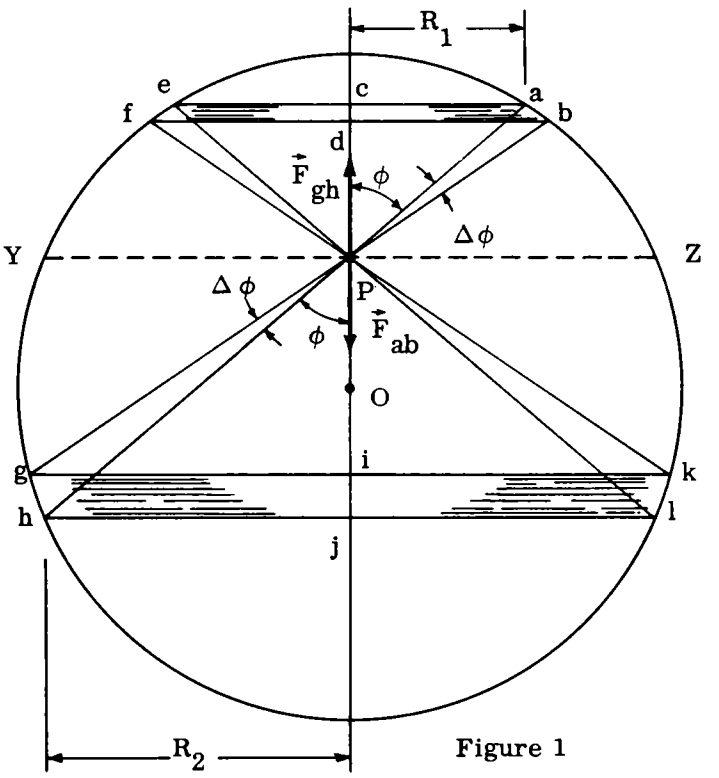


Figure 1

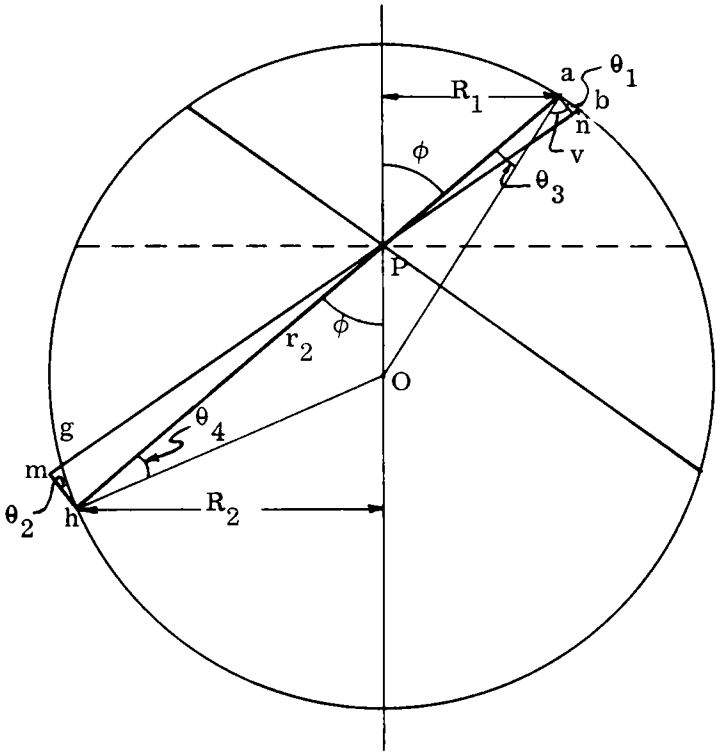


Figure 2

The factor $\cos\phi$ occurs because we are interested only in the *component* of the force along *OP*.

Q, the total charge on the ring in question, is equal to σ, the charge per unit area on the spherical shell, times A, the area of that ring. But A is equal to the circumference of the ring, $2\pi R_1$, times its width, W_1. The force due to the other ring must similarly be

$$F_{gh} = \frac{\sigma 2\pi R_2 W_2 q}{r_2{}^n}\cos\phi.$$

These forces must be equal in magnitude to give the observed null answer. Thus,

$$\frac{\sigma 2\pi R_2 W_2 q}{r_2{}^n}\cos\phi = \frac{\sigma 2\pi R_1 W_1}{r_1{}^n}\cos\phi, \quad \text{or}$$

$$\frac{R_2 W_2}{r_2{}^n} = \frac{R_1 W_1}{r_1{}^n} \quad \text{or} \quad \frac{R_2 W_2}{R_1 W_1} = \left(\frac{r_2}{r_1}\right)^n.$$

A glance at Figure 2, representing a cross section of the sphere, shows that $R_1 = r_1 \sin\phi$, $R_2 = r_2 \sin\phi$.

And $W_1 = \widehat{ab} = \dfrac{\overline{an}}{\cos\theta_1}$, $W_2 = \widehat{gh} = \dfrac{\overline{mh}}{\cos\theta_2}$.

But $\theta_2 = \theta_4$;
$\theta_1 = \theta_3 (\theta_1 + v = 90°, \theta_3 + v = 90°; \therefore \theta = \theta 3)$
and $\theta_3 = \theta_4$ (base angles of isosceles triangle)
$\therefore \theta_1 = \theta_2$.

Thus

$$\frac{W_1}{W_2} = \frac{\overline{an}}{\cos\theta_1} \times \frac{\cos\theta_2}{\overline{mh}} = \frac{\overline{an}}{\overline{mh}} = \frac{r_1}{r_2}.$$

Now, taking

$$\frac{F_{ab}}{F_{gh}} = \frac{R_2 W_2}{R_1 W_1} = \left(\frac{r_2}{r_1}\right)^n,$$

and knowing that $\dfrac{W_1}{W_2} = \dfrac{r_1}{r_2}$; and that $R_1 = r_1 \sin\phi$

and $R_2 = r_2 \sin\phi$, we can write

$$\frac{F_{ab}}{F_{gh}} = \frac{R_2 W_2}{R_1 W_1} = \frac{r_2 \sin\phi \times r_2}{r_1 \sin\phi \times r_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^n.$$

The only value of n that will make the above equation a true identity is $n \equiv 2$.

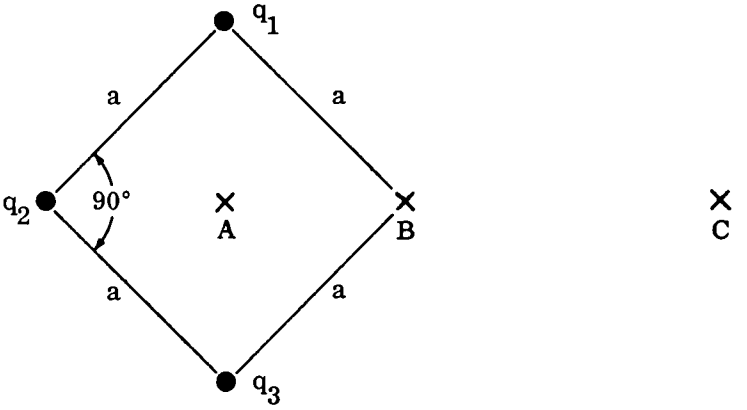
Thus the observation of zero force acting on a point charge inside a charged spherical shell leads uniquely to the result that the exponent of the distance in Coulomb's law must be two. The exactness of the determination of the exponent depends on the accuracy of the measurement of a null force. This kind of an experiment can be done very precisely.

Much more sophisticated arguments lead to a proof that even if the outer conductor is not a sphere the electric forces inside it are still zero.

A corollary of this proof is that if a *charged* conducting object is touched to the inside of a hollow conductor it will lose all its charge to the outer conductor. The proof is simple. The particles of charge repel one another and thereby try to get off the object. If there is no force from a charge on the outer conductor, as soon as the two conductors touch, all the charge will leave the inner one. This charge immediately flows to the outside of the outer conductor and by our demonstration exerts no counterforce to keep any remaining charge on the inner conductor. Why the outside? The *inside* of the outer conductor is just like a conductor inside of another hollow conductor so we know that all the charge will leave it, etc.

The fact that all the charge goes to the outside of a *spherical* conductor is often erroneously explained as due to the fact that like charges repel and surely they will get as far as possible away from each other, i.e., to the outside. This will only happen if the law of repulsion is an inverse r^2 one. Any other law of force will leave some charge inside. To see how this might work, think of two charged beads sliding on a wire. They will slide to the ends under a mutual repulsion no matter what the law of force. But put two more on. They do not go to the ends because the ones already there repel the new ones. A similar thing would happen in three-dimensional space if the law of force were not exactly inverse r^2.

APPENDIX 2. SUPPLEMENTARY WORK FOR SECTION 27-3, ELECTRIC FORCE FIELDS



The second paragraph of Section 27-3 introduces the idea that the effect of the forces due to several charges acting on a given charge can be found by adding the individual forces vectorially. In this course it is not intended to give major emphasis to this idea. If you have the time (!) some practice along these lines will stand your students in good stead in their college work. A sample problem might be the following: given four equal charges, q_1, q_2, q_3, and q_4, with q_1, q_2, and q_3 fixed as shown in the diagram, what is the force on q_4 when it is at A? B? C? The force between any two of these charges at a distance, a, is 0.10 newton. When the charge q_4 is at A, the forces are as shown at (a) below, The magnitude of each force was found from Coulomb's law.

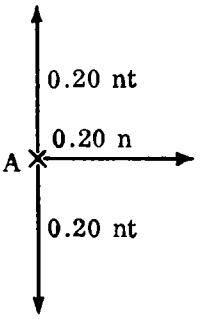
$$F = k\frac{q^2}{\left(\frac{a}{\sqrt{2}}\right)^2} = k\frac{q^2}{a^2/2} = 2 \times 0.10 = 0.20 \text{ newton.}$$

A vectorial addition gives the result, 0.2 newton to the right. At (b) the forces are shown when the charge is at B. The two diagonal forces are due to two charges q at a distance a and are therefore 0.1 newton each. The force from the charge on the left is

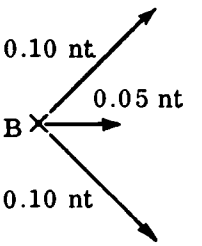
$$F = k\frac{q^2}{(\sqrt{2}\,a)^2} = \frac{1}{2}k\frac{q^2}{a^2} = 0.05 \text{ newton.}$$

Doing necessary vector addition geometrically gives $(0.10 \times \sqrt{2} + 0.05)$ to the right or 0.101 newton.

The point at C can be anywhere far away. This part of the problem is meant to be done graphically. The idea is to show that the force at C approaches that due to a single point charge, $3q$, placed somewhere near the center of the three charges. This last idea is important. At a long distance from a group of charges, the electric force on a test charge is the same as that which would arise from a point charge which is equal to the algebraic sum of the group of charges and is placed somewhere near the center of the group. That is, at long distances from the group of charges, the force is an inverse square force. Close to a group, the force is not necessarily an inverse square force and must be calculated by vectorial addition.



(a)



(b)

APPENDIX 3. DISCUSSION OF SUPPLEMENTARY INFORMATION ON PAGE 585

The supplementary material in the box on page 585 is a necessary step for good understanding of the quantitative discussion of the speed of propagation of electromagnetic waves. The following material may help clarify some of the points presented there.

The text begins by discussing a point charge case which the students can understand but which may not sound much like a changing electric field between conducting plates, which was our original model for discussion. If the question should arise, the connection between these two models can be made in the following way.

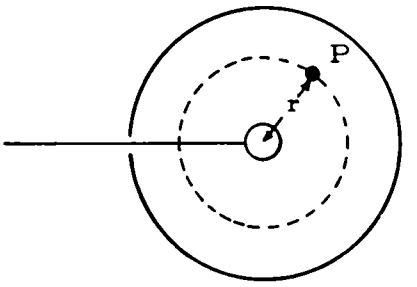
Section 7 of the text discusses electric fields between parallel plates. Such fields are easy to think about, but hard to handle quantitatively with the techniques we have available. Therefore, we consider the situation shown below.

On the left below we have the experimental arrangement of Section 7. Two parallel conducting plates are being charged through wires as shown. In order to charge the left-hand plate positively, a current is flowing onto the left-hand plate. Consequently a current is leaving the right-hand plate, leaving it negatively charged. The electric field is clearly from the left plate *toward* the right plate and is increasing as long as the current continues to flow. If the current stops the field remains; it simply stops changing. Thus it is the change of field which is the continuation of the current.

In the arrangement shown at the right above we have a slightly altered situation. Through a wire, a charge is supplied to a small conducting ball at the center of a hollow spherical conductor. The outer conductor has a small hole to admit the wire to the inner sphere. As current flows onto the ball, current also leaves the outer conductor, leaving it negatively charged. The two charges are equal as long as the two currents are always equal. An electric field exists between the two conductors. The field is radially out from the center (i.e., a small positive test charge placed between the spheres would be forced out away from the center). The center conductor thus plays the role of the left-hand plate in the previous diagram, and the outer conductor that of the right-hand plate.

Suppose at time $t = 0$ we start with neither sphere charged and allow a *constant* current I to flow onto the inner sphere. This means that there are I elementary charges per second flowing along the wire to the sphere. After a time, t, there are then $I \times t$ elementary charges on the sphere. They spread out uniformly. Suppose we consider a small test charge of one elementary charge placed at point P, a distance r from the center of the inner sphere as shown below. By Coulomb's law we know that the force on it will be

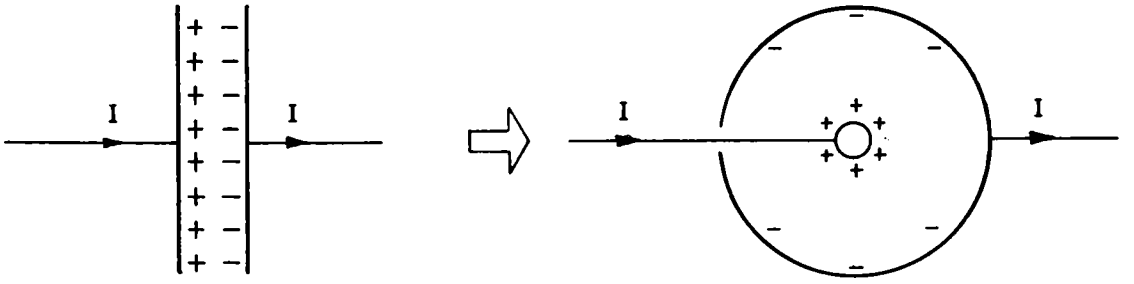
$$F = k \frac{It \times 1}{r^2}.$$



Since k is the appropriate constant, it is the charge at the center and 1 is the magnitude of the test charge. Our definition of electric field is simply the force on one elementary charge so that we conclude that the electric field at a distance r from the center is

$$E_r = \frac{kI}{r^2} t.$$

We note that the electric field is symmetrical and everywhere the same over a sphere of radius r, shown



dotted in the figure. We introduce the term electric flux ϕ_E, which is defined to be the electric field multiplied by the area we are considering. We do not need to introduce this term, but it makes things a bit easier to discuss and facilitates comparison with the magnetic flux. The electric flux over the dotted sphere is thus the area of the sphere times the field or

$$\phi_E = 4\pi r^2 \frac{kI}{r^2} t = 4\pi kIt.$$

Now it can be seen that the electric flux is changing at a rate

$$\frac{\Delta\phi_E}{\Delta t} = 4\pi kI$$

$$\left[\begin{array}{l} \Delta\phi_E = 4\pi kIt_2 - 4\pi kIt_1 = 4\pi kI(t_2 - t_1) \\ \dfrac{\Delta\phi}{t_2 - t_1} = \dfrac{\Delta\phi}{\Delta t} = 4\pi kI \end{array}\right]$$

Thus,

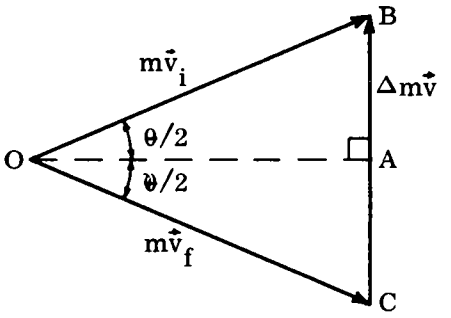
$$I = \frac{1}{4\pi k} \frac{\Delta\phi_E}{\Delta t}$$

Notice that we began by talking about the electric flux across a particular sphere of radius r but that r does not appear in our answer. The flux is the same across *any* sphere drawn between the spherical conductors. This is necessary if the rate of change of electric flux is to be considered a "current" flow. Thus it appears that the rate of change of flux can be considered as an extension of the current flowing into the center sphere and out of the hollow sphere. Although the statement is not rigorous, it seems that the current I "turns into" a current, $\frac{1}{4\pi k} \frac{\Delta\phi_E}{\Delta t}$, to cross empty space.

APPENDIX 4.

PART 1. THE RUTHERFORD SCATTERING FORMULA. APPROXIMATE DERIVATION FOR SMALL ANGLES OF SCATTERING.

If you have the time you can make the results of Section 3 more plausible by working out the relationship between scattering angle and aiming error in an approximation which is valid for small deflections (large aiming errors). The deflection is easily calculated if we know the vector change of momentum in the collision; we only need to draw the vector sum triangle showing the initial momentum $m\vec{v}_i$, the final momentum $m\vec{v}_f$, and the vector change $\Delta(m\vec{v})$. By energy conservation we know that the magnitudes of the initial and of the final momentum are equal $\left(\frac{mv_i^2}{2} = \frac{mv_f^2}{2} \text{ implies } mv_i = mv_f\right)$. The scattering angle θ is of course the angle between the initial and final momenta:



$$m\vec{v}_f = m\vec{v}_i + \Delta m\vec{v}$$

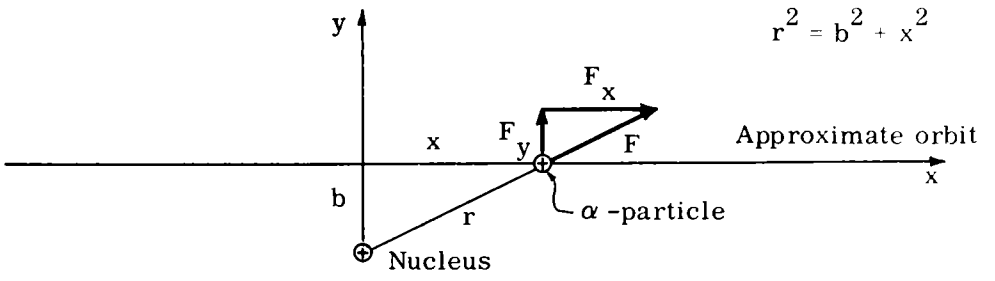
$$\frac{AB}{OB} = \frac{AC}{OC} = \sin\frac{\theta}{2}$$

$$OB = OC = mv$$

$$|\Delta m\vec{v}| = CB = AB + AC = 2mv \sin\frac{\theta}{2}$$

The momentum change in the collision is equal to the impulse of the force acting on the particle. Since the force is not constant, we divide the collision time into a large number of small intervals, Δt, short enough so that the force is nearly constant over each interval. The impulse, and therefore the momentum change, in each time interval, Δt, is given by $\vec{F}\,\Delta t$. Taking the vector sum of all the partial impulses, $\vec{F}\,\Delta t$, we get the total impulse and therefore the momentum change for the whole collision. We must of course take into account the change in $\mathbf{F}$ with time and use the value appropriate to each interval of time.

Everything that we have said so far is exact. However, if we try to evaluate the result we run into a difficulty. The force, $\vec{F}$, on the particle at time, t, depends on where the particle is at that time, and we do not know this until we have worked out the orbit. We can get around this difficulty by noting that if the aiming error is large the particle will move in an orbit which is approximately straight and at a speed which is nearly constant. We can therefore get an approximate value for the impulse by working out the impulse which the particle would experience if it were to move in a straight line at a constant speed, v_0. If the aiming error is b, the approximate orbit is a straight line passing a distance b from the nucleus. The force on the particle is $F = \frac{kq_\alpha q_N}{r^2}$. In our approximation the x-component of momentum does not change in the complete collision (the impulse to the left in the first half of the orbit is cancelled by an equal impulse to the right in the second half).



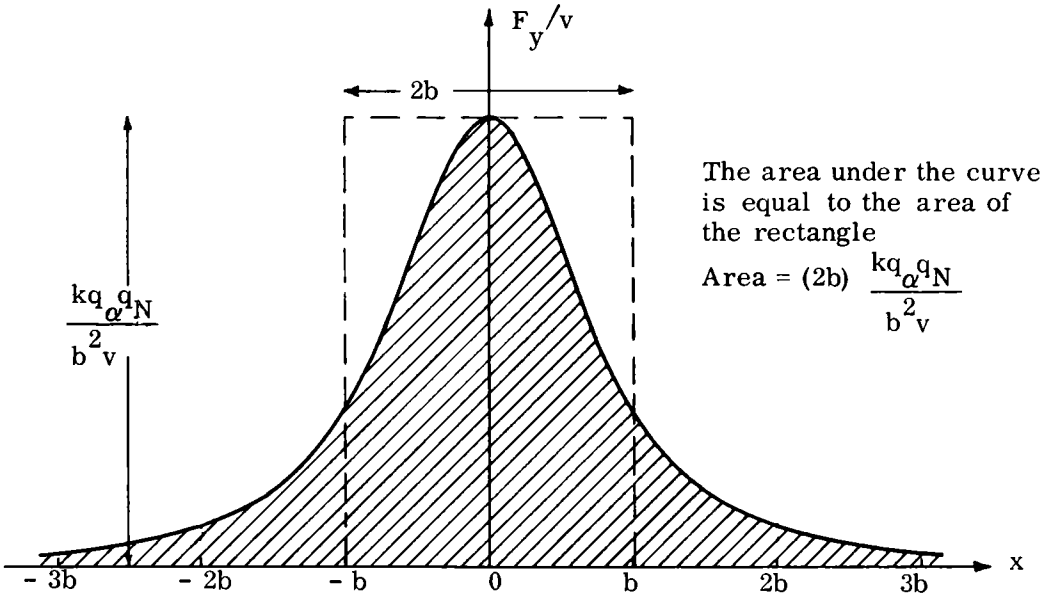
To calculate the y-component of impulse we consider the y-component of the force:

$$F_y = \frac{b}{r} F = \frac{kq_\alpha q_N}{r^2} \frac{b}{r} \text{ (by similar triangles).}$$

The particle moves a distance Δx in a time $\Delta t = \frac{\Delta x}{v}$, therefore the y-component of impulse acting on the particle as it moves a distance Δx is given by:

$$F_y \Delta t = \frac{F_y}{v} \Delta x = \left(\frac{kq_\alpha q_N b}{vr^3}\right) \Delta x.$$

The sum of all the little impulses is given by the area under the curve of $\frac{F_y}{v} = \frac{kq_\alpha q_N}{v} \times \frac{b}{r^3}$ vs. x. We have drawn such a graph (using $r^2 = b^2 + x^2$ to calculate r). We can measure the area under the curve (or evaluate it by using calculus). The result is:



$$\text{Impulse} = \frac{2kq_\alpha q_N}{bv}$$

Equating the impulse to the momentum change $2mv \sin \theta/2$, (see first figure on the previous page) we find:

$$2 \sin \frac{\theta}{2} = \frac{r_0}{b},$$

where $r^0 = \frac{kq_\alpha q_N}{E_k}$ is the distance of closest approach for a head-on collision (see 32–4).

The exact result is:

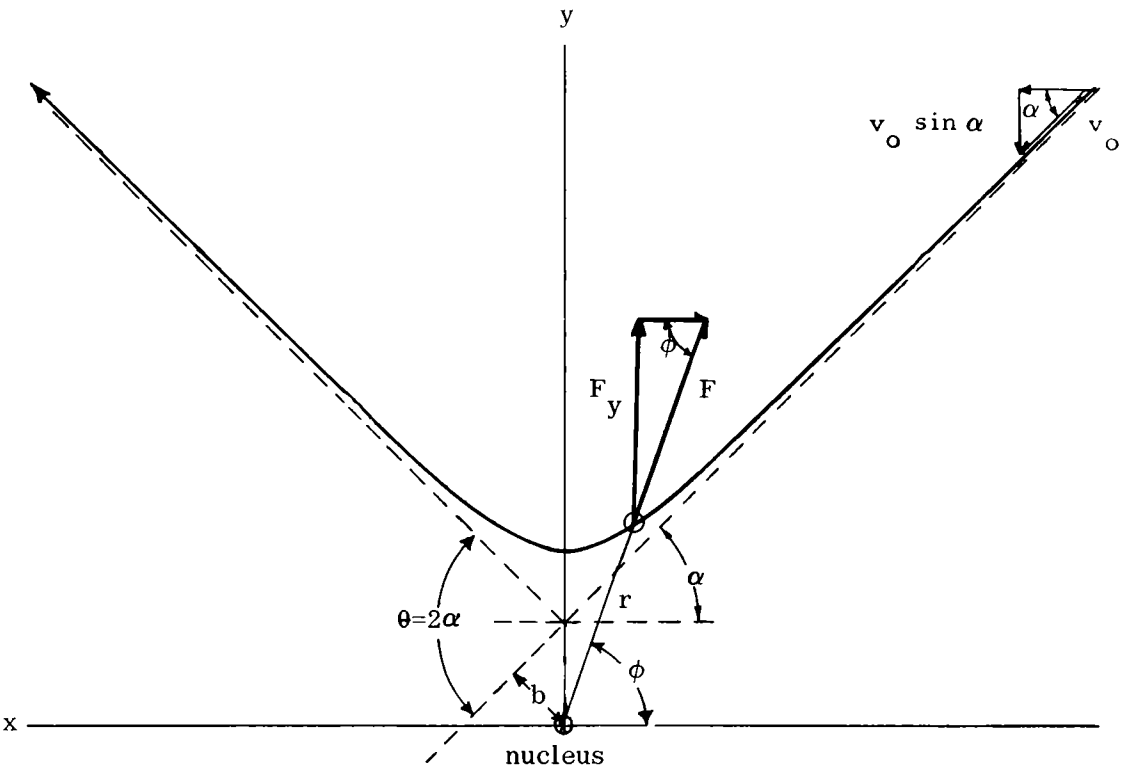
$$2 \tan \frac{\theta}{2} = \frac{r_0}{b}.$$

For small angles $\sin \frac{\theta}{2}$ and $\tan \frac{\theta}{2}$ are nearly the same.

PART 2. DERIVATION OF THE RUTHERFORD SCATTERING FORMULA.

A derivation of the Rutherford scattering formula is given here for your reference. Although the derivation is fairly simple, it uses calculus and conservation of angular-momentum, and is therefore too advanced for most high school students.

We suppose that the mass of the nucleus is much larger than the mass of the alpha particle, so that we may consider the nucleus to remain at rest. Since the orbit lies in a plane which contains the nucleus we can describe the position of the alpha particle by two cartesian coordinates, x and y, measured from the nucleus as origin. For convenience, we choose the y-axis to divide the orbit symmetrically. Using New-



ton's law of motion and Coulomb's law of force, we calculate the y-component of the acceleration:

$$\frac{dv_y}{dt} = \frac{F_y}{m_\alpha} = \frac{F \sin \phi}{m_\alpha} = \frac{kq_\alpha q_N}{m_\alpha} \frac{\sin \phi}{r^2}. \quad \text{(See figure.)}$$

Since the force on the alpha particle acts along a line through the nucleus, there is no torque (moment of force) about the nucleus; therefore the angular momentum (moment of momentum) about the nucleus is constant. The angular momentum is given at any instant by the product $mr^2 \frac{d\phi}{dt}$ of the distance r, from the nucleus and of the component of momentum $\left(mv_\perp = mr \frac{d\phi}{dt}\right)$ perpendicular to the line drawn from the nucleus to the alpha particle. The alpha particle moves initially with momentum mv_0 along a straight line which passes at a distance b from the nucleus; therefore the initial angular momentum is $b(mv_0)$. Since the angular momentum is constant we have:

$$mr^2 \frac{d\phi}{dt} = mv_0 b, \quad \text{or} \quad \frac{d\phi}{dt} = \frac{v_0 b}{r^2}.$$

We use the last result to eliminate the time from the equation for v_y:

$$\frac{dv_y}{d\phi} = \frac{\frac{dv_y}{dt}}{\frac{d\phi}{dt}} = \frac{\frac{kq_\alpha q_N}{m_\alpha} \frac{\sin \phi}{r^2}}{\frac{v_0 b}{r^2}} = \frac{1}{2} v_0 \frac{r_0}{b} \sin \phi;$$

with the abbreviation:

$$r_o = \frac{kq_\alpha q_N}{\frac{m_\alpha v_0^2}{2}} = \frac{kq_\alpha q_N}{E_\alpha}.$$

We know that $\sin \phi = -d \cos \phi / d\phi$; we also know that $v_y = 0$ at $\phi = \frac{\pi}{2}$; we can therefore integrate the equation for $dv_y/d\phi$ to obtain

$$v_y = -\tfrac{1}{2} v_0 \frac{r_0}{b} \cos \phi.$$

When the alpha particle is still a long way from the nucleus, $\phi = \alpha$ and our last equation gives $v_y = -\frac{1}{2} v_0 \frac{r_0}{b} \cos \alpha$. Since at $\phi = \pi/2$ the motion is $\perp$ to y, $v_y = 0$ at that point. We also have $v_y = -v_0 \sin \alpha$. In order that the two results agree we must require:

$$-\tfrac{1}{2} v_0 \frac{r_0}{b} \cos \alpha = -v_0 \sin \alpha.$$

Since, as shown on the figure, $\alpha = \theta/2$, we get

$$2 \tan \alpha = 2 \tan \frac{\theta}{2} = \frac{r_0}{b}.$$

NUMBER	EXPERIMENT	BEST TIME	PRIORITY
IV–1	Electrified Objects	Before the study of Chapter 26	**
IV–2	Electrostatic Induction	Same time or shortly after Section 26–5	**
IV–3	The Force Between Two Charged Spheres	Before Chapter 27	***
IV–4	The Addition of Electric Forces	After Experiment IV–3	*
IV–5	Driving Force and Terminal Velocity	Before Section 27–4	**
IV–6	The Millikan Experiment	Before Section 27–5	**
IV–7	The Charge Carried by Ions in Solution	After discussion of Section 28–3	**
IV–8	The Charge on a Capacitor	During Chapter 29	**
IV–9	Energy Transfer by an Electric Motor	During Chapter 29	*
IV–10	The Magnetic Field of a Current	Before Section 30–3	**
IV–11	The Measurement of a Magnetic Field in Fundamental Units	After Section 30–4	***
IV–12	The Mass of the Electron	After doing Experiment IV–11 and discussing Section 30–6	***
IV–13	The Magnetic Field Near a Long Straight Wire	Before Section 30–9	**
IV–14	Randomness in Radioactive Decay	During Chapter 33 or later	***
IV–15	The Spectrum of Hydrogen and Planck's Constant	During Section 34–4	***

*** essential
** desirable
* optional

IV–1 Electrified Objects

This experiment requires little time and equipment and may be done at home. It is best done before the study of Chapter 26. If your students are already familiar with the simple behavior of positively and negatively charged objects, it may be done as a demonstration. In any case, it prepares the student for the three following experiments, which assume a familiarity with simple electrostatics.

The success of Experiments IV–1 to IV–4 depends greatly upon the relative humidity in the laboratory. If it is about 50 percent or less, the experiments will work well, but if it is about 80 percent or more, charges will leak off across insulators, making the experiments difficult to do. To reduce leakage in high humidity, store the apparatus in a large box heated by an electric bulb or in a heated room, and remove it just before use. Vinylite, which takes a negative charge when rubbed with wool, and cellulose acetate, which becomes positively charged when rubbed with cotton, develop strong charges and work better than glass, Bakelite, or other plastics when the humidity is high. However, if the surface becomes dirty, it may be difficult to keep charge on a rubbed plastic strip. Charge cannot be easily removed from vinylite and cellulose acetate by rubbing the surface with the hand. Passing the strip quickly over a flame will discharge it.

The two different strips can be hung from a crossbar attached to a ringstand. They should be far apart and far enough from the metal of the ringstand so they will not interact with each other or with the induced charges on the ringstand.

Students may find that a charged plastic strip will always attract an uncharged strip, although they are not told to look for this effect until the end of the experiment. This inductive attraction is not easy to understand until induction has been studied in the text and in Experiment IV–2. The explanation should probably be deferred until later.

ANSWERS TO QUESTIONS

A vinylite strip rubbed with wool repels another similarly rubbed vinylite strip, and attracts a cellulose acetate strip that has been rubbed with cotton. There appear to be at least two kinds of electric charge.

As a result of bringing acetate rubbed with cotton near another similarly charged strip, one can conclude that the like charges we used repel each other and the unlike charges attract.

The wool that charged the vinylite has a charge unlike that on the vinylite. Holding the wool close to the acetate shows that they repel each other and hence probably have the same charge. (At this point

the student may make the assumption that there are only two kinds of charge, which can be given names such as *a* and *b*, *p* and *q*, or plus and minus. He can then try to explain all further results on the basis of this assumption.)

Almost any reasonably good insulator when rubbed will become charged to some extent, and the student can identify its charge as being the same as or opposite to the charges he has found by observing its effect on charged vinylite and cellulose acetate strips.

The general conclusions resulting from the experiment are:

1. There are two kinds of charge.
2. Like charges repel, opposite charges attract.
3. Two neutral objects, rubbed together, become oppositely charged by close contact.

The names are of no importance. It makes no difference which we call "plus" or which we call "minus" (or "a" or "b").

If a student has observed that both kinds of charged plastic attract an unrubbed object, he is justified in concluding that an unrubbed object has a "charge" that attracts all rubbed objects but does not affect another unrubbed object. He will later find that there is really no new kind of charge involved.

Unless the student is sophisticated in matters electrical, you should not expect him to give an adequate explanation of how uncharged paper or thread can be attracted. The question is given to him simply to make him aware that all is not so simple as it seems.

Students should be encouraged to try charging various substances with different materials. They will conclude that there is no easy way to predict what charge substance *x* will gain when rubbed with material *y*. They are right! In fact, depending on the humidity and cleanliness of the surface, some substances can be charged with either sign when rubbed with the same material.

The difference between insulators and conductors can be learned by having the student try charging metal rods. The charge will leak off as fast as it is formed; but if the rubbed metal rod is held by a good insulator, it will become charged like the plastic but the charge will spread all over the rod and will not stay localized as on an insulator.

APPARATUS

1 Cotton cloth
1 Wool cloth
2 Cellulose acetate strips
2 Vinylite strips
1 Ringstand, 36″
1 Rod, 24″
1 Clamp holder

Cellulose or masking tape

USEFUL HINTS ON LOW-MOLECULAR-WEIGHT POLYETHYLENE INSULATION

When the humidity is high, charge leakage occurs across the surface of otherwise good insulating materials because dust and dirt on the surface absorb water and form a thin conducting layer of electrolyte. Low-molecular-weight polyethylene does not pick up dust and dirt as easily as many other insulators and so has a very low surface leakage in humid conditions *if the surface is kept clean.*

Low-molecular-weight polyethylene is a waxlike substance resembling paraffin in appearance and texture. Although harder and stronger than paraffin, it is not as hard or as strong as Bakelite or other common plastic insulators. It can be melted in a clean pan at 112°C until it is fluid enough to be poured into molds to form cylinders, plates, etc. Its tendency to shrink upon cooling will make it easy to remove from molds. Do not use polyethylene that has become discolored through overheating.

Like all insulators, after it has been cast it must be kept clean. If it becomes dirty and shows excessive leakage with use, it may be cleaned by washing with soap and water or by scraping with a knife to give it a clean, new surface.

The material can be easily sawed and trimmed. Holes less than about 1 inch deep can be easily drilled, but deeper holes must be drilled slowly to avoid softening the material so it sticks to the drill. It is better to drill deep holes in steps, drilling a short distance, withdrawing the drill, and cleaning it before continuing.

IV-2 Electrostatic Induction

Electrostatic induction is not easy for students to understand. It may be best to do this experiment at the same time or shortly after the topic is studied in the text. Very able students however, may find it a challenging problem before they have learned about induction. The purpose of the experiment is to familiarize the student with the separation and transfer of charges. In the last part of the experiment he will become familiar with the behavior and use of an electroscope.

If metal rods about 10 cm long and a centimeter in diameter are not available, other similar objects such as spoons, table knives, or pieces of metal tubing may be used. Wires also may be used, but their capacity is so low that they produce very small deflections of the suspended foil. In any case, try to avoid using objects with sharp points or edges since they lead to corona discharge or spark discharges between the objects and the charging plastic strips. If such effects occur, the rods must be discharged and the experiment repeated.

If the humidity is high, to prevent leakage it may be necessary to support the rods on pieces of low-molecular-weight polyethylene.

The sign of the charge on the electroscope can be determined by bringing charged vinylite and cellulose acetate strips close to the instrument.

A leaf electroscope or a "soda straw" electroscope is most sensitive for small deflections. If it is charged to more than about a 45° deflection, it will show little change when another moderately charged object is brought close.

If there is sufficient time, students may try the following variation on the separation of charges by induction: charge two insulated metal rods by induction as in the first part of the experiment, then charge one electroscope by contact with one rod and another electroscope with the other rod. The two electroscopes will then discharge completely when their terminals are connected. The electroscopes must be quite sensitive to show this clearly.

It might be pointed out to students that when a charged object picks up bits of insulating materials like paper and thread, the attraction is primarily the result of the separation of charge like that in a conductor rather than polarization, the distortion of charges in individual molecules. A perfect insulator would show only the weak attraction resulting from polarization. In ordinary insulators there is sufficient conduction so the greater charge separation resulting from conduction masks the effects of polarization.

APPARATUS

2 Metal rods, 6″
2 Beakers (200 to 500 ml)
1 Cotton (or wool) cloth
1 Cellulose acetate (or vinylite) strip
1 Ringstand, 36″
1 Rod, 24″
1 Clamp holder

Thread, aluminum foil, thin cellulose or masking tape
Electroscopes, several

(Added material for higher humidity)
4 Insulators
1 Capacitor, 20 kv, 500 mmf

High-voltage source kit:
4 No. 6 Dry cells, $1\frac{1}{2}$-volt
3 Connecting wires, 6″

A SPARK-COIL HIGH-VOLTAGE SOURCE

When the humidity is high, or when consistent charging voltage is required, a Ford vibrator spark coil can be used as a source of electric charge. Don't try to explain its operation; it can be introduced as a "black box."

Connect a Ford ignition coil to batteries and a protective resistor as shown in Fig. (a). The coil and resistor should be enclosed in a box that permits access only to the protected high voltage.

A high-voltage capacitor is normally used to accumulate a charge from the ignition coil. The capacitor is then used to apply the charge to the desired object by direct contact.

To charge the capacitor, hold it by its molded case and touch its two terminals to the tips of the high-voltage wires while the vibrator is running. Withdraw the capacitor so that sparks jump to its terminals as it is moved away. The vibrator should be turned off only after the capacitor has been withdrawn. The

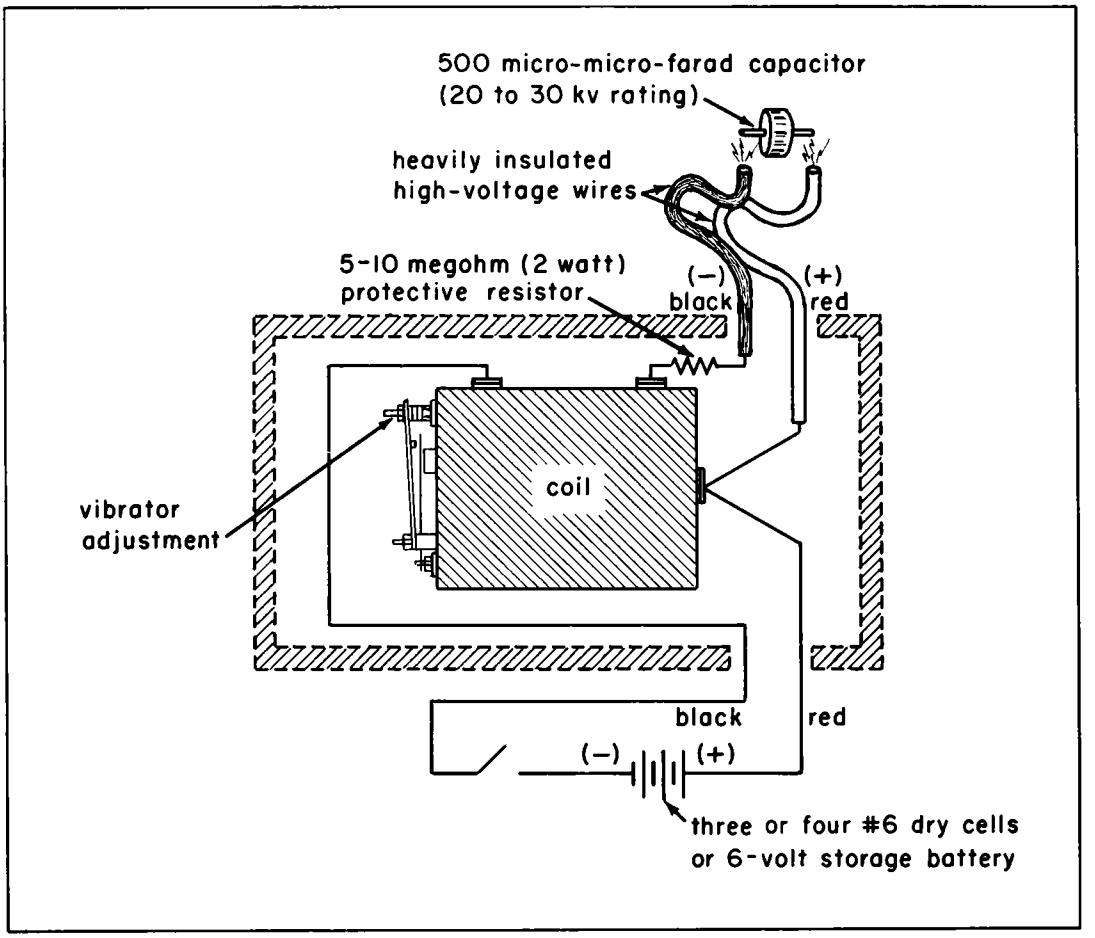
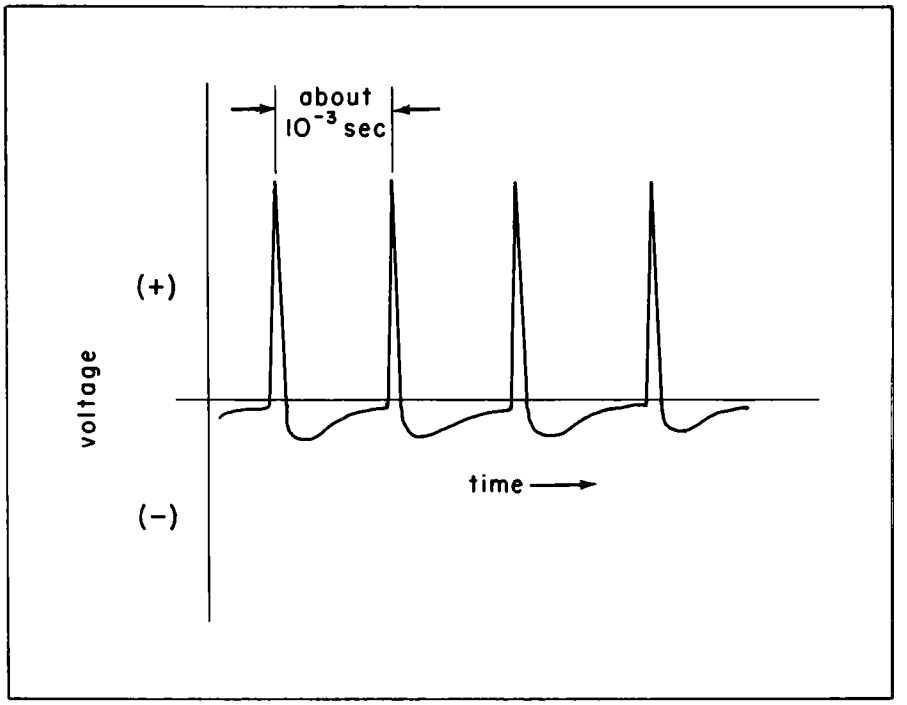


Figure (a)

Figure (b)



capacitor is now charged to a potential of about 3000 volts. The terminal that received sparks from the red high-voltage wire will be positively charged. The other terminal, which was near the black wire, will be negatively charged. Grasp either terminal (*but not both simultaneously*) depending upon which polarity you wish grounded, and touch the other to the electroscope or other object that you wish to charge. A single charging of the capacitor will charge a neutral electroscope to nearly full deflection about forty times in succession.

The alternating voltage produced by the coil is asymmetrical; the peak voltage on one half of the cycle is many times greater than the peak voltage on the other half [Fig. (b)]. As the capacitor is removed from the coil terminals while the coil is still on, sparks continue to jump until the capacitor terminals are nearly a centimeter away from the coil terminals. At this distance, the capacitor is charged only on the highest-voltage half of the cycle (which is sufficiently high to spark across). As a result, each time the capacitor is withdrawn from the terminals (with the coil on), it is charged with the same polarity and to nearly the same potential. Reversing the battery polarity reverses the asymmetry and hence the output polarity of the coil.

The coil can be used directly to charge electroscopes and other objects by the jumping-spark method. The capacitor serves only as a convenient, transportable reservoir of charge.

When voltages higher than about 3000 are required, this value may be doubled by screwing two capacitors together in series and charging them in succession. The voltage may be tripled by using three capacitors and the same charging method.

IV–3 The Force Between Two Charged Spheres

This is an important experiment and should not be omitted unless the humidity is too high. If it is done before Chapter 27 is studied, the student will discover for himself the inverse-square nature of electric forces.

The experiment is delicate and difficult to perform, and the results will probably have to be analyzed at home. A few practice runs, taking data but not analyzing it, are advisable so a quick final run can be made before leakage becomes excessive.

Under very humid conditions you can set up the pith-ball arrangement and apply heat to the inside of the carton for about three minutes with an infra-red drying lamp (reflector type) before doing the experiment. If the effect of this drying wears off before the students can get enough data, you can apply the lamp again.

The pith balls are coated with a colloidal suspension of graphite in alcohol (commercially known as "Dag 154") or with India ink. (Aluminum paint is *not* a good conductor.) A single nylon fiber is used to support the hanging pith ball because it is a good insulator and weighs much less than the pith ball. If the supporting threads are too heavy, they will hang in a curve (a catenary) when the pith ball is deflected. As the deflection increases, the curve will become more and more pronounced and the horizontal deflection will not be proportional to the force exerted; large errors will occur. The nylon fiber can be straightened by stretching it with a firm pull. The pith ball is then attached by threading the fiber through a fine needle hole poked in the ball. Apply a bit of glue to the fiber and pull the glue inside the ball. After the glue has dried, the short end of the fiber may be cut off. For the two-filament suspension, both fibers should come out of the same side of the ball. If the humidity is high, it may be advisable to support the suspending fibers from pieces of low-molecular-weight polyethylene. The support for the pith ball that is moved along the base should also be of a good insulating material.

Both spheres are given charges of the same sign by conduction from the electrophorus. If the spheres are given opposite charges and attract each other, they may swing together, touch and discharge, making the experiment more difficult to perform and yielding results which are not very satisfactory. The two spheres should be at the same level above the base, and when taking readings they should not be close to the walls of the box, the hand, or other objects since induced charges will cause errors.

The student may find that if he brings the spheres very close together, the inverse-square relation seems to break down. This is because the inverse-square law holds for point charges. The spheres behave like point charges only when separated by distances several times their diameters. When very close, the balls cease to act like point charges because of the separation of charge on the spheres by induction. [See Fig. (a).]

Since the distance between corresponding sides of the spheres equals the separation of the centers of the spheres, the readings of the spheres' positions are taken from these sides because they can be read more accurately than the centers.

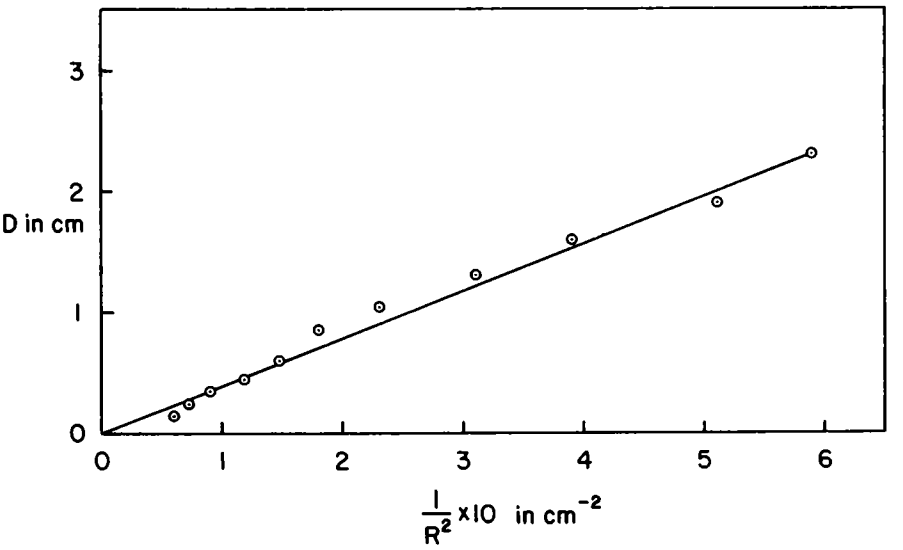


Figure (a)

ANSWERS TO QUESTIONS

At the end of the run, the sphere on the base can be moved to the position where the first reading was made. If there has been any leakage, the deflection of the hanging ball will be less than it was originally.

The graph of the forces as a function of separation will not, of course, tell the student at a glance the algebraic form of the function. However, the force at any distance r should be close to one-quarter the force at $\frac{1}{2}r$ and close to $(\frac{2}{3})^2 = \frac{4}{9}$ the force at $\frac{2}{3}r$. This suggests an inverse-square dependence, and a graph of F as a function of $(1/r^2)$ should give a straight line.

APPARATUS

1 Coulomb's Law Apparatus Kit, consisting of
- Box with mirror, scale, base, and lid
- Styrofoam spheres, 0.5 cm diameter
- Nylon fibers
- Plastic strip with charging cloth
- Electrophorus disc with handle
- Two blocks with spheres on insulating rods and keys
- Dag 154 suspension of graphite

IV-4 The Addition of Electric Forces

The purpose of this experiment is to show the vector nature of electric forces. This is an important concept. Nevertheless, the experiment is given a low priority because it is difficult to perform and analyze; charge leakage limits the accuracy to 10 percent at best.

Charge leakage during the experiment should be checked in the same way as in Experiment IV-3. The students should record the positions of the shadow of the suspended ball on a second piece of graph paper outside the box, so that it will not be necessary to reach inside the box.

When both spheres A and B are used simultaneously, the distance between A and C (or B and C) will be different from the distance obtained when either A or B alone interacts with C. To compute the forces between A and C and between B and C at these new distances, the student must use the inverse-square law. It may be worth while to present the following method of analysis to the class:

We first measure the deflection of sphere C by B alone [Fig. (a)]; with a separation r_B we find a deflection d_B. Similarly we measure the deflection of C by sphere A alone [Fig. (b)] and find a deflection d_A with a separation r_A.

If both spheres A and B act on C [Fig. (c)], we measure a deflection D with the separations R_A and R_B. (Note that when spheres A and B both act on C, they do not have to be in the same positions they occupied when each acted alone on C).

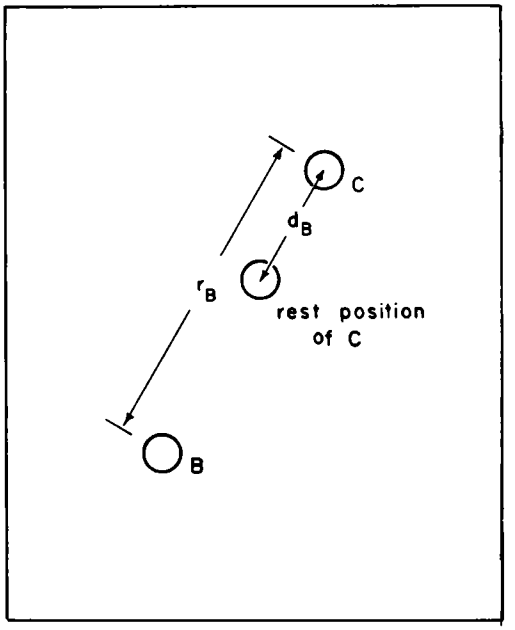


Figure (a)

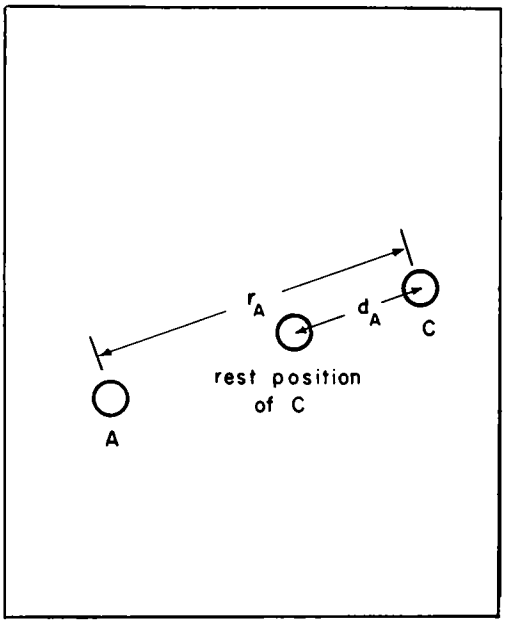


Figure (b)

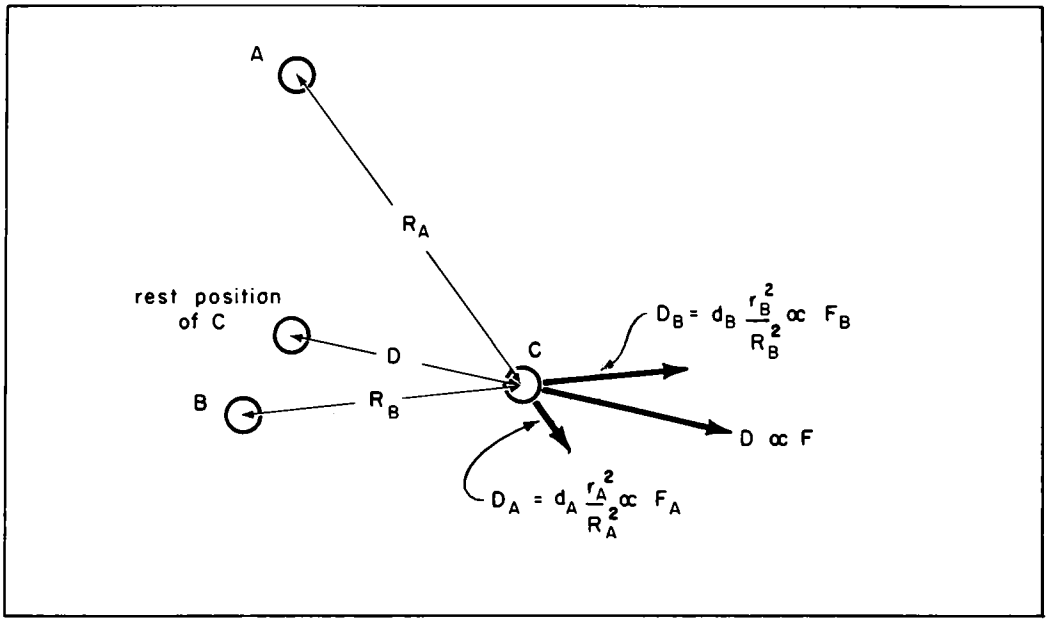


Figure (c)

Using Coulomb's law, we can express the force F_A that A exerts on C when B is present in terms of the force f_A it exerts when B is not present,

$$\frac{F_A}{f_A} = \frac{{r_A}^2}{{R_A}^2}.$$

Since the forces are proportional to the deflection:

$$\frac{F_A}{f_A} = \frac{D_A}{d_A} = \frac{{r_A}^2}{{R_A}^2}$$

where D_A is the deflection of C if A and C were a distance R_A apart and B were not present. D_A is, of course, not observed directly because A and C are R_A apart only when B is present.

Similarly, for sphere B we have

$$\frac{D_B}{d_B} = \frac{{r_B}^2}{{R_B}^2}.$$

The two hypothetical displacements, D_A and D_B which are proportional to the forces exerted by A and B, are vector components of the net displacement D of ball C. We can find the vector sum of D_A and D_B and compare it with the observed deflection D.

ANSWERS TO QUESTIONS

To measure the effect of one charge only, the other one should be moved to infinity. In practice, "infinity" is reached as soon as the force from the charge being moved away is small, say $\frac{1}{100}$ the force exerted by the charge under investigation. This will be the case if the distances are in the ratio of about 10 : 1. It will suffice to move one charge out of the box.

Induced charges on a hand will deflect the suspended sphere. The hands should be kept away from the box to avoid disturbing the position of the suspended ball.

From the analysis, using the inverse-square law, one concludes that electric forces add vectorially.

APPARATUS

1 Coulomb's Law Apparatus Kit, consisting of
  Box with mirror, scale, base, and lid
  Styrofoam spheres, 0.5 cm diameter
  Nylon fibers
  Plastic strip with charging cloth
  Electrophorus disc with handle
  Two blocks with spheres on insulating rods and keys
  Dag 154 suspension of graphite
1 Point-source lamp with socket and cord
1 Ringstand, 24″
1 Flask clamp
1 Clamp holder
  Protractor
  Ruler
  Graph paper

IV-5 Driving Force and Terminal Velocity

This experiment is designed to familiarize the student with the Millikan apparatus, to help him feel at home with this kind of measurement, and to show evidence for the proportionality of driving force and terminal velocity. The experiment first establishes that the spheres reach a terminal velocity very quickly and thereafter move at constant velocity. This constant terminal velocity is then seen to be directly proportional to the driving force.

Data can be taken most efficiently if one student watches a sphere and also starts and stops the stopwatch as the sphere passes two reference marks. A second student should note down the watch reading, the direction of motion, and whether the plates are charged. Note that the telescope inverts as well as magnifies: therefore, spheres will appear to "fall up" under the action of gravity.

The spheres are supplied as a concentrated (10 percent solids) water suspension that should be diluted in the ratio of one part suspension to two parts water. One teaspoonful or less of this mixture is enough to put in the atomizer jar for best results. The diluted mixture can be poured into a clean container and saved for future use. The jar and particularly the atomizer nozzle should be cleaned frequently. The jar is cleaned by rinsing it several times in clean water, and the nozzle by blowing water through it until the spray is clear.

The plates should be charged from a well-filtered D.C. power supply or battery giving about 500 volts. The source impedance should be as high as possible for safety reasons; the current drain is small, because the only load is a 10-megohm bleeder resistance across the plates. Five or six 90-volt "B" batteries will serve equally well. If you use batteries, be sure the connectors are well insulated and that a 1-megohm safety resistor is connected in series with the output to increase the source impedance.

It is most satisfactory to perform the experiment in a darkened room. The contrast between spheres and background can be controlled by rotating the light source around its pivot.

ANSWERS TO QUESTIONS

The spheres move upward in the field of view.

All move in the same direction.

No acceleration is apparent; they move at constant velocity.

With the charging switch centered, gravity and the air resistance are the only forces acting on the spheres. They are equal and opposite.

Putting charge on the plates makes some spheres move faster, some slower, and some reverse their direction of motion.

Reversing the direction of the electric field will reverse the direction of motion of most spheres.

The value V^+ is greater than the terminal velocity under gravity alone, while V^- is less. Often V^- is in the opposite direction.

Each graph is a straight line. They differ in their slope because there are different charges on the spheres shown in Figure (a).

The speed is directly proportional to driving force.

APPARATUS

1 Millikan apparatus
1 Stopwatch
1 Power supply (500-volt D.C.; 6.3-volt A.C. for lamp

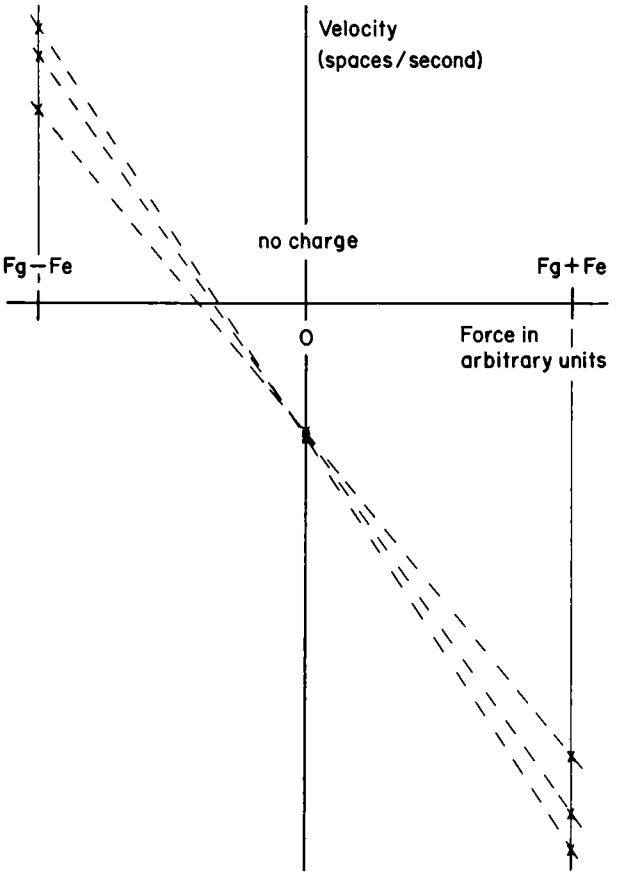


Figure (a)

IV-6 The Millikan Experiment

In this experiment the student makes measurements of the speeds of a number of small plastic spheres in an electric field. The distribution of velocities is seen to cluster around multiples of a definite unit velocity, which suggests that the charges also come only in multiples of a unit charge.

The apparatus is the same as in the previous experiment, and the same words of advice and caution apply. Again, a team of two is essential. Note that there is no need to make measurements of speeds in free fall. It is advisable, however, to make several (say, three) pairs of measurements on each sphere and average them.

Remind your students first to let the electric field clear away the spheres that are moving too fast for accurate timing and carry too many charges to offer good evidence for the discreteness of charge. After a while only a few spheres will be visible. It is best to track the slowest ones. Having chosen a sphere, the student puts it through its paces by reversing the field after each timed run. The student who is observing the spheres should operate the stopwatch, while his partner reads it and records the results. The observer must keep the sphere in view while the watch is being read.

Once the optical adjustments are made and the spheres are clearly visible as bright points of light, the only real difficulty lies in the interpretation of the results. In simplest terms, the most direct way to present the data would be in a bar graph, using an arbitrary horizontal scale and a vertical scale proportional to speeds measured for different spheres in one given electric field. This direct approach is not the best method for two reasons: (1) the weight of the spheres appears as a kind of background force in all measurements; (2) fragments of spheres or clumps of spheres have different weights and shapes that make the gravitational background nonuniform, thereby tending to wash out the signal.

The method of analysis proposed here reduces these two difficulties: First, the velocity obtained as $V^+ - V^-$ is proportional to $(F_g + F_e) - (F_g - F_e) = 2F_e$. In other words, it depends only on the electric force. But plotting a bar graph of $V^+ - V^-$ does not eliminate all the problems. We have not eliminated, for instance, the fact that the air resistance on clusters and fragments of spheres is different from that on a single sphere. If we take the *sum* of the velocities

$$V^+ + V^- \propto (F_g + F_e) + F_g - F_e) \propto 2F_g,$$

we have a quantity which is a measure for the mass. When we make this addition for each sphere, it becomes evident that there is one most common mass value with some low and high variants. If we reject the variants, it is highly probable that the remaining data refer only to single, unbroken spheres.

ANSWERS TO QUESTIONS

The difference $V^+ - V^-$ is proportional to $(F_g + F_e) - (F_g - F_e) = 2F_e$. Since F_e depends only on the charge $V^+ - V^-$ can be used as a measure for the charge.

A quick trial suffices to show that a crumpled paper ball has a much higher terminal velocity than a sheet of paper. It is highly unlikely that a fragment will be spherical; a clump will very likely have a shape quite different from that of a single sphere, and therefore $V^+ - V^-$ for single spheres will be different from $V^+ - V^-$ for clumps even if they have the same charge.

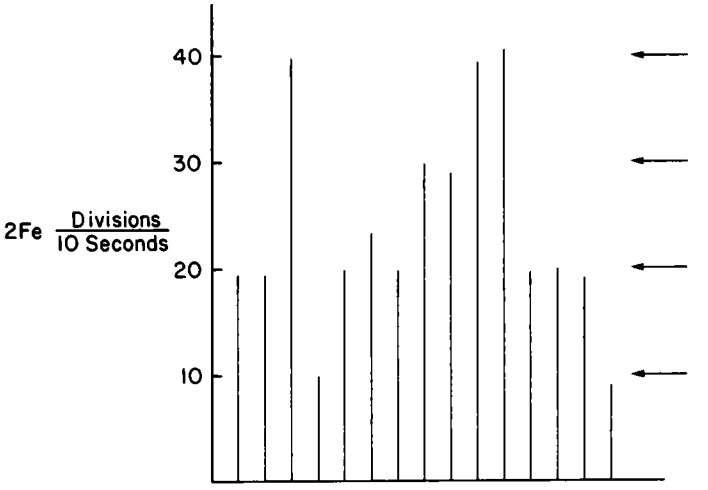
The sum

$$(V^+) + (V^-) \propto (F_g + F_e) + (F_g - F_e) \propto 2F_g$$

The quantity $2F_g$ is a measure of the mass and enables us to reject clumps and fragments of spheres.

Figure (a) shows the results of a set of readings.

Figure (a)



The clustering of the readings around multiples of a smallest speed is good evidence for the existence of a natural unit of charge.

In this set of data the smallest charge consisted of one elementary charge. If single charges were absent from the sample, the ratio of the two lowest speeds would be 3 : 2, and not 2 : 1.

The speeds corresponding to one elementary charge may vary from group to group because of slightly different plate spacings and potential differences. Therefore, to make it possible to compile the readings of the whole class, each group has to normalize its data and express them in terms of the speed corresponding to one elementary charge.

APPARATUS

1 Millikan apparatus
1 Stopwatch
1 Power supply (500-volt D.C.; 6.3-volt A.C. for lamp)

IV–7 The Change Carried by Ions In Solution

This is the only experiment that deals directly with the atomicity of electric charges, and thus has a high priority. It should be done after Section 28–3 has been discussed.

It is best to prepare the dilute acid yourself before the laboratory period. Be sure to add the acid slowly to the water. Large amounts of that are generated when concentrated sulfuric acid is diluted. If the water is added to the acid, spattering will occur. Caution your students on the proper handling of acids.

The anode should be cut from very thin copper sheet so the small loss in its mass which occurs with the recommended current and time will be a reasonable fraction of its original mass. Soldered connections to the anode and cathode may cause unwanted chemical reactions. To avoid having solder in the liquid, cut the anode in the shape shown in Fig. (a), making the narrow strip just long enough to hang over the edge of the dish. For a cathode connection, wrap the copper wire around the end of the cathode and pinch the wire tight with pliers. The copper wire to the cathode should be covered with waterproof plastic insulation so that hydrogen will not be liberated in the acid outside the flask.

The copper anode and cathode can be scoured with fine sandpaper or steel wool and then wiped clean. The anode should be weighed *after* cleaning. At the end of the experiment the anode should be rinsed and cleaned with water and a soft rag and then dried carefully. If a brown coating appears on the anode, it should be rubbed off, but the anode should not be scoured lest some copper be removed.

The power supply used must be capable of delivering 5 amperes at about 12 volts. The anode will lose approximately 1.5 grams if 5 amperes flows through the solution for about 15 minutes. During the same time about 500 cm^3 of hydrogen will be collected. The accuracy of the experiment does not warrant determining the loss in mass to more than two significant figures.

If the students are sufficiently familiar with the general gas law, corrections can be made for pressure and temperature. The pressure inside the flask includes the saturated vapor pressure of the water, which must be subtracted from the total pressure to obtain the true hydrogen pressure. The saturated vapor pressure of water as a function of temperature is given in Table 1.

Table 1. Saturated Vapor Pressure of Water

TEMPERATURE	VAPOR PRESSURE
15°C	12.8 mm of mercury
20	17.5
25	23.7
30	31.7
35	42.0
40	55.1

The following is a sample analysis of actual data obtained in this experiment:

Data:

Current $I = 4.8$ amp
Time $\Delta t = 13.5$ min $= 810$ sec
Decrease in mass of anode $\Delta M = 9.8$ gm − 8.4 gm $= 1.4$ gm
Volume of hydrogen $V_1 = 0.525$ liter
Temperature $T = 21$°C
Atmospheric pressure $P_A = 750$ mm of mercury

Charge Transferred:

$q = 6.25 \times 10^{18}$ It $= 6.25 \times 10^{18} \times 4.8 \times 810 = 2.4 \times 10^{22}$ elementary charges

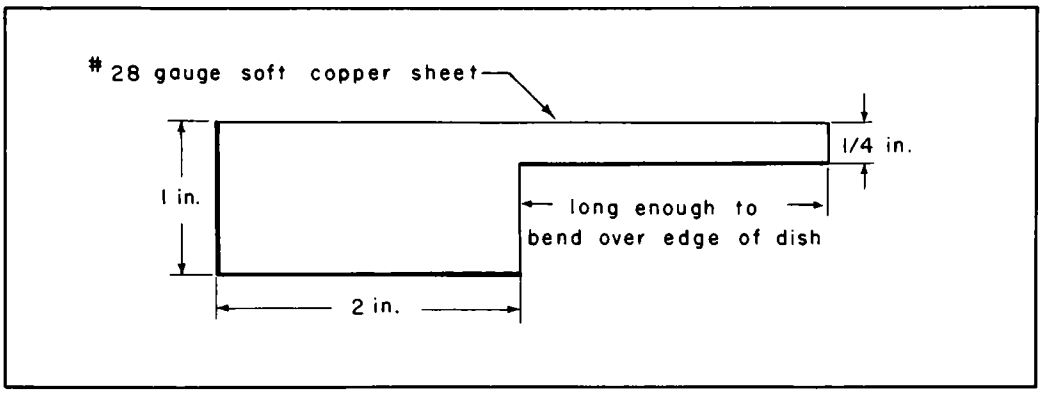


Figure (a)

Moles of Copper Ions:

$$n_{Cu} = \frac{\Delta M}{\text{at. mass of Cu}} = \frac{1.4}{63.5} = 0.022 \text{ mole}$$

Number of Copper Ions:

$$N_{Cu} = (\text{Avogadro's No.}) \times n_{Cu}$$
$$= 6.02 \times 10^{23} \times 0.022 = 1.3 \times 10^{22} \text{ ions}$$

Number of Elementary Charges per Copper Ion:

$$\frac{q}{N_{Cu}} = \frac{2.4 \times 10^{22}}{1.3 \times 10^{22}}$$
$$= 1.8 \approx 2 \text{ elementary charges per ion}$$

Number of Elementary Charges per Hydrogen Ion:

Without making temperature and pressure corrections, the charge per hydrogen *molecule* is

$$\frac{q}{n_H} = \frac{q}{6.02 \times 10^{23} \times \frac{V}{22.4}} = \frac{2.4 \times 10^{22} \times 22.4}{6.02 \times 10^{23} \times 0.525}$$

$= 1.7$ elementary charges per molecule.

But two hydrogen ions are needed to form one molecule; therefore, the charge carried to the cathode by each hydrogen ion is $\frac{1.7}{2} = 0.85 \approx 1$.

Pressure and Temperature Corrections for the Volume of Hydrogen

From Table 1, we find a water-vapor pressure of 19 mm of mercury at 21°C. Thus the partial pressure P_1 of hydrogen in the flask is the atmospheric pressure P_A minus the water-vapor pressure. Substituting in the general gas law gives the volume of hydrogen V_2 at S.T.P.

$$V_2 = \frac{P_1 T_2}{P_2 T_1} V_1 = \frac{(750 - 19) \times 273}{760 \times (273 + 21)}$$
$$\times 0.525 = 0.470 \text{ liter}$$

This corrected value for the volume gives 1.9 charges per molecule, a little closer to 2 than the uncorrected value.

It should be noted that all but the most expensive ammeters are likely to have systematic errors due to inaccurate calibration. Though meter errors are likely to be small compared to other errors, your meters should be checked against some reasonably accurate standard.

ANSWERS TO QUESTIONS

The number of elementary charges that passed through the solution equals the product of the current in amperes and the time in seconds multiplied by the conversion factor 6.25×10^{18}.

The number of moles of copper ions formed equals the mass of copper in grams lost by the anode divided by 63.5, the atomic mass of copper. The number of copper ions equals the number of moles of copper times 6.02×10^{23} (Avogadro's number).

The number of elementary charges carried by each copper ion equals the number of elementary charges passing through the solution divided by the number of copper ions.

The most important assumption made in calculating the number of charges carried by each copper ion is that all copper ions carry the same number of elementary charges. Students who have had chemistry know that there are two common copper ions, Cu^+ and Cu^{++}. There is nothing in this experiment or in the text to tell them which ion forms in this experiment and, if they get a value between 1.0 and 2.0 for the charge carried by copper ions, they are justified in making the hypothesis that both Cu^+ and Cu^{++} are formed to give an average value between 1.0 and 2.0.

One elementary charge is carried by each hydrogen ion. If a value of less than 1 is obtained, i.e., 0.86 in the sample analysis, it must be concluded that one charge is carried per ion because if some ions carried one charge and others none, the "ions" without charge could not be attracted to the cathode and would not be liberated (they wouldn't be ions!). A value greater than 1 could not indicate the existence of hydrogen ions of charge +2 since hydrogen atoms have only one electron. Another important assumption to be noted is that both copper and hydrogen ions are single atoms, not clusters of two or more. This is no assumption, however, for students who have had chemistry.

To show that $m = \alpha q$, the experiment should be repeated with different currents and times. Fresh acid should be used each time the experiment is done to prevent copper from depositing on the cathode.

The value of α for copper is the atomic mass of copper divided by the number of charges carried by copper ions; the value of α for hydrogen is the atomic mass of hydrogen divided by the number of charges carried by hydrogen ions. The ratio α_{Cu}/α_H is 36.8.

We have

$$m = \text{mass of ion} \times \text{number of ions}$$
$$q = \text{charge on ion} \times \text{number of ions}$$

Hence,

$$\alpha = \frac{\text{mass of ion}}{\text{charge on ion}}.$$

The charge on the ion expressed in elementary charges turns out to be a small whole number, showing that it comes in multiples of the elementary charge.

Very little copper deposits on the cathode if there are few copper ions in solution and the anode is not close to the cathode. If copper should deposit, electrons taken up by the copper ions will not be available for liberating hydrogen, and too little hydrogen will be

collected, giving a value for the charge carried by each hydrogen ion that is greater than 1.

APPARATUS

1 500-ml flask
1 Ringstand, 24″
1 Ring clamp (to fit flask as shown)
1 Safety bottle (see Fig. 1)
1 Rubber stopper, 2-hole (to fit safety bottle)
1 Flask clamp
2 Clamp holders
1 Brick or C clamp
3 Pieces standard glass tubing (bent as shown in Fig. 1)
Rubber tubing, 1′ to 2′ (to fit glass tubing)
1 Pinch clamp
Copper sheet, No. 28, about 1″ × 6″
1 Glass tray (see Fig. 1)
1 Insulated wire, 2′
3 Clip leads, 2′
1 Liter 5% sulfuric acid
DC current source: 5 to 10 amp (6–12 volt); this can be used simultaneously for several setups
1 Ammeter, 0–10 amp
1 Variable resistor (to regulate the current above)

Several beakers, 500 or 1000 ml
Several graduates, 500 or 1000 ml
Fine sandpaper or steel wool
Pan balance
Paper towels
Soft rags
Pliers
Clock

IV–8 The Charge on a Capacitor

Measuring electric charge directly is rather hard. Therefore, an indirect method is used. It consists of measuring an average current and multiplying it by the time it flows.

In this experiment the amount of current passing through the milliammeter for one discharge of a capacitor is very small, and the meter movement has too much inertia to give a reading. However, if many discharges occur in a short time the meter will measure the average current and the charge placed on the capacitor each time can be calculated.

For this method to work, the switch must make contact long enough for the capacitor to charge and discharge completely each time. Later, resistors are placed in the circuit to slow down the charging rate of the capacitor so that the student can think about the transient time during charge and discharge.

There is also the problem of the timer circuit battery sending current through the vibrating armature and developing an EMF in it. The resistance and current in the armature are low, so there is no apparent interference between the timer and capacitor circuits.

It is important to have the timer running smoothly and steadily. The clapper should touch the metal clips for a short time at the ends of the vibrating motion. A steady reading on the discharge milliammeter indicates proper adjustment. If the vibrating rate changes, the current reading will change; therefore, readings can be compared directly only at the same vibration rate. You may want to include a rheostat to allow the students to adjust the potential difference applied to the timer.

An extension is included concerning capacitors connected in series and parallel.

ANSWERS TO QUESTIONS

The milliammeter will jump only slightly on a single discharge of the capacitor, because the momentary flow of current through the meter gives only a small impulse to the meter movement.

The timer battery does not cause any current to flow through the milliammeter.

The inertia of the meter movement averages out the individual pulses of current.

The charge and discharge rate will be the same as the vibrating rate of the timer, about 50 to 60 times a second.

The number of coulombs placed on the capacitor in each charge cycle depends on the potential difference applied to the capacitor. See data.

The ratio of the charge to the potential difference applied remains constant.

The 100 and 200-ohm resistors have no effect on the current. The contact time is long enough to completely charge and discharge the capacitor if the timer is adjusted properly. The presence of 10^3, 10^4, and 10^5-ohm resistors in the charging circuit results in less and less current passing through the meter, because the capacitor does not become fully charged during the contact time.

There will be the same current when the resistors are in the discharge circuit if the contact times on both are the same in both circuits. This is unlikely to be the case with the bell timers used.

When two capacitors are connected in parallel, their capacities are added. When they are in series, the total capacity is less than that of either alone. In this case $\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2}$ or $C = \frac{C_1 C_2}{C_1 + C_2}$.

Sample data:

TRIAL	VOLTS	CURRENT 10^{-4} amp	FREQUENCY (cycle/sec)	CAPACITY in μfd	MANUFACTURER'S VALUE OF CAPACITY in μfd
1	0	0	60		
2	1.5	1.0	60	1.1	1.0
3	3.0	2.0	60	1.1	1.0
4	4.5	3.1	60	1.1	1.0
5	6.0	4.2	60	1.2	1.0
6	6.0	1.0	60	.28	.25

Resistor in Circuit

Capacitor used = 1.0 μfd
Voltage applied = 6.0 v

TRIAL	RESISTOR (ohms)	CURRENT in amp: RESISTOR IN CHARGE SIDE	CURRENT in amp: RESISTOR IN DISCHARGE SIDE	FREQUENCY cycles/sec
1	100	4.2×10^{-4}	4.2×10^{-4}	60
2	200	4.2	4.2	60
3	10^3	1.4	1.5	60
4	10^4	.51	.56	60
5	10^5	.06	.09	60

A 1.0 μfd and a 0.25 μfd were connected in parallel and then in series

CONNECTION	VOLTS	CURRENT 10^{-4} amp	FREQUENCY cycle/sec	MEASURED cap'y in μfd	CALCULATED cap'y in μfd
Parallel	6.0	5.1	60	1.4	1.25
Series	6.0	.8	60	.22	.20

APPARATUS

1 Timer with electric switch modification
1 Milliammeter 0 – 1 ma. Full scale
4 1.5-volt dry cells
1 1.5-volt No. 6 dry cell
1 Capacitor paper 1.0 μfd
1 Capacitor paper 0.5 μfd
5 Resistors, $\frac{1}{4}$-watt carbon, 100r, 200r, 10^3, 10^4 10^5

IV–9 Energy Transferred by an Electric Motor

Mechanics and electricity are too often kept apart in the mind of the student. This experiment offers an opportunity to relate the two by concentrating on the energy transfer rather than on the internal working of the motor.

The motors used vary considerably in their characteristics and the data your students obtain may differ quite a bit from the sample data given in this Guide.

ANSWERS TO QUESTIONS

The EMF of the cell is 1.5 volts. This will not change very much under load if the cell is fresh.

From the sample data, at a speed of 0.42 m/sec it takes 3.6 second to lift 6 washers 1.5 m.

The energy supplied by the battery is $\mathcal{E}It$ = 1.5 volts × 0.81 amp × 3.6 sec = 4.4 joule.

The change in potential energy of the washers is 1.5 m × 0.02 kg × 9.8 m/sec^2 = 0.29 joule.

The energy supplied by the battery is about 15 times the change in potential energy of the washers.

The current is nearly constant for different speeds and the same load. (See sample data.)

A graph of current as a function of speed is shown in Fig. (a) and of efficiency as a function of speed in Fig. (b).

A graph of efficiency as a function of the number of

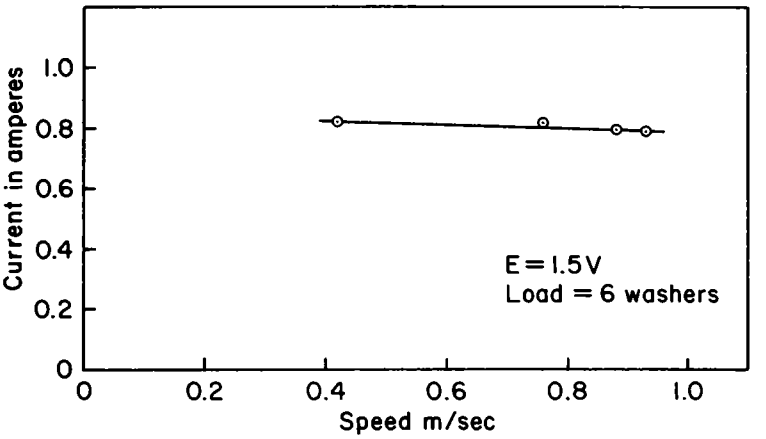


Figure (a)

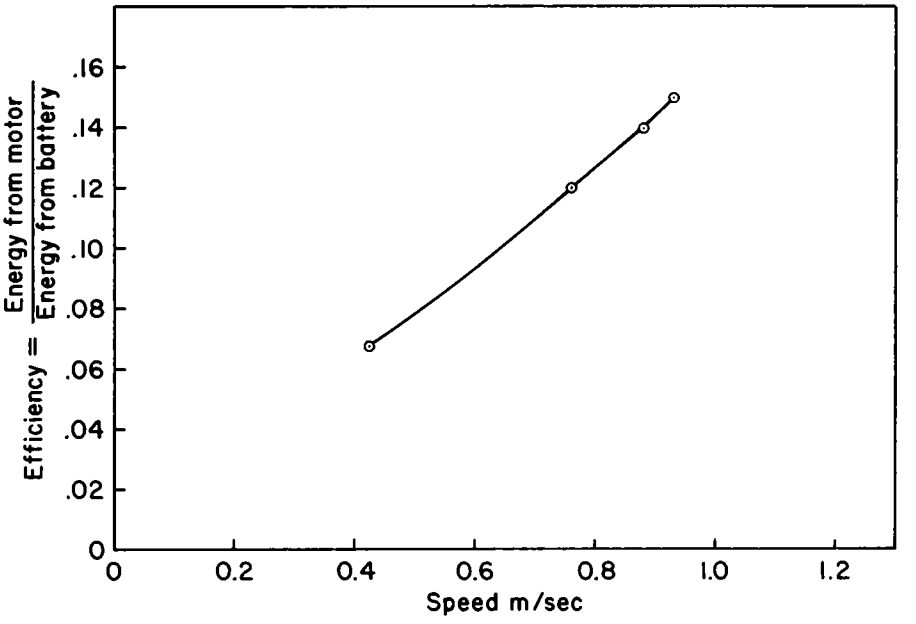


Figure (b)

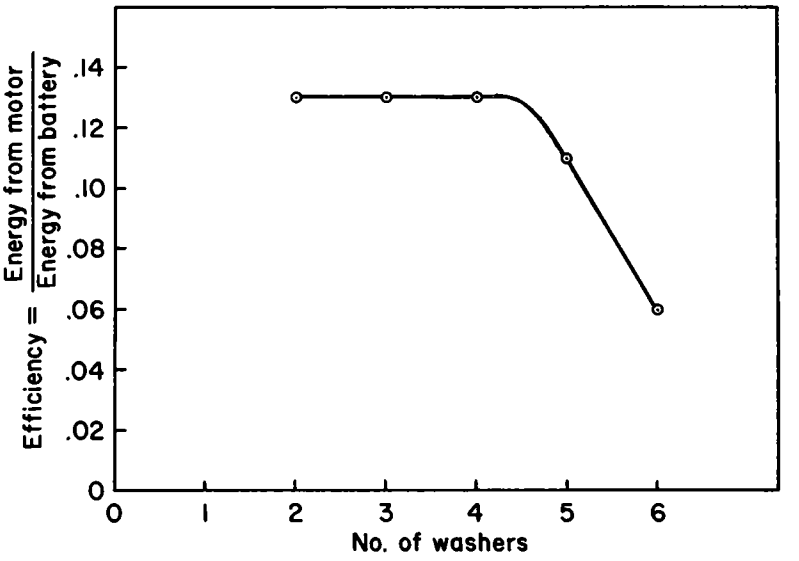


Figure (c)

washers is shown in Fig. (c). The efficiency is nearly constant up to about 4 washers and then drops rapidly.

The energy that is not converted into potential energy in lifting the washers appears as thermal energy in the resistance and motor.

APPARATUS

1 Energy transfer kit
1 Ringstand, 24″
1 Ringstand clamp as shown
1 No. 6 dry cell
1 Ammeter, 0–3 amps
1 Stopwatch

Sample Data:

Mass = 6 washers = 0.020 kg
Distance = 1.77 m

RESISTANCE SETTING	CURRENT in amp	TIME in sec	AVERAGE SPEED in m/sec
1	0.81 0.81 0.81	4.3 4.4 4.0	0.42
2	0.82 0.82 0.82 0.82	2.2 2.4 2.6 2.1	0.76
3	0.80 0.80 0.80	2.0 2.0 2.0	0.88
4	0.80 0.80 0.80	1.9 1.9 1.9	0.93

TRIAL	WASHERS	MASS in kg	CURRENT in amps	TIME in sec	AVERAGE SPEED in m/sec*
1	6	.020	.82	4.6	0.39
2 3	5 5	.017 .017	.70 .70	2.5 2.5	0.71
4 5 6	4 4 4	.013 .013 .013	.60 .60 .60	1.8 2.0 1.9	0.93
7 8	3 3	.010 .010	.55 .54	1.7 1.6	1.1
9 10	2 2	.007 .007	.45 .45	1.4 1.4	1.3

* Distance = 1.77 m

IV–10 The Magnetic Field of a Current

This experiment should be done before Section 30–3 of the text. It is easy to do, and the analysis by vector diagrams or simple trigonometry is short. The experiment shows that magnetic fields add vectorially and are proportional to the current that produces them.

A square rather than a circular loop of wire is used because it is easier to make. The entire 30 feet of wire should be used throughout the experiment. This will insure a constant current in the wire provided the total battery drain is not excessive. The small (No. 41) flashlight bulb limits the current to about 0.3 ampere with a single No. 6 dry cell.

As an indicator for the magnetic field, one can use either a small magnetic compass of about 5 cm diameter or the compass whose construction is described below.

The angles can be read directly from a small compass if it has a scale divided in degrees. The compass should be tapped lightly before taking each reading to prevent errors due to friction. If the pivot is not centered with respect to the scale, the deflections can be read at both ends of the needle and the average of the two readings recorded. The experiment can be done using only graphic methods in the taking of data

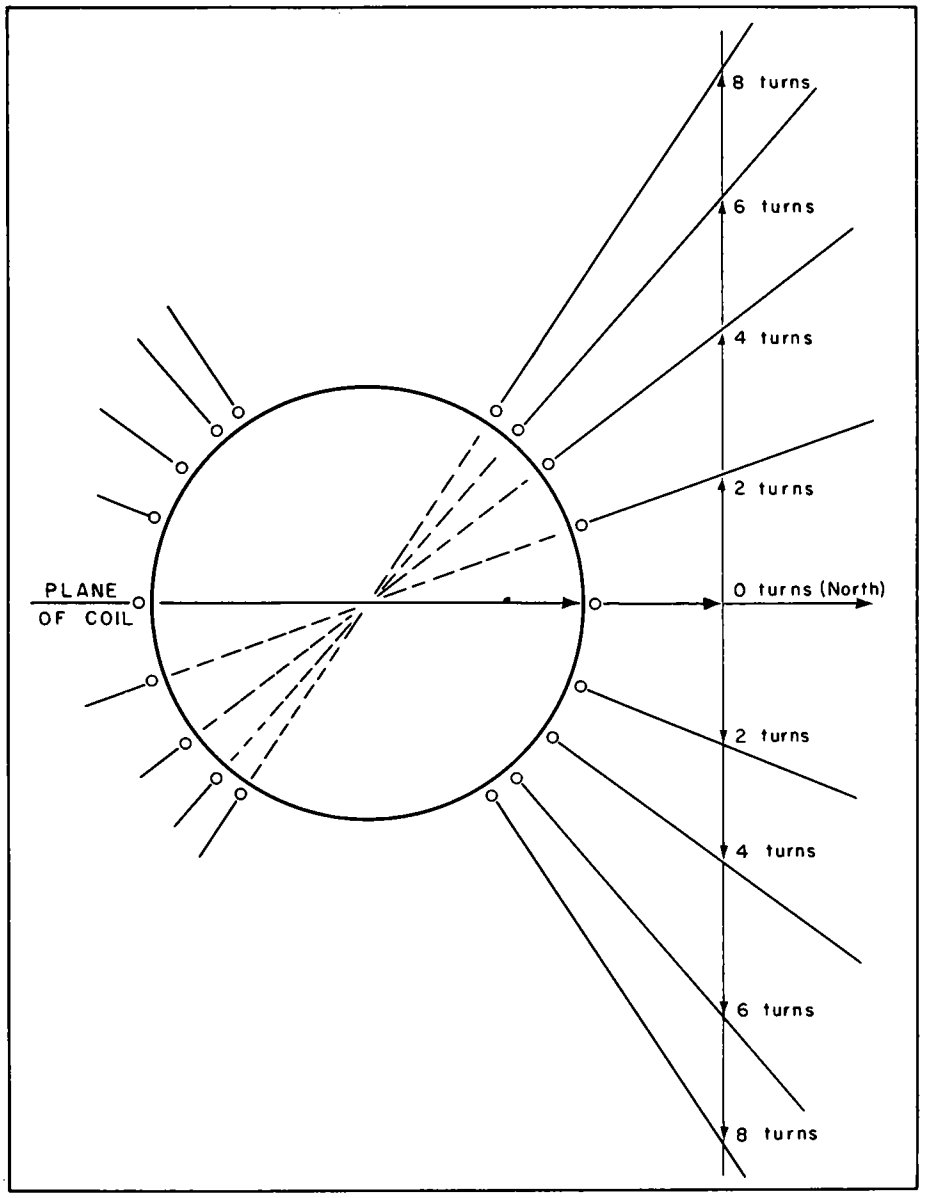


Figure (a)

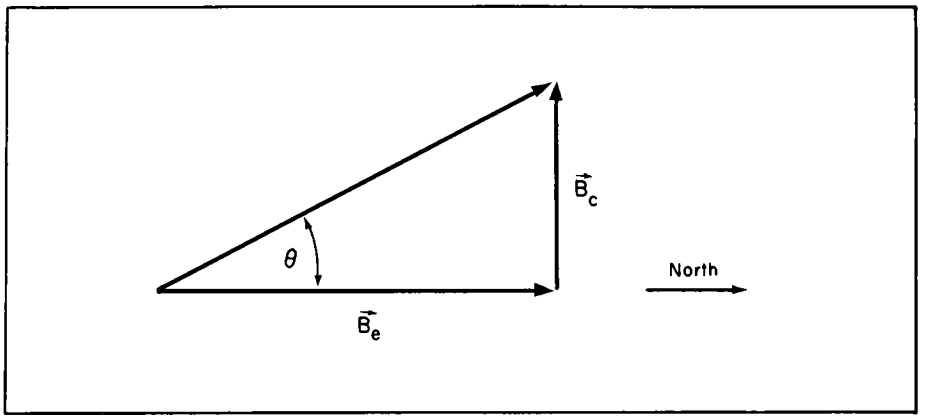


Figure (b)

and in the analysis, or by reading angles in degrees and using simple trigonometry to find the field strength.

The direction of the compass needle can be indicated by putting dots on the paper at the ends of the needle.

The data can be analyzed as shown in Fig. (a). The direction lines of the compass needle for different numbers of turns should mark off equal segments on a line drawn perpendicular to the zero current line. Each of these segments corresponds to the field added as another turn of wire is added to the coil.

If the deflection angles of the compass are measured in degrees, the data can be analyzed [Fig. (b)]; the constant horizontal component of the earth's field B_e, the field due to the current B_c, and the angle which the needle makes with its zero position are related by $B_c = B_e \tan \theta$. Plotting tan θ as a function of the number of turns of wire will give a straight line.

Some students may object to the statement in the Laboratory Guide that the current in the coil can be increased by increasing the number of turns. The important point is that, even though the current in the wire remains constant, the current flowing around the coil in three wires close together is the same as three times the current in one wire. This method of increasing the current around the loop in measurable amounts eliminates the need for an ammeter.

ANSWERS TO QUESTIONS

The field in the center of the coil due to the current is perpendicular to the plane of the coil. Reversing the current through the coil reverses the direction of the field.

The magnitude of the magnetic field is proportional to the total current flowing around the coil.

Winding turns in opposite directions will produce fields in opposite directions. The net field will be due to the excess number of turns in one direction.

It is possible to measure the field even if the compass needle is not parallel to the plane of the coil, but reversing the current will no longer give equal deflections [Fig. (c)]. In this case, the graphical method is much easier to do.

The strength of the magnetic field of the compass needle does not influence the results of the experiment.

APPARATUS

1 Tangent galvonometer kit which includes:
  Insulated wire, 30′
  1 Magnetic compass
1 No. 6 Dry cell, $1\frac{1}{2}$-volt
1 Flashlight bulb, No. 41, and socket
1 Sheet polar or rectilinear graph paper

Alternate for light bulb:
1 Variable resistor, 10 ohm, 4 watt
1 Ammeter, 0–3 amp

HOMEMADE COMPASS

A cork, a needle, copper wire and a circular plastic box will make a sensitive compass that can be used in Experiments IV–10 and IV–13.

Magnetize a sewing needle (about 3 cm long) by stroking it with a strong permanent magnet. Or you can hold it next to a pencil and wind about 20 turns of insulated wire around both; connecting the ends of the wire for a second or less across a 1.5 to 3-volt battery will magnetize the needle.

Cut off the upper end of a small cork ($\frac{1}{2}$″ diameter at bottom) so its height is $\frac{1}{2}$″. Center the magnetized needle in a knife cut made on the bottom of the cork along a diameter, as shown in Fig. (d).

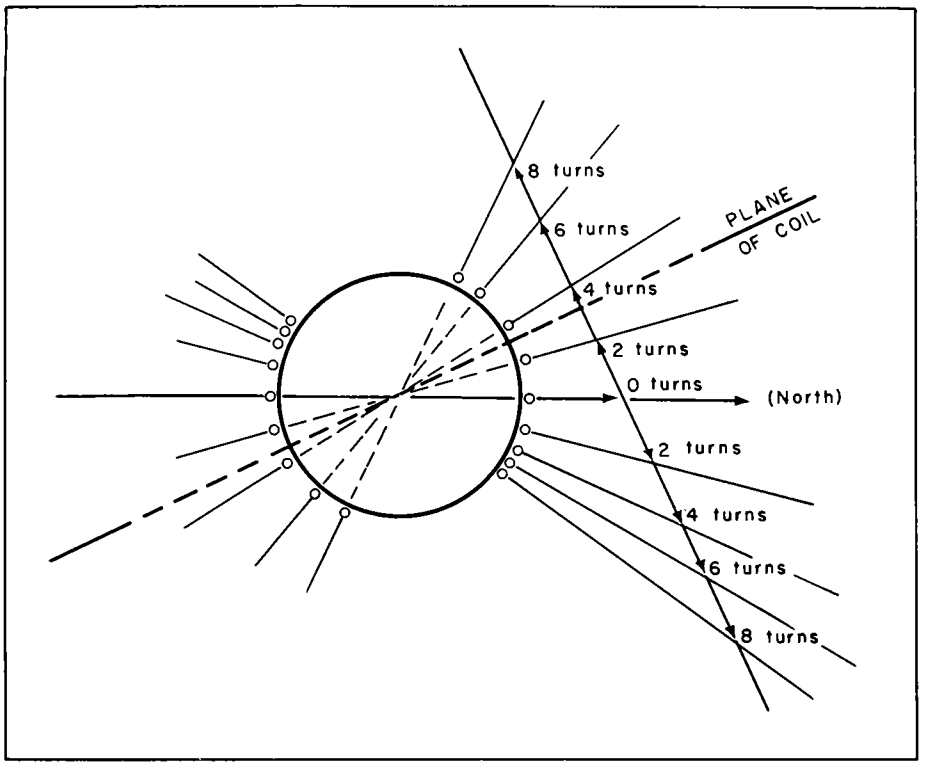


Figure (c)

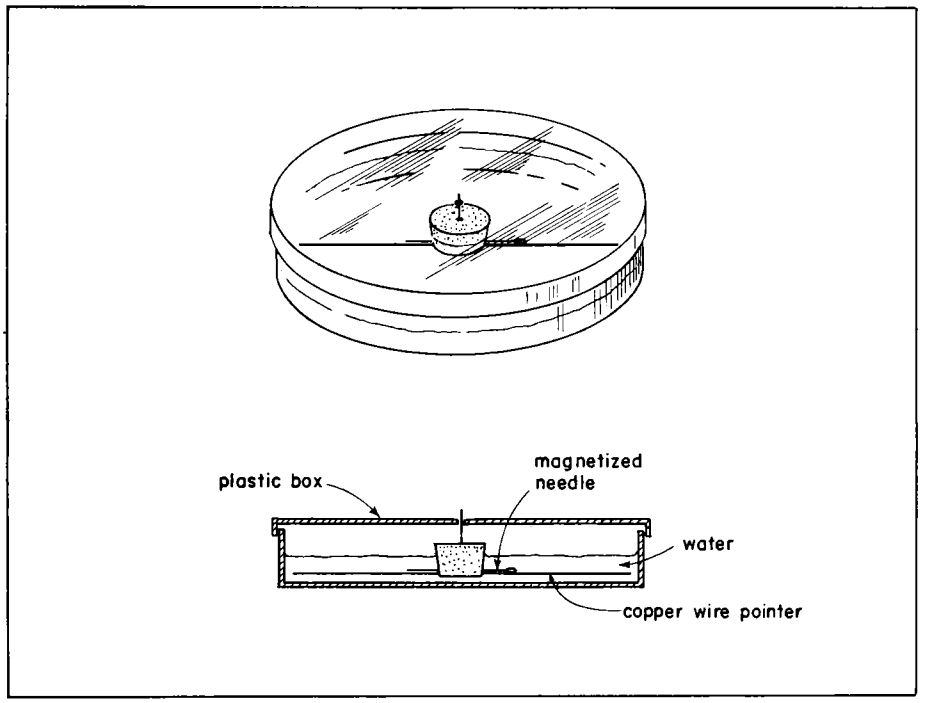


Figure (d)

Straighten a length of No. 18 or No. 20 bare copper wire by fastening one end in a vise and pulling on the other end until the wire starts to stretch. Cut a length of straightened wire about $\frac{1}{4}$ inch shorter than the inside diameter of the plastic container and center it in the slot at the bottom of the cork just beneath the magnetized needle.

Float this assembly in water in a circular plastic box. To keep it centered in the box, fasten a piece of copper wire vertically to the center of the cork so it protrudes through a small hole drilled in the lid of the box.

The question may be asked why a long magnetized steel wire cannot be used in this compass instead of the short magnetized needle and the long nonmagnetic indicator. This is because the magnetic fields investigated are usually not uniform over large areas. A long magnetized wire would not measure the field at the center of the compass. Since it is difficult to determine directions with a short magnet, a long nonmagnetic indicator is used as a pointer.

IV–11 The Measurement of a Magnetic Field in Fundamental Units

In addition to establishing a unit of magnetic field strength in familiar units, this is the only experiment in which magnetic forces are measured. Consequently, it has a high priority. This experiment can be done after the topic is studied in the text.

The coil calibration determined in this experiment will be needed in Experiment IV–12. The coils should therefore be numbered if their characteristics are different, so they can be identified when used again.

In Fig. 2 of the Laboratory Guide, the loop and coil are shown connected in parallel to the source of current, each having a variable resistor and an ammeter in its circuit to permit independent control of their currents. This arrangement will be most easily understood by students, but if you are short of rheostats and ammeters, the apparatus can be connected in series as shown in Fig. (a). If this circuit is used, it will not be possible to make several determinations of a given field in the coil by varying the current in the loop.

For the accuracy of the experiment, the wire or string can be assumed to be uniform in cross section, and the mass of the short length of string can be calculated from the mass of several meters of string.

The length L of the straight end of the loop (A in Fig. 1) should be measured as the distance between the centers of the strips forming the sides of the loop.

Notice that a systematic error can occur if the ammeter's calibration is off.

ANSWERS TO QUESTIONS

The ratio of the field strength of the coil to the current in the coil is constant.

Although the current-carrying loop can be balanced in the magnetic field near a small magnet, the measured value of the field strength will be hard to interpret since the field about the magnet is far from uniform over distances comparable to the dimensions of the loop. We would have no idea at what point near the magnet the field would have the strength we calculated from the balancing force on the loop.

In principle, the balance could be used to measure the field of the earth, which is uniform over large distances, but the earth's field is so weak that, without using excessive currents through the balance, the force is too small to measure. Students might try this, however, to see if any motion of the balance can be observed. If more than 5.0 amperes are used in the loop, the contacts will become hot and corroded. A piece of fine sandpaper or emery cloth will clean the contacts.

If iron wire is used in the loop, it becomes magnetized and adds an unknown field to the field being measured.

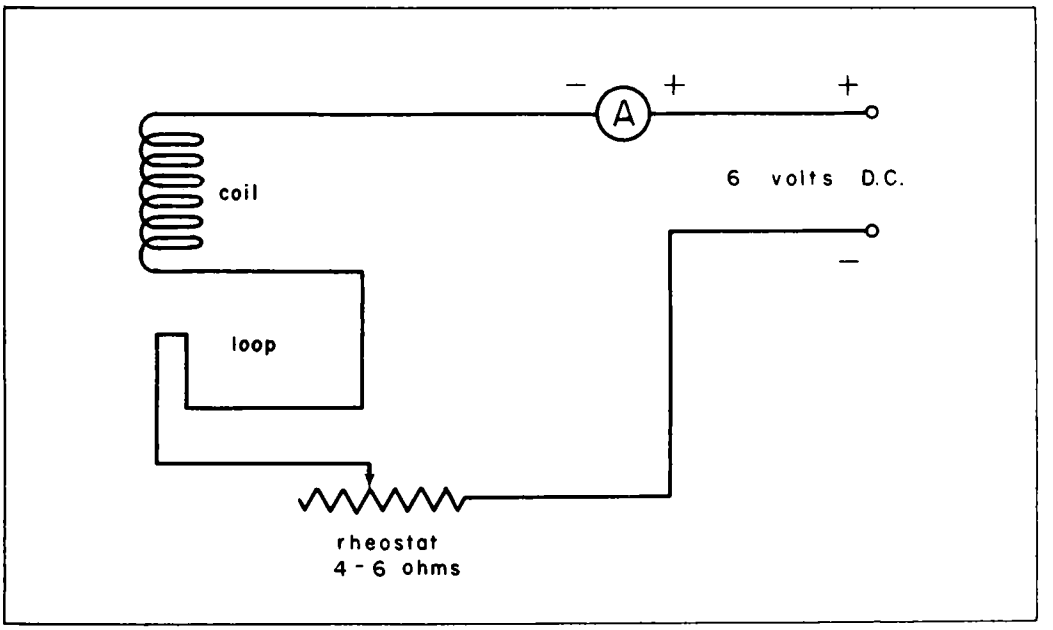


Figure (a)

APPARATUS

1 Air-core solenoid
1 Current balance and contacts
1 DC current source, 0–10 amp (6 to 12 volts) this can be used simultaneously for several setups
2 Ammeters, 0–10 amp
2 Variable resistors (to regulate the current above)
8 Clip leads, 2′ or 3′

Pan balance (sensitive to 0.1 gram)
String
Ruler or meter stick

IV-12 The Mass of the Electron

This is a very fundamental experiment which clearly shows how the mass of the electron can be determined by simple means. The best time to do the experiment is after Section 30–6 has been discussed in class.

The conditions inside the 6AF6G tube are more complicated than as described in the Laboratory Guide. The tube elements and their supports are slightly magnetic, introducing a small amount of distortion in the magnetic field. In addition, since the accelerating electric field extends from the cathode to the anode, electrons in the beam are not moving at constant speed while being deflected by the magnetic field. However, owing to the "geometry" (the size, shape, and spacing of the elements) of the tube, most of the increase in speed takes place before the electrons move out from under the center cap. Both the magnetic distortion and the change in speed during deflection result in a mass determination in error by a factor of 2 or 3, but the mass of the electron is determined to the right order of magnitude, 10^{-30} kg. This is a very important accomplishment in measuring such a small mass.

Note that the pattern we see is not the path of the electrons, but the *end point* of the paths of many different electrons as they hit the conical target.

It may be asked why the helical grid surrounding the cathode cannot be connected to the anode so the electrons will gain all their kinetic energy between the cathode and this grid and move at constant speed from it to the anode. As a careful examination of Fig. 1(b) shows, this grid is internally connected to the cathode.

The filament supply for the 6AF6G tube is 6.3 volts at 0.15 amp. Batteries or a filament transformer can be used, or the filament can be operated from the 110-volt A.C. line, using a 15-watt, 110-volt incandescent lamp as a potential-dropping resistor as shown in Fig. (a).

It is also possible to use 110 volts A.C. for the anode voltage. One still sees a sharp shadow with the rapidly changing anode voltage owing to the persistence of vision and the increase in brightness of the beam as the anode potential increases. The peak voltage should be used as the potential difference in

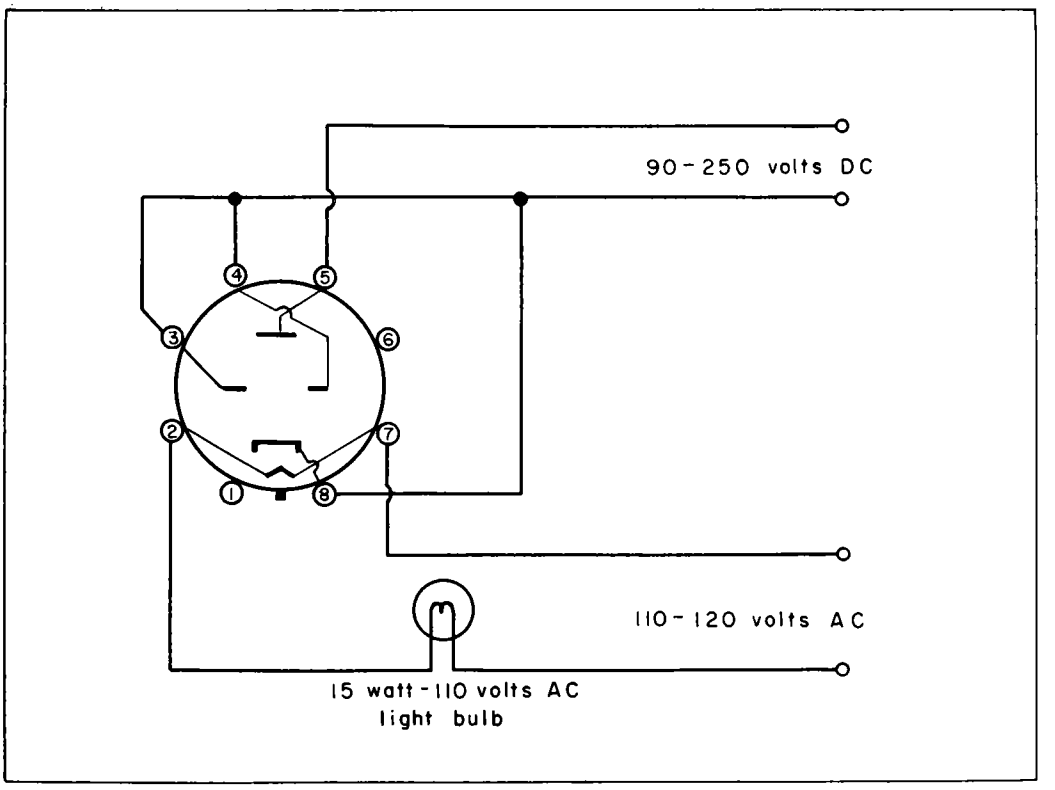


Figure (a)

the calculation. These points are difficult to explain to the student at this time, and therefore D.C. is to be preferred for the anode voltage.

When wiring the circuit, remember that the pin numbers on the tube socket shown in Fig. 4(a) of the Laboratory Guide apply when looking at the *bottom* of the socket, not when looking down at the top.

The voltages used in this experiment are dangerous. Be sure precautions are taken to avoid shocks.

The solenoid used here is the same as that used in Experiment IV–11, and currents up to 5 amperes will be needed. A low-voltage D.C. supply operating from the 110-volt A.C. line can be used for the solenoid current, but these supplies are sometimes poorly filtered. In Experiment IV–11 this was not noticeable because of the inertia of the wire loop, but electrons will follow the rapid fluctuations in the magnetic field and the edge of the shadow will be fuzzy. Hence, it may be necessary to use a 6-volt storage battery. Do not use dry cells; they won't last long with the currents used.

Consistent units must be used in the analysis. To obtain the mass m in kilograms, measure R in meters, q in coulombs, B in newtons/amp-meter and V in volts. The following data were taken in an experiment:

Coil constant,

$$B/I = 3.6 \times 10^{-3} \text{ newton/amp}^2\text{-meter}$$
$$I = 3 \text{ amperes}$$
$$B = 10.8 \times 10^{-3} \text{ newton/amp-meter}$$
$$V = 135 \text{ volts}$$
$$R = 5 \times 10^{-3} \text{ meter}$$

Substituting in the equation for m in the Laboratory Guide, we find a value for m of

$$m = \frac{B^2qR^2}{2V}$$
$$= \frac{1.2 \times 10^{-4} \times 1.6 \times 10^{-19} \times 25 \times 10^{-6}}{270}$$
$$m = 1.8 \times 10^{-30} \text{ kg.}$$

ANSWERS TO QUESTIONS

It is possible to use the earth's magnetic field to deflect an electron beam. With a vertical component of the earth's field of about 5×10^{-5} newton/amp-meter and an accelerating voltage of 100 volts, the radius of the circle in which an electron beam will be bent is about 67 cm. Consequently, we would need a tube at least one meter in diameter if we wanted to use the same method. However, such a deflection can easily be measured in a straight tube.

Using the free fall of an electron to determine its mass is theoretically sound but impractical. An electron gains a very high velocity for even very small accelerating potentials, and it would travel a very long horizontal distance before "falling" a measurable distance. We would have to determine its "initial" velocity from the distance it had fallen and compute the mass from $mv^2/2 = qV$. An electron accelerated through a potential difference of only 1 volt will acquire a velocity of nearly 6×10^5 meter/sec. During $\frac{1}{100}$ sec it will travel a horizontal distance of 6000 meters and will fall under gravity a distance of 4.9×10^{-4} meter, or about 0.5 mm. Obviously, this will make it impossible to perform any experiments of this nature.

APPARATUS

- 1 Air-core solenoid
- 1 Mass-of-the-electron apparatus
- 4 45-volt batteries (dry cells) or power supply
- 1 D.C. source; 0–5 amp with at least a 6-volt tap for filament supply; this may be used simultaneously for several setups
- 1 Ammeter, 0–10 amp
- 1 Variable resistor (to regulate the current above)
- 4 Clip leads, 2′
- 1 Ruler or meter stick
- 1 Set of wood dowels of various diameters

DESIGN OF SOLENOID

You can make your own coil as in Fig. (b). The ratio of the field strength in the center of the coil to the current is about 3.6×10^{-3} newton/meter(amp)^2 (plus or minus 10 percent). Computing this ratio from the geometry of the coil and assuming an infinitely long coil gives 4.0×10^{-3} newton/meter(amp)^2. If corrections are made for the finite length of the coil, this computed value will come closer to 3.6×10^{-3} newton/meter(amp)^2.

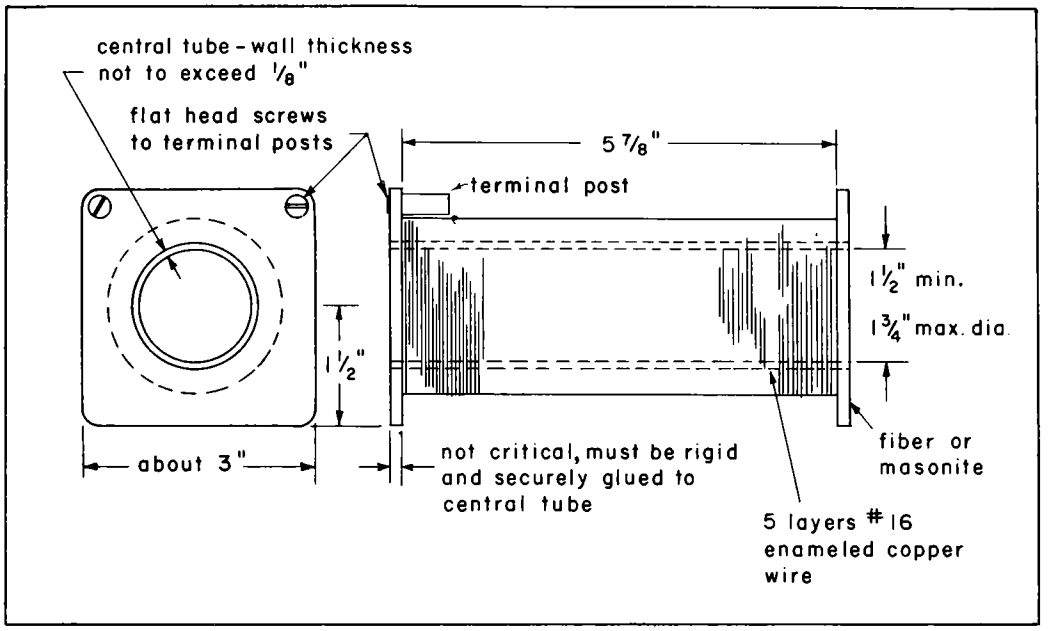

central tube - wall thickness
not to exceed 1/8"
flat head screws
to terminal posts
5 7/8"
terminal post
1 1/2" min.
1 3/4" max. dia.
1 1/2"
about 3"
not critical, must be rigid
and securely glued to
central tube
fiber or
masonite
5 layers # 16
enameled copper
wire

Figure (b)

IV–13 The Magnetic Field Near a Long, Straight Wire

This is the only experiment in which the dependence of magnetic forces on distance is investigated. Furthermore, it is easy and gives accurate results. It is most appropriately done before Section 30–9 is assigned.

The vertical section of wire must be long, extending, if possible, as far above the table as the table is above the floor. The effect of iron objects in the vicinity can be checked by moving the compass along the graph paper with no current flowing in the wire. If the compass shows no change in the direction of the earth's field as it is moved out 20 cm along the paper from the wire, it can be assumed that there will be no error due to neighboring iron. If there is much iron in the table, it may help to use the nonmagnetic coil frame used in Experiment IV–10.

The compass should be handled as in the preceding experiment. For the reading at minimum distance, the homemade compass is placed with its case in contact with the wire, since the magnetic needle is short and will be far enough from the wire to avoid the errors mentioned in the Laboratory Guide. If time permits and greater accuracy is desired, the current through the wire can be reversed at each position of the compass to give deflections in opposite directions.

If a 5-ampere D.C. source with a self-contained rheostat and ammeter is not available, batteries or other power supplies can be used. Students may have to be shown how to connect a rheostat and ammeter in series with the vertical wire and power supply. Dry cells are not recommended for this experiment, since they are expensive and deteriorate rapidly when supplying large continuous currents.

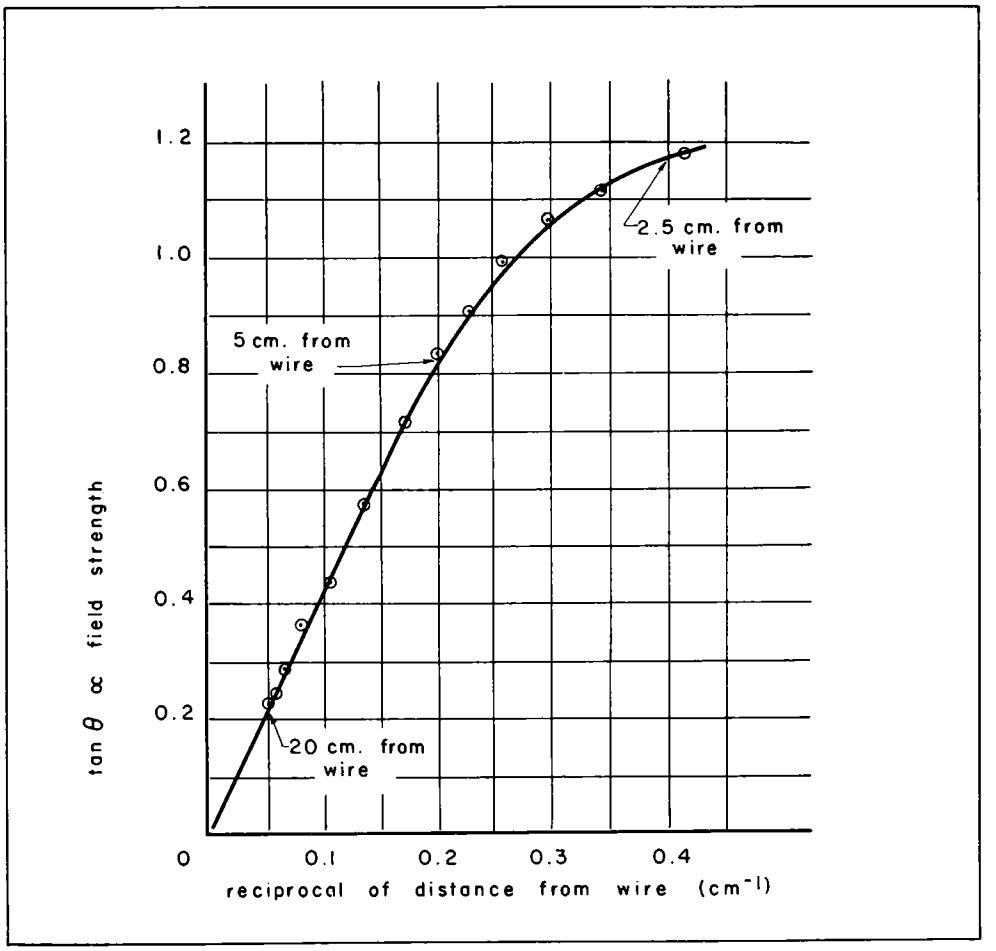


Figure (a)

The strength of the field in terms of the horizontal component of the magnetic field of the earth is inversely proportional to the distance from the wire. This can be found by determining the field strength at different distances and then drawing a graph of field strength as a function of the reciprocal of the distance from the wire. If data is taken with the compass too close to the wire, the curve will deviate from a straight line at short distances as shown in Fig. (a). The shorter the compass needle, the less the error will be.

Because iron becomes magnetized even by the earth's field alone, close proximity will produce an additional unknown field. Similarly, the close proximity of the rest of the wire in the circuit would add unwanted fields to the field being measured.

If the wire were only 20 cm long, the field would drop off faster than inversely with the distance. Students should be encouraged to check this experimentally if time allows. Fig. (b) illustrates the situation. The field at the point *M* is primarily due to the current in the wire in the vicinity of *O*, because the distances *MB* and *MC*, for example, are already several times larger than *MO*, and therefore the current at *B* and *C* will contribute very little to the field at *M*. Whether the wire extends to *A* and *D* will have little effect on the magnetic field at *M*. On the other hand, the distances *AN*, *ON*, and *DN* are about the same; therefore the sections *AB* and *DC* of the wire will contribute significantly to the magnetic field at *N*. Shortening the wire will decrease the field at *N*. Hence, for the short wire the field will drop off faster than inversely with the distance.

A current one hundred times as large would make the field about the wire very large and give compass deflections so close to 90° that the small changes in angle as the compass is moved out to 20 cm from the wire could not be measured accurately. If the current

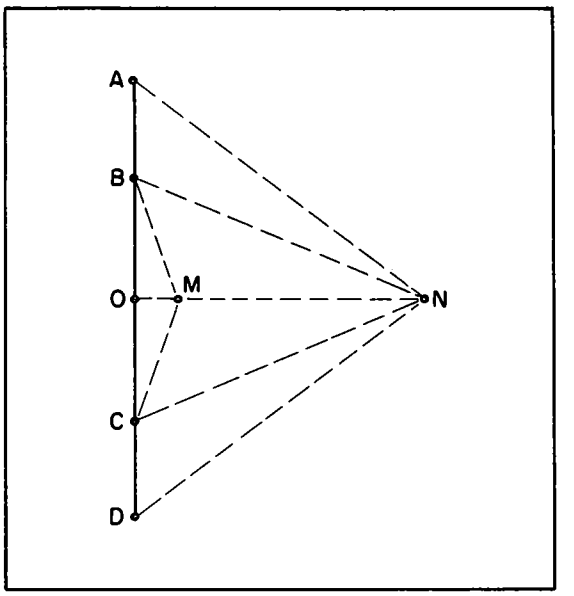


Figure (b)

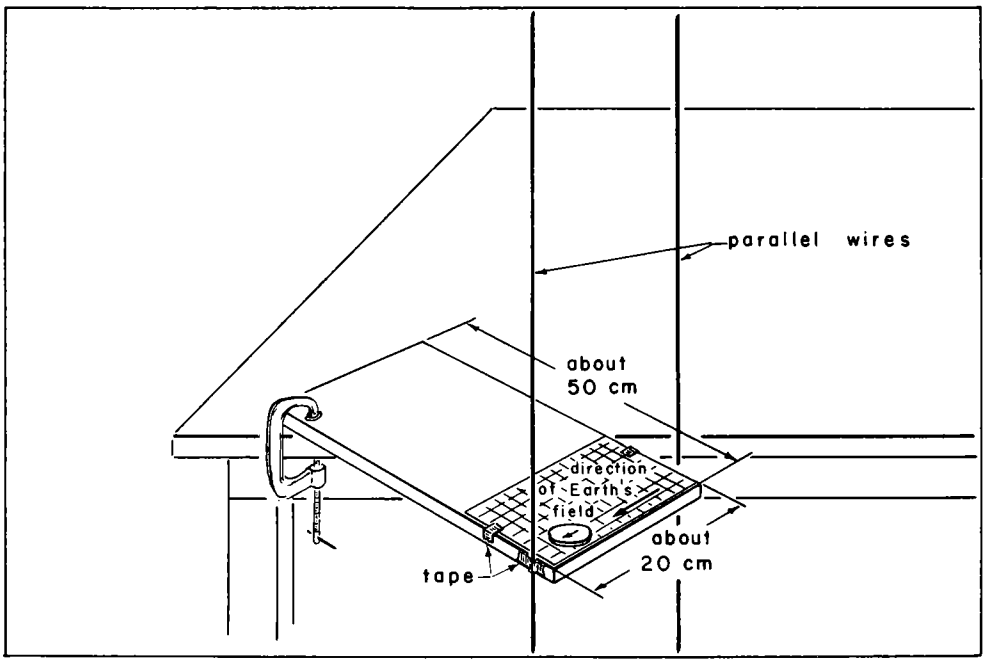


Figure (c)

were one hundred times smaller, the maximum deflection would be only a fraction of a degree and the changes in deflection could not be measured accurately.

If the laboratory tables cannot be moved to make their edges parallel to the earth's field, it will be necessary, in the last part of the experiment, to clamp a properly oriented board to the table edge as shown in Fig. (c). Computed graphs of the field strength as a function of distance are shown in Fig. (d). This part of the experiment is intended only for those students who wish to find out more about the fields surrounding wires. Some students will be able to predict the results by the superposition of the two fields before they do the experiment, as was done in drawing Fig. (d).

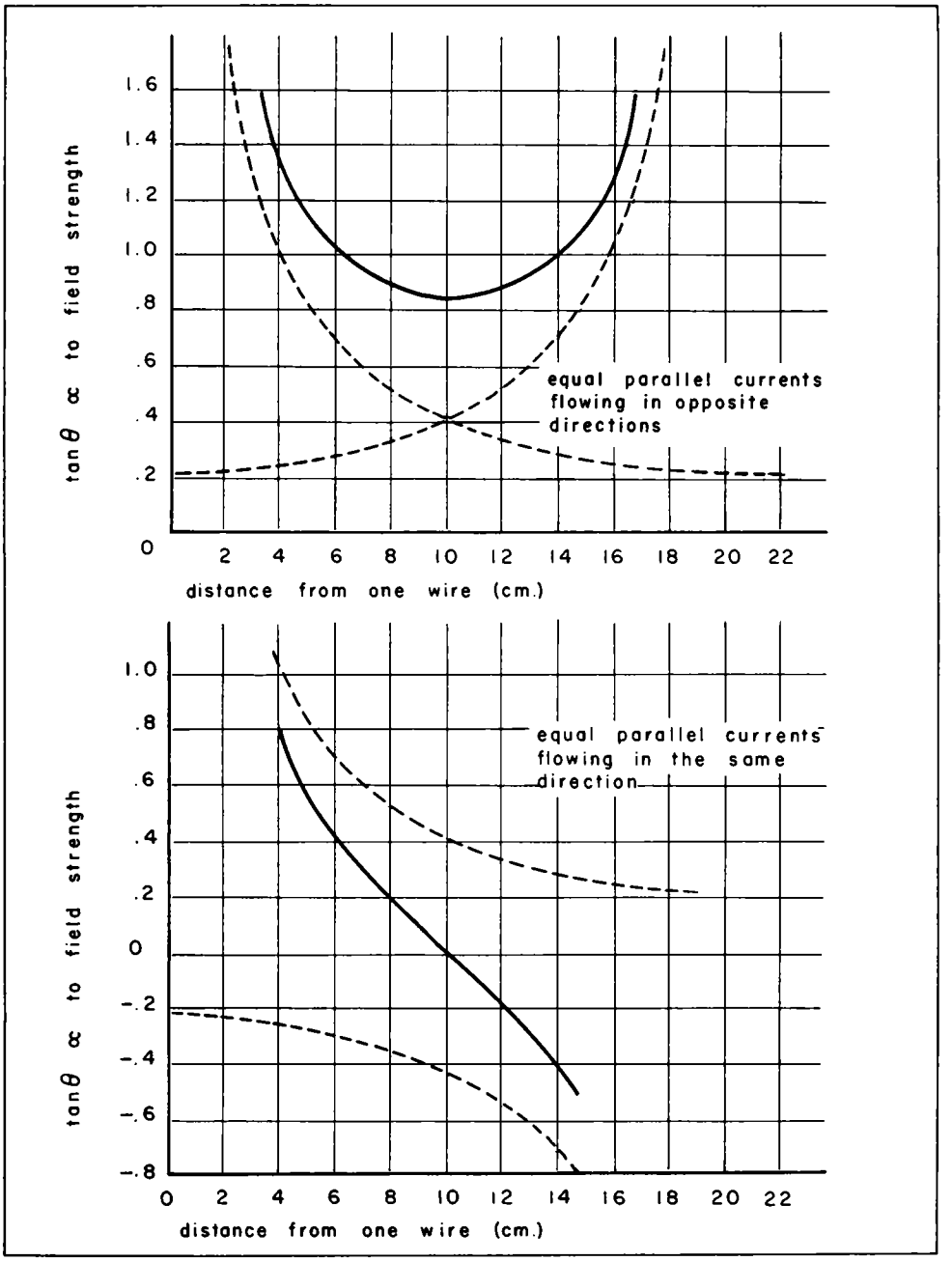


Figure (d)

APPARATUS

1 Magnetic compass
Insulated wire, 30′ No. 18 stranded
2 Sheets rectilinear graph paper
1 Ringstand, 36″
1 Clamp holder
1 Meter stick
1 DC source, 5 amp (6 to 12 volt); this source may be used simultaneously for several setups
1 Ammeter, 0–10 amp
1 Variable resistor (to regulate the current above)
2 Bricks

Cellulose or masking tape

---

1 C clamp
1 Board $\frac{1}{2}'' \times 8'' \times 12''$

IV–14 Randomness in Radioactive Decay

Radioactive decay and the emission of light by atoms have the common property that they can be described only in statistical terms. That is, individual atoms or nuclei decay at random; the time of individual decay can be neither predicted nor controlled. We can predict only the average behavior of a large number of them. This is the only experiment in the entire course where the student can investigate this randomness. That is the sole purpose of the experiment. The experiment may be done during the study of Chapter 33 if one limits the emphasis to the randomness, or it may be postponed until one wants to study radioactivity in particular.

To get a reasonable number of counts per second without running the danger of missing counts in a burst, have the radioactive sample at such a distance from the counter as to give about one count per second.

One counter can be used and the clicks relayed through a loudspeaker for the whole class. (If you have a counter that does not include an amplifier and loudspeaker, use the sound amplifier in a movie projector.) In this case, students should start their graphs of average counting rate versus total number of counts at different initial times, say a minute apart. Comparing these graphs, they will then find that, although the individual graphs look very different at the beginning, they all converge to the same average counting rate.

It is not necessary to count continuously for 20 minutes. A few breaks will make it easier, and the duration of the breaks will have no effect on the final counting rate.

The analysis takes a long time. To speed up the numerical work, have students work in pairs, one adding and one dividing, to find the averages. Fig. (a) and (b) show the results of an actual experiment.

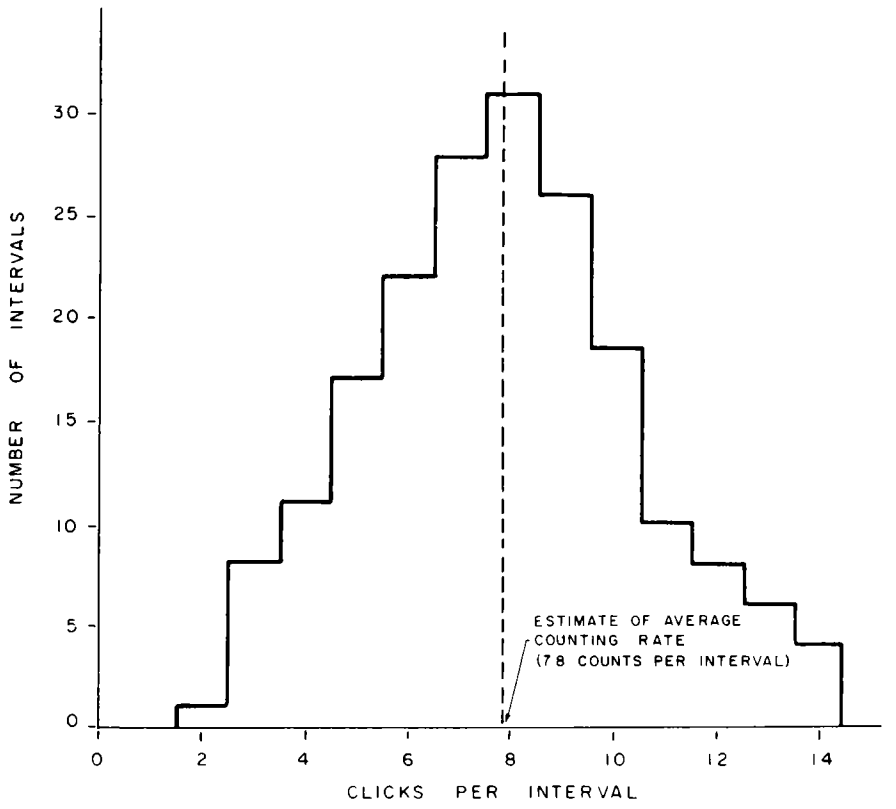


Figure (a)

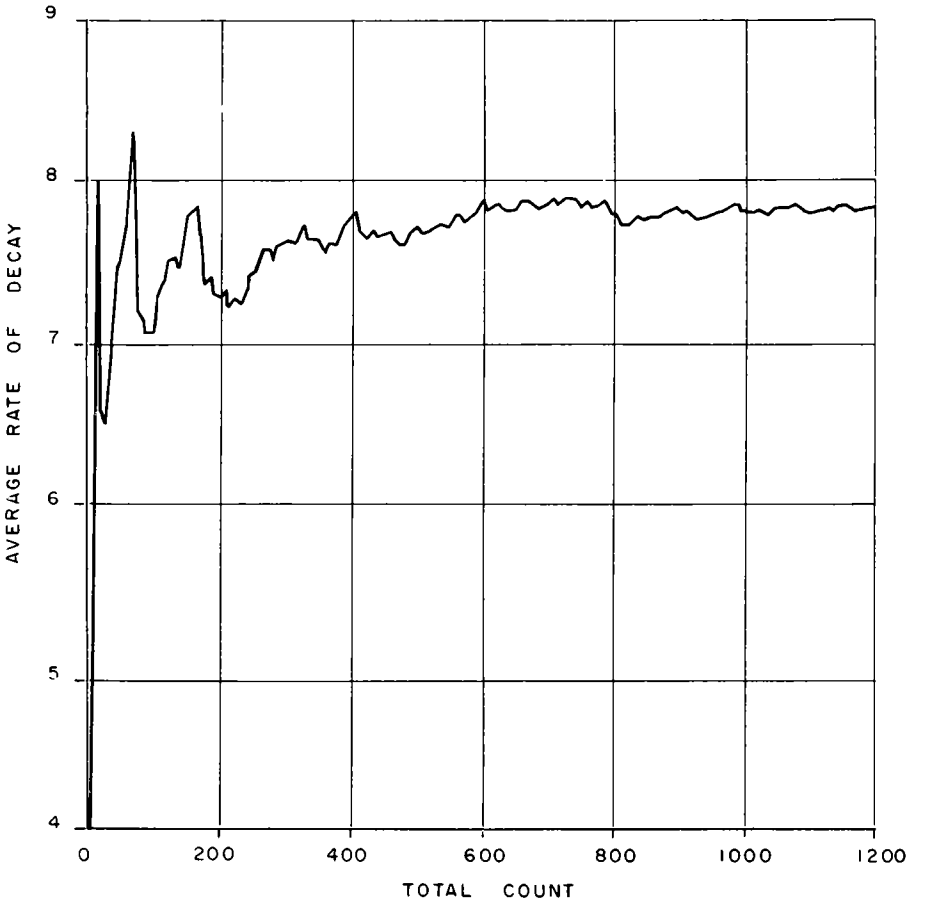


Figure (b)

ANSWERS TO QUESTIONS

The average counting rate obtained from the bar graph is within about 10 percent of the computed value.

For a small number of counts, the average counting rate changes drastically from point to point; as the number of counts increases, the fluctuations in the average counting rate decrease. The accuracy of the determination of the average counting rate increases with the total number of counts used in the computation. From the small and nearly constant fluctuations during the last few minutes, the student can conclude that counting for two more hours will not yield a substantially higher accuracy. He is not required to know that the accuracy is proportional to the square root of the total number of counts.

Assuming that the sample emits particles evenly in all directions, we can estimate the fraction hitting the counter by estimating the solid angle intercepted by the counter. To do this, we draw an imaginary sphere, centered at the sample, whose surface passes through the counter. We then estimate the cross-sectional area of the counter and divide it by the area of the imaginary sphere $4\pi R^2$. The fraction of the total number of emitted particles that hit the counter will be equal to this ratio. Another point to consider is the efficiency of the counter (though most students should not be expected to know about this). Only a fraction of the particles hitting the counter register counts, and the efficiency varies widely with different counters and different sources. Alpha particles are not counted by most Geiger counters.

We cannot determine the half-life of the sample from our measurements. The half-life of the sample is too long to show a measurable change in the rate of decay in an hour. To find it, we must know the number of radioactive nuclei present in the source as well as the overall efficiency of the counting. Knowing the

number of nuclei N in the source and the number of disintegrations per second $\Delta N/\Delta t$, the half-life is

$$T = \frac{0.693N}{\Delta N/\Delta t}$$

if ΔN is small compared to N. Students, of course, are not expected to know this equation.

A half-life of the order of an hour can be found directly by plotting the counting rate as a function of time. It can then be seen from the graph that the time it takes for the counting rate to drop to half its value (its half-life) is independent of the counting rate itself. Plotting the log of the counting rate yields a straight line, the slope of which is 0.693/(half-life).

APPARATUS

1 Geiger counter
1 Radioactive source (wrist watch with radium dial)
2 Sheets rectilinear graph paper
1 Meter stick
1 Stopwatch or watch with sweep second hand

Amplifier and loudspeaker
Cloud chamber

IV–15 The Spectrum of Hydrogen and Planck's Constant

The study of the spectrum of hydrogen offers a unique opportunity to combine precise measurements with the checking of a theory and the determination of a universal constant.

Students should be given sufficient time to align the spectrometer carefully and practice using it to locate not only strong but weak lines, particularly at the ends of the visible spectrum. A hard and sharp pencil is a must if the maximum precision of the instrument is to be exploited. A slide rule is not sufficiently accurate for the computations.

Care has to be taken to focus the telescope on infinity and to place the grating directly over the pivot point of the telescope.

Slight errors in alignment are reduced and the general accuracy is increased by observing all lines on both sides of the central maximum.

Particular care should be used in measuring the position of the mercury green line, since this line is used to calibrate the spectrometer. Any error in measuring its position will result in an error in the proportionality constant and thus be reflected as a systematic error in all hydrogen wavelength determinations.

The useful life of the hydrogen tube source is only a few hours. After this time, lines in addition to the hydrogen lines will begin to appear, owing to contamination of the gas by elements present in the materials of which the tube is constructed. It is wise, therefore, to warn students to operate the tube only while actually making measurements with the spectroscope.

ANSWERS TO QUESTIONS

The image of the slit is at infinity if the telescope has been properly focused on infinity beforehand. When the image is sharpest, the slit is in the focal plane of the collimator lens, and the light from the slit emerges from the lens as a parallel beam.

The grating is parallel to the slit when the spectral line under observation is diffracted by the largest angle. When the grating is tilted with respect to the slit, the effective distance between the lines is increased, and therefore the diffraction angle is reduced.

From Fig. (a),

$$\lambda = d \sin \theta = d \cdot \frac{x}{L}$$

where L is the distance from the center of rotation to the notch at the center of the base, and x is the displacement perpendicular to the central maximum. The constant of proportionality is d/L.

We can express the constant of proportionality in terms of the calibration wavelength, in this case the green mercury line:

$$\lambda_{Hg} = x_{Hg} \cdot \frac{d}{L}$$

$$\frac{d}{L} = \frac{\lambda_{Hg}}{x_{Hg}}$$

Data obtained with the spectrometer are shown in the table. Error in λ may be about 0.3 percent, but should never be above 1 percent.

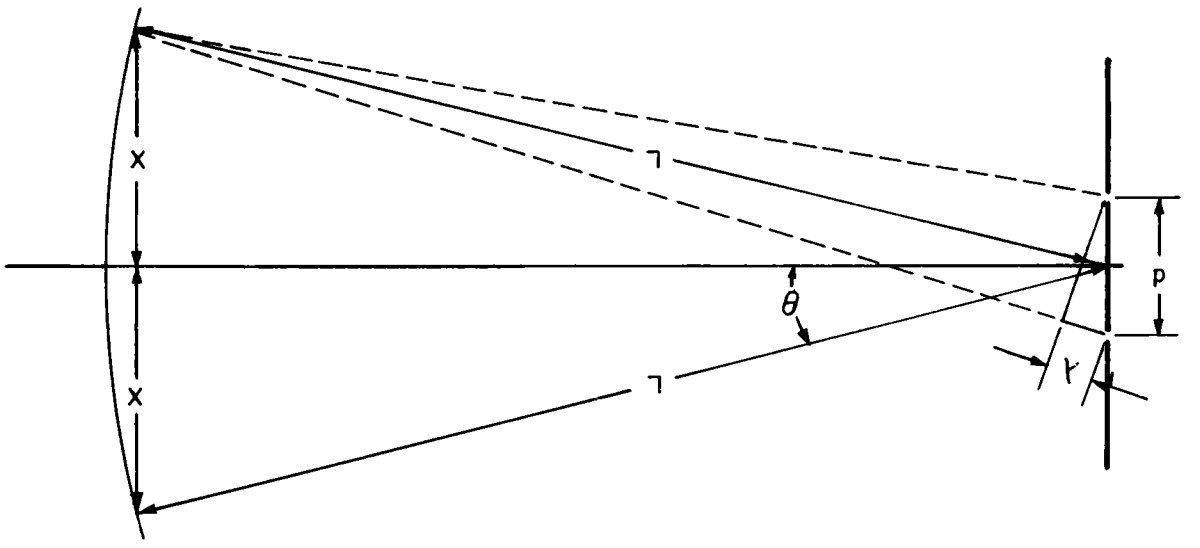


Figure (a)

DATA OBTAINED WITH SPECTROMETER

SOURCE	COLOR	DISTANCE	λ (MEASURED)	λ (HANDBOOK DATA)
Mercury	Green	18.66 cm	Standard	5461 A
Hydrogen	1. Deep violet	14.03	4107 A	4102
	2. Violet	14.81	4335	4340
	3. Blue-green	16.62	4865	4861
	4. Red	22.39	6554	6563

Wavelengths from a handbook were included for comparison.

One can readily see three of the bright lines of hydrogen. By darkening the room and waiting until the eyes have become accustomed to darkness, one may be able to see a fourth line of hydrogen in the far violet. Some students may have physiological difficulty in seeing the deepest violet line.

The table below shows both the calculated ratios of the reciprocals of the wavelengths and the ratios obtained from wavelengths measured with the spectroscope. The calculated ratios were found from

$$\frac{\frac{1}{\lambda_1}}{\frac{1}{\lambda_2}} = \frac{\frac{1}{n_i^2} - \frac{1}{n_f^2}}{\frac{1}{n_i'^2} - \frac{1}{n_f^2}}$$

n	n_i'	n_f	CALCULATED RATIO	MEASURED RATIO
2	3	1	.8438	
2	4	1	.8000	
2	5	1	.7813	
3	4	2	.7407	.7426
3	5	2	.6614	.6615
3	6	2	.6250	.6267
4	5	3	.6836	
4	6	3	.5833	
4	7	3	.5359	

Planck's constant is determined from the relation

$$\frac{1}{\lambda} = \frac{2\pi^2 k^2 m}{h^3 c}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Using each value of measured wavelength in turn with the appropriate values of n_i and n_f gives an average value for h of 6.622×10^{-34} joule/sec. The error introduced from the measurement of λ is very small — about 0.3 percent. Since $h\alpha\lambda^{1/3}$, the error in determining h in this experiment is about 0.1 percent. This is comparable with the experimental errors in the other physical constants used in the above equation as given in handbook tables of physical constants.

APPARATUS

1 Spectrometer
1 Ruler
Masking tape
1 Sheet plane paper
1 Hydrogen spectrum tube
1 Mercury spectrum tube
1 High-voltage source kit